GRAIN SIZE EFFECT ON GROWTH THRESHOLD FOR SMALL SURFACE-CRACKS AND LONG THROUGH-CRACKS UNDER CYCLIC LOADING

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1. INTRODUCTION

The growth behavior of fatigue cracks has been studied extensively with long through-cracks, and it has been found that there is a threshold condition for a crack to grow. In the region near the threshold, the crack growth behavior is strongly dependent on the microstructure of material, the crack length, load sequence and the stress ratio. As to the crack length effect on the threshold, Kitagawa and Takahashi1 reported that the value of threshold range of the stress intensity factor (SIF) decreased when the crack length is shorter than about 0.13 mm in a high strength steel and that the threshold value of the stress amplitude approached the fatigue limit of the smooth specimen at very short cracks. A similar tendency was reported with other steels2, 3.

A few models have been proposed for explaining the dependence of the threshold SIF on the crack length. Haddad and others4 introduced the concept of an effective crack length which was given by the sum of the real crack length and the intrinsic crack length. The intrinsic crack length was obtained from the threshold value of SIF and the fatigue limit of the smooth specimen. Usami and Shida5 proposed that the threshold condition was given by the critical size of the plastic zone at the crack tip. These studies, however, lack the considerations on the physical mechanism and on the crack closure phenomenon.

In order to explain the effect of the grain size on the threshold stress intensity for long through-crack in low-carbon steels, the authors6 proposed in the previous study a model of the crack-tip slip band blocked by grain boundary, called the BSB model. In this model, the threshold value is estimated from the frictional stress and the critical value of SIF at the tip of the slip band blocked by the grain boundary. In the present study, the BSB model is further extended to predict the effects of grain size and crack length on the threshold condition, and the analytical prediction is compared with the experimental results. The experiments to determine the growth threshold for small surface-cracks as well as for long through-cracks were carried out with low-carbon steels with different grain sizes which were the same materials used in the previous study7.

2. APPLICATION OF BSB MODEL TO GROWTH THRESHOLD OF FATIGUE CRACKS

In the previous study of near-tip deformation of through cracks, the slip band formed at the crack tip of long through-cracks was found to be blocked by the grain boundary in the region near the threshold7. The situation was modeled by the piling up dislocations at grain boundary expanding coplaner to the crack plane6. The singular stress field near the tip of the slip band is characterized by SIF in a microscopic sense. In the application of the model to fatigue crack growth, it is assumed that the threshold condition for crack growth is determined by the condition whether the slip band near the crack tip propagates into an adjacent grain or not, which is expressed by the critical value of the microscopic stress intensity factor (MSIF). The model will be used to derive the threshold stress amplitude for specimens with small cracks and the fatigue limit of the smooth specimen as below.

In the case of two-dimensional crack with coplaner slip band as shown in Fig. 1, the critical value of the stress \( \sigma_{th} \) is given by

\[
\sigma_{th} = \frac{K_m}{\sqrt{b} \sigma_f} + \frac{(2/m) \sigma_f \cos^{-1}(a/b)}{2} \tag{1}
\]

where \( K_m \) is the critical value of MSIF and \( b \) is the crack length \( a \) plus the size of the blocked slip band zone \( \omega_0 \). The threshold value of SIF \( K_{th} \) for long cracks can be obtained by taking the limit of \( \sigma_{th} \sqrt{\omega_0} \) as \( \omega_0/a \) tends to zero. The result is

\[
K_{th} = \frac{K_m}{\sigma_f} + 2\sqrt{2\pi} \frac{\sigma_i}{\sqrt{\omega_0}} \tag{2}
\]

