

# BLOW-UP FOR A NON-LOCAL DIFFUSION PROBLEM WITH NEUMANN BOUNDARY CONDITIONS AND A REACTION TERM.

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ABSTRACT. In this work we study the blow-up problem for a non-local diffusion equation with a reaction term,

$$u_t(x, t) = \int_{\Omega} J(x - y)(u(y, t) - u(x, t)) dy + u^p(x, t).$$

We prove that nonnegative and nontrivial solutions blow up in finite time if and only if  $p > 1$ . Moreover, we find that the blow-up rate is the same that the one that holds for the ODE  $u_t = u^p$ , that is,  $\lim_{t \nearrow T} (T - t)^{\frac{1}{p-1}} \|u(\cdot, t)\|_{\infty} = (\frac{1}{p-1})^{\frac{1}{p-1}}$ . Next, we deal with the blow-up set. We prove single point blow-up for radially symmetric solutions with a single maximum at the origin, as well as the localization of the blow-up set near any prescribed point, for certain initial conditions in a general domain with  $p > 2$ . Finally, we show some numerical experiments which illustrate our results.

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