Backward blow-up profile in a Supercritical Nonlinear Heat Equation

Hiroshi Matano (University of Tokyo)

In this talk I will show the existence of a backward blow-up profile for a nonlinear heat equation with supercritical power nonlinearity.

More precisely, let $u(x,t)$ be a radially symmetric solution of the equation

$$u_t = \Delta u + |u|^{p-1}u,$$

where $p > p_S := \frac{N+2}{N-2}$. Assume that u blows up at $t = T$ and that the blow-up is of type I. Suppose that u has a limit L^1 continuation $\tilde{u}$ beyond the blow-up time T. Then the limit

$$w_*(y) := \lim_{t \searrow T} t^{\frac{1}{p-1}} \tilde{u}(\sqrt{t}\, y,\, t) \qquad (*)$$

exists locally uniformly in $y \in \mathbb{R}^N$, and is a solution of the equation

$$\Delta w + \frac{1}{2} y \cdot \nabla w + \frac{1}{p-1} w + |w|^{p-1} w = 0.$$

In other words, u approaches asymptotically a forward self-similar solution as $t \searrow T$.

In my earlier paper with Frank Merle [1], we showed that $t^{\frac{1}{p-1}} \tilde{u}(\sqrt{t}\, y, t)$ remains bounded in L^∞ as $t \searrow T$, but we did not show the existence of the limit $(*)$. One of the difficulties was that, unlike the usual "backward rescaled equation", the "forward rescaled equation" does not have a good Lyapunov functional. Another difficulty is that the intersection-number argument is not easy to apply since the intersection-number may tend to infinity as $t \searrow T$.

Out method is based on some estimate in [1] combined with a new zero-number technique employed in the paper [2].

References

[1] H. Matano and F. Merle: *Classification of Type I and Type II behaviors for a supercritical nonlinear heat equation*, to appear in J. Funct. Anal.

[2] Y. Du and H. Matano: *Convergence and a sharp threshold for propagation in a bistable reaction-diffusion problem*, preprint.