The fatigue limit of the smooth specimen is obtained by substituting \( a=0 \) in Eq. (1) as

\[
\sigma_{\omega_0} = \frac{\sigma_f^*}{K_m^*} + \frac{\sigma_f^*}{\sqrt{\omega_0}} \tag{3}
\]
The threshold stress $\sigma_{th}$ is given from Eq. (1) as

$$\sigma_{th} = K_{th}/\sqrt{\pi(a + \alpha_{o})} \tag{10}$$

Usami and others$^{5}$ assumed the constant value of the plastic zone size calculated through BCS model for $K_m=0$. The values of $\sigma_{th}$ and $K_{th}$ are derived from Eqs. (1) and (2) by substituting $K_m=0$ as

$$\sigma_{th} = (2/\pi) \sigma_{fr}^h \cos^{-1}(a/b) \tag{11}$$

$$K_{th} = 2\sqrt{2/\pi} \sigma_{fr}^h \omega_{o} \tag{12}$$

They assumed that the $\sigma_{th}$ value is cut off when $\sigma_{th}$ is larger than $\sigma_{WO}$. A logical extension of the critical slip band zone model will result in

$$\sigma_{WO} = \sigma_{fr}^h \tag{13}$$

The value of $\alpha_{o}$ defined by Eq. (7) is given as

$$\alpha_{o} = \left[ \frac{1 - 1 - 2\sqrt{2/\pi}}{1 + K_m^m \omega_{o} \sqrt{\pi \omega_{o}}} \right] \omega_{o} \tag{14}$$

for BSB model, and

$$\alpha_{o} = (8/\pi^2) \omega_{o} \tag{15}$$

for the critical plastic zone size model.

3. EXPERIMENTAL PROCEDURE

1. Experimental Material and Specimen

The experimental material used in the present experiment is the same low-carbon steel with the carbon content 0.20% as used in the previous study. The microstructure of the material is a mixture of ferrite and pearlite grains. The ferrite grain sizes measured by the linear intercept method are 7.8 and 55 $\mu$m, and the material with each grain size is designated as A and C respectively. The heat-treatment condition and the mechanical properties derived from simple tension tests are summarized in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Heat-treatment post machining</th>
<th>Grain size $d$ ($\mu$m)</th>
<th>Yield strength $\sigma_y$ (MPa)</th>
<th>Tensile strength $\sigma_u$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Annealed at 1173 K for 10 min followed by air cooling</td>
<td>7.8</td>
<td>366</td>
<td>528</td>
</tr>
<tr>
<td>C</td>
<td>Annealed at 1473 K for 5 hr followed by furnace cooling</td>
<td>55</td>
<td>194</td>
<td>433</td>
</tr>
</tbody>
</table>
Prior to machining the specimens, the plates of experimental material with 5 mm thickness were first quenched into water from 1173 K and then tempered at 723 K for 20 min. The specimens were re-heat-treated after machining according the condition given in Table 1. The decarburized layer of the specimen surface due to heat-treatment was taken off by electropolishing. The final shape and the dimension of the specimens are shown in Fig. 2, where (a) is the centered-notched plate for through-crack propagation tests and (b) is the smooth specimen for surface-crack propagation tests.

The specimens for surface crack propagation tests were prepared as follows. First, the smooth specimen shown in Fig. 2 (b) was pre-cracked in a resonance type bending fatigue testing machine operated at a speed of 33.3 Hz under the condition of \( R = -1 \). By considering the fatigue limit \( \sigma_{U}=235 \) MPa for Material A and \( \sigma_{U}=163 \) MPa for Material C obtained in the previous study, the stress amplitude in pre-cracking was prescribed as 275 MPa for Material A and 245 MPa for Material C. The pre-cracked specimens were annealed at 923 K for 1 hr before propagation tests, and then the surface layer of about 1 \( \mu \)m was taken off by electropolishing. The crack growth tests were conducted with the pre-cracked specimen in the same testing machine used for pre-cracking. The stress amplitude was increased after each \( 10^6 \) cycles to obtain the growth behavior of a crack in the region with rates less than \( 10^{-10} \) m/cycle.

(3) Crack Length Measurement and Stress Intensity Factor

In through-crack growth experiments, the length of a fatigue crack was measured with a traveling microscope at 400 magnification. The value of SIF \( K \) was calculated by the secant formula

\[
K = \sqrt{\sec(\pi a/2W) \cdot a^{\sqrt{\pi a}}}
\]

where \( W \) is the half specimen width and \( a \) is the half crack length.

