PARTIAL REGULARITY AND BLOW-UP ASYMPTOTICS OF WEAK SOLUTIONS TO KELLER-SEGEL SYSTEMS

Yoshie Sugiyama
Department of Mathematics, Tsuda College,
2-1-1, Tsuda-chou, Kodaira-shi, Tokyo, 187-8577, Japan
sugiyama@tsuda.ac.jp

Abstract

We deal with the Keller-Segel systems:

$$(\mathrm{KS})_m \begin{cases} \partial_t u &= \Delta u^m - \nabla \cdot \left(u^{q-1}\nabla v\right), \quad x \in \mathbb{R}^N,\ t > 0, \\ 0 &= \Delta v - \gamma v + u, \quad x \in \mathbb{R}^N,\ t > 0, \\ u(x,0) &= u_0(x), \quad x \in \mathbb{R}^N \end{cases}$$

for critical case of $q = m + \frac{2}{N}$ with $N \geq 2,\ m \geq 1,\ q \geq 2$. Based on our ε-regularity theorem, we first show that the set S_u of blow-up points of the weak solution u has at most the zero-Hausdorff dimension if $u \in C_w([0,T]; L^1(\mathbb{R}^N))$. Next, we give various conditions on the weak solution u so that the set S_u consists of finitely many points. Furthermore, we obtain an explicit constant for ε in such a way that if the local concentration of mass around some point $x \in S_u$ is less than ε, then u is in fact locally bounded around x, which may be regarded as a removable singularity theorem. Simultaneously, we shall show that the solution u in $C([0,T]; L^1(\mathbb{R}^N))$ can be continued beyond $t = T$, which gives an extension criterion in the scaling invariant class associated with $(\mathrm{KS})_m$.