

An analytic approach to the normalized Ricci flow-like equation*

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Abstract

This paper is concerned with the nonlocal parabolic equation

$$\frac{\partial e^w}{\partial t} = \Delta w + \lambda \left(\frac{e^w}{\int_{\Omega} e^w dx} - \frac{1}{|\Omega|} \right), \quad x \in \Omega, \quad t > 0, \quad (0.1)$$

defined on a compact Riemannian surface Ω (without boundary) associated with initial conditions $w(x, 0) = w_0(x)$, $x \in \Omega$. If the total mass of the solution, $\lambda = \int_{\Omega} e^w dx = \int_{\Omega} e^{w_0} dx$, is equal to 8π and Ω is the standard sphere, then (0.1) describes Hamilton's normalized Ricci flow. In this work, we obtain the global in time existence of the solution to this problem for $0 < \lambda \leq 8\pi$. Moreover for $0 < \lambda < 8\pi$, the orbit is compact, while if $\lambda = 8\pi$, there is a time sequence along which the solution converges to a stationary solution.

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