The growth behavior of surface crack was measured on the specimen surface at a higher magnification 1000, and the profile of a surface crack in thickness direction was observed in the broken specimens. The shape of the crack was found to be semi-elliptical or quarter-elliptical where the major axis is parallel on the surface as described in the next chapter. The value of SIF used in representing the growth data is evaluated at the specimen surface as described in the Appendix.

4. EXPERIMENTAL RESULTS

(1) Growth Characteristics of Long Through-Crack

The growth curve of a fatigue crack from a pre-crack in an annealed specimen of Material C is shown in Fig. 3, where the load amplitude was varied step-wise. As far as the load amplitude is from 3.92 to 6.08 kN, the crack begins to grow and then stops after a small amount of crack length increment. When the load amplitude is 6.37 kN, the crack continues to grow. Therefore, the SIF value for the growth initiation of a crack in an annealed specimen is lower than that required for growth continuation. A similar tendency was observed in Material A. The data above the condition of growth continuation will be used for later analysis.

In Fig. 4, the growth rate \( da/dN \) against
the range of SIF $\Delta K$ is plotted with open marks both for $K$-decreasing and $K$-increasing tests, where $\Delta K$ is taken to be that for the tension part of the cycle only. After a crack continues to grow in the latter test, the $da/dN$ vs $\Delta K$ relation is identical to that in the $K$-decreasing test. In the region near the threshold, the rate is lower in material with larger grain size than in smaller grain-sized material when compared at the same $\Delta K$ value. The threshold value of SIF $\Delta K_{th}$ obtained in $K$-decreasing test are summarized in Table 2. The SIF value at the point of crack opening was measured by the compliance method, and the effective value of the SIF range $\Delta K_{eff}$ is also shown with solid marks in Fig. 4. As seen in Fig. 4, the rate is a power function of $\Delta K_{eff}$ down to the rate of $5 \times 10^{-11}$ m/cycle. In the region where the rate is lower than this value, the relation between $da/dN$ and $\Delta K_{eff}$ deviates from the power function, and the rate is higher and the threshold value $\Delta K_{effth}$ is lower in smaller grain-sized material. The value of $\Delta K_{effth}$ is also included in Table 2.

(3) Growth Characteristics of Small Surface-Crack

The shape of the surface crack measured in broken specimens was found to be nearly semi-elliptical or quarter-elliptical. Figure 5 indicates the variation of the aspect ratio with the length of semi-axis measured on the specimen surface, in which the data on Material B with the grain size 20.5 $\mu$m are included. Its variation is roughly independent of the stress amplitude and the grain size. In calculating SIF, the aspect ratio was estimated from the measurement of the major semi-axis $a$ on the specimen surface by using the dashed line in Fig. 5 for semi-elliptical cracks, while the

![Fig. 3. Crack propagation curve.](image)

![Fig. 4. Relation between crack propagation rate and stress intensity range for through-crack.](image)

![Fig. 5. Aspect ratio of surface-crack under plane bending: (S.C.; Semi-elliptical surface crack, C.C.; Corner crack).](image)

Table 2. Experimental results and values calculated by BSB model.

<table>
<thead>
<tr>
<th>Material</th>
<th>Grain size $d(\mu m)$</th>
<th>Fatigue limit $\sigma_{uO}(MPa)$</th>
<th>$\Delta K_{th}(MPa\sqrt{m})$</th>
<th>$\Delta K_{effth}(MPa\sqrt{m})$</th>
<th>$K_{th}(MPa\sqrt{m})$*</th>
<th>Intrinsic crack length $a_0(\mu m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7.8</td>
<td>235</td>
<td>5.21</td>
<td>0.61</td>
<td>0.68</td>
<td>156</td>
</tr>
<tr>
<td>C</td>
<td>55</td>
<td>163</td>
<td>6.20</td>
<td>1.30</td>
<td>1.28</td>
<td>461</td>
</tr>
</tbody>
</table>

* Calculated by Eq. (2)  ** Calculated by Eq. (14)
Fig. 6. Relation between crack propagation rate and stress intensity range for surface-crack (T.C.; Through-crack).

