Threshold solutions for a semilinear heat equation with Sobolev critical exponent

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We will consider the behavior of positive solutions to the Cauchy problem

$$\begin{cases} u_t = \Delta u + u^p, & x \in \mathbf{R}^N, \ t > 0 \\ u(x,0) = u_0(x;\alpha), & x \in \mathbf{R}^N, \end{cases} \tag{1}$$

where $p = (N+2)/(N-2)$, $N \geq 3$, $\alpha \geq 0$ is a parameter, and $u_0(x;\alpha)$ has the form

$$u_0(x;\alpha) = \phi_0(|x|) + \alpha\phi_1(|x|) \tag{2}$$

with $\phi_0, \phi_1 \in C([0,\infty))$. We assume in (2) that ϕ_0 satisfies

$$\phi_0(r) > 0 \quad \text{for } r \geq 0 \quad \text{and} \quad \lim_{r\to\infty} r^{2/(p-1)}\phi_0(r) = \mu$$

with some $\mu > 0$, and that ϕ_1 satisfies

$$\phi_1(r) > 0 \quad \text{for } 0 \leq r < r_0 \quad \text{and} \quad \phi_1(r) \equiv 0 \quad \text{for } r \geq r_0$$

with some $r_0 > 0$. We will study the behavior of solutions to (1) under suitable conditions on ϕ_0 and μ, and classify their global in time behavior. We are particularly interested in the behavior of threshold solutions lying on the borderline between global existence and blowup.