Nonexistence of Backward Self-similar Blowup Solution to a Supercritical Semilinear Heat Equation

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We consider a Cauchy problem for a semilinear heat equation

$$\begin{cases} u_t = \Delta u + u^p & \text{in } \mathbf{R}^N \times (0,T), \\ u(x,0) = u_0(x) \geq 0 & \text{in } \mathbf{R}^N \end{cases}$$

with $p > p_S$ where p_S is the Sobolev exponent. If $u(x,t) = (T-t)^{-1/(p-1)}\varphi((T-t)^{-1/2}x)$ for $x \in \mathbf{R}^N$ and $t \in [0,T)$, where φ is a regular positive solution of

$$(P) \qquad \Delta\varphi - \frac{y}{2}\nabla\varphi - \frac{1}{p-1}\varphi + \varphi^p = 0 \quad \text{in } \mathbf{R}^N,$$

then u is called a backward self-similar blowup solution. It is immediate that (P) has a trivial positive solution $\kappa \equiv (p-1)^{-1/(p-1)}$ for all $p > 1$. Let p_L be the Lepin exponent. Lepin obtained a radial regular positive solution of (P) except κ for $p_S < p < p_L$. We show that there exist no radial regular positive solutions of (P) which is spatially inhomogeneous for $p > p_L$.