The growth rate of small surface cracks was measured on the specimen surface above the condition of growth continuation described in the previous section. The rate is plotted against the SIF range in Fig. 6. The threshold range of SIF is defined as the value corresponding to the rate $10^{-11}$ m/cycle. The crack length $a_{eq}$ indicated in the figure is the equivalent through-crack length given by

$$a_{eq} = \frac{\Delta K_{th}/\sigma_{th}}{\pi}$$  \hspace{1cm} (17)

The change of the threshold stress amplitude $\sigma_{th}$ with the equivalent crack length $a_{eq}$ is shown in Fig. 7. The dashed line in the figure is the relation calculated by substituting the threshold SIF of long through-crack for $\Delta K_{th}$ in Eq. (17). As the crack length approaches zero, the value of the threshold stress amplitude approaches the fatigue limit of the smooth specimen. The threshold condition for long surface cracks is equal to the threshold condition for long through-cracks. The crack length $a_{eq}$ corresponding to the point of intersection of two straight lines given by Eq. (7) is longer in larger grain sized material.

Fig. 7. Variation of threshold stress amplitude with equivalent crack length.

(a) Through crack, Material A
$\Delta K=4.02$ MPa/m, $a=5.54$ mm

(b) Surface crack, Material C
$\Delta K=2.96$ MPa/m, $a_{eq}=0.217$ mm

Photo. 1. Optical micrographs taken from near crack tips.
5. DISCUSSION

From the finding that the slip band zone size near the crack tip is less than the grain size for various type of cracks, it can be said that the slip band made from the crack tip is blocked by the grain boundary at the threshold. Therefore, the BSB model can be applied.

According to the previous experimental data on the fatigue limit of the smooth specimen of the low-carbon steel, which was the same material used in the latter experiment of the present paper, the fatigue limit $\sigma_{\omega_0}$ (MPa) was expressed by

$$\sigma_{\omega_0} = 114 + 3.29 \times 10^{-1}/\sqrt{d}$$

(18)

where $d$ (m) is the grain size of the material. Since fatigue slip bands formed under stress amplitudes below $\sigma_{\omega_0}$ were found to be constrained within one grain, the $\omega_0$ value in Eq. (6) can be assumed equal to half of the grain size. Then we have

$$\sigma_{\omega_0} = \sigma_{\omega}^* + 1.30/\sqrt{2 \pi C_{\omega}}$$

(19)

From Eqs. (18) and (19), the value of $\sigma_{\omega}^*$ is obtained as 114 MPa and $C_{\omega}$ as 0.318 MPa$m^{-1}$. By substituting these values into Eq. (2), the $K_{th}$ value is obtained as 0.68 MPa$m^{1/2}$ for Material A and 1.28 MPa$m^{1/2}$ for Material C. With the comparison of the threshold value for long through-cracks shown in Table 2, the calculated value is nearly equal to the threshold value of the effective component of SIF. The dependence of threshold stress amplitude on the crack length is derived by substituting $K_{th}$ and $\sigma_{\omega}^*$ into Eq. (1). Figure 9 indicates its variation. In comparison of Fig. 7 with Fig. 9, it can be seen that the values of $\sigma_{\omega}$ and $K_{th}$ for long through-crack are much smaller in theoretical calculation than in the experimental data, although the tendency of the variation $\sigma_{\omega}$ with crack length is similar in both cases.

From the discussion given above, the crack closure seems to be responsible for this discrepancy. The values of $K_{th}$ and $\sigma_{\omega}$ calculated by BSB model give the effective component of the apparent $\delta K_{th}$ and $\sigma_{\omega}$. Although a further study is needed on the crack closure of small surface cracks, it is worthy to note that the tendency of the change in $\sigma_{\omega}$ with $\omega$ is similar in Figs. 7 and 9. In Fig. 10, $\sigma_{\omega}$ is normalized by $\sigma_{\omega_0}$ and $\omega$ by $\omega_0$. The change of ratio $\sigma_{\omega}/\sigma_{\omega_0}$ with $\omega/\omega_0$ is similar in both Materials A and C. Based on BSB model, the theoretical relation between $\sigma_{\omega}/\sigma_{\omega_0}$ and $\omega/\omega_0$ can be derived from Eqs. (1) and (3) as

$$\sigma_{\omega} = \frac{1}{\sqrt{\omega_0/\omega}}$$

If the value of $K_{th}^* \sigma_{\omega}^* / \omega_0$ is given, the $\sigma_{\omega}/\sigma_{\omega_0}$ vs $\omega/\omega_0$ relation can be predicted from Eqs. (14) and (20). Equation (20) can be written for $\sigma_{\omega}^* = 0$, as assumed in Haddad's model,

$$\sigma_{\omega} = \sqrt{\omega_0 / (\omega + \omega_0)}$$

(21)

where Eq. (9) is used, and from Eqs. (15) and (20) we have

$$\sigma_{\omega} / \sigma_{\omega_0} = (2/\pi) \cos^{-1} [\omega / (\omega + \pi^2/8 \omega_0)]$$

(22)
for the model of the critical plastic zone size, i.e., $K_{0}^m = 0$. It is interesting to note that the $\sigma_{th}/\sigma_{0}$ vs $a/a_0$ relation calculated by substituting arbitrary value for $K_{0}^m/\sigma_{0}^m$ is nearly independent of the value substituted. The curve is shown in Fig. 10 with the solid line. The experimental data are well approximated by the theoretical relation. This will be checked for the cases of other materials in the future.

Apart from the problems on crack closure and physical mechanisms, the model proposed by Haddad is the simplest method among the three methods in predicting the growth threshold of small cracks for engineering purposes. The physical meaning is probably given by combining the BSB model with crack closure consideration. Then, it will become possible to predict the effect of the material microstructure on the growth threshold of fatigue cracks and moreover on the fatigue limit of the smooth specimens.

6. CONCLUSION

The effects of grain size and crack length on the threshold condition were analysed both theoretically and experimentally. The experiments were carried out with low-carbon steels having different grain sizes. In the $K$-increasing test of annealed specimens with pre-cracks, the stress intensity range for the growth initiation of a crack is lower than that required for growth continuation. After a crack continue to grow, the growth behavior is identical to that in the $K$-decreasing test, and the rate is lower in material with larger grain size than in smaller grain sized material when compared at the same stress intensity range. The crack length found to have a significant influence on the threshold condition. The threshold stress range approached the fatigue limit of smooth specimen as the crack length approaches zero. These effects were explained micromechanically by the model of the crack tip slip band blocked by the grain boundary. Through the model, the threshold value of the effective component of stress intensity range was successfully estimated from the fatigue limit of smooth specimen. The observed variation of the threshold stress amplitude with the crack length was found to agree well with the theoretical prediction derived as an extension of the BSB model.

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APPENDIX

Stress Intensity Factors for Elliptical Cracks under Bending

The value of SIF is evaluated at the specimen surface as follows. For a quarter-elliptical surface crack subject to bending, the computational result of $K$ reported by Kobayashi and Enetanya is approximated by

$$K_A = [1.3 - 0.6(c/t)][\lambda E(\kappa)]^{1/2}$$

where $t$ is the specimen thickness, $\lambda$ aspect ratio, and $E(\kappa)$ is the complete elliptic integral of the second kind with the modulus $\kappa^2 = 1 - \lambda^2$.

For a semi-circular surface crack, the following equation approximates the result of Smith and others

$$K_B = [1.21 - 0.360(c/t)][\lambda E(\kappa)]^{1/2}$$

Equations (24) and (25) are equivalent for $\lambda = 1$ because $E(0) = \pi/2$.

Grandt and Sinclair reported the value for a semi-elliptical crack

$$K_C = 0.69 \sigma \sqrt{\alpha}$$

for the cases of $0.22 < c/t < 1.0$ and $0.1 < c/t < 0.5$. In comparison with the experimental results done by Schroedl and Smith, Eq. (25) underestimates $K$ for small $\lambda$ and Eq. (26) underestimates $K$ when $\lambda$ approaches unity. From above considerations, the SIF value used in this paper is

$$K = \max[K_B, K_C]$$

for a semi-elliptical surface crack, and

$$K = \max[K_A, (K_B/K_B)K_C]$$

for a quarter-elliptical corner crack.

REFERENCES