# MULTI-STEP DIRECT-REACTION ANALYSIS OF CONTINUOUS SPECTRA OF 14 MeV (n, n′) DATA

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The multi-step direct-reaction theory proposed by Tamura et al. was applied, with some modifications, to the continuous spectra of 14 MeV (n, n′) reaction. The calculated results reproduced well the experimental 14 MeV (n, n′) data for both the energy and angular distributions with a reasonable normalization procedure.

The energy and angular distributions of inelastically scattered neutrons to continuum states have aroused interest to test the reaction mechanisms because of the absence of the Coulomb effect in the incident and outgoing channels. Recently, the theoretical calculations on these processes have become more important to improve the accuracy in fusion reactor design.

The pre-equilibrium model [1,2] has succeeded in reproducing the high energy tail of the angle-integrated energy spectrum in the reactions with neutrons, protons and alpha particles in the energy range of 10–60 MeV. This model, however, cannot predict the angular distribution of the emitted particles.

Recently some attempts [3,4] have been made to extend the pre-equilibrium approach to the calculation of the angular distribution. The extension was carried out by combining a somewhat crude application of the concept of direct reaction with those of the pre-equilibrium model. Agreement between the experimental and calculated angular distributions is not so good.

More recently Tamura et al. [5,6] proposed a multi-step direct-reaction (MSDR) theory to describe reactions that leave the residual nucleus in a highly excited continuum state. They demonstrated the capability of this approach by successfully fitting spectra of the (p, p′) and (p, α) reactions at 62 MeV.

In the present work we apply the MSDR analysis, with some modifications, to the 14 MeV (n, n′) reaction. Since Tamura et al. analyzed data at excitation energies higher than 10 MeV, they assumed, for simplicity of the presentation, that the target is an even–even $0^+$ nucleus with a doubly closed (sub-)shell configuration. In our case data at excitation energies between 3 and 10 MeV are analyzed, so that the following three effects which are expected to influence the low-lying continuum states are taken into account:

(1) extension from a completely filled shell to partly filled shells,

(2) pairing correlation,

(3) difference between n–p and n–n effective interactions.

Moreover a normalization of the absolute cross sections was performed using the transition strengths of the low-lying $2^+$ states.

The DWBA cross section can be written [5] in the form

$$\sigma^{(1)}_{j_1 j_1';L}(E'_p;\theta) = d^2_{j_1 j_1';L}\,\sigma^{(1)}(E'_p;\theta)\,, \qquad (1)$$

where $d_{j_1 j_1';L} = \langle j_1 \| Y_L \| j_1' \rangle$, the reduced matrix element of the spherical harmonics $Y_{LM}$, is a purely "geometric" factor, and $\sigma^{(1)}_L(E'_p;\theta)$ is the DWBA cross section obtained after setting $d_{j_1 j_1';L} = 1$.

For transitions from a completely filled shell the reduced matrix element can be written as

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$$d_{j_1 j_1';L} = [(2j_1 + 1)(2j_1' + 1)/4\pi]^{1/2} \times (j_1 \tfrac{1}{2} j_1' - \tfrac{1}{2} | L\, 0),$$

where $(j_1 \tfrac{1}{2} j_1' - \tfrac{1}{2} | L\, 0)$ is the Clebsch–Gordon coefficient. The present calculation was extended to transitions involving partly filled shells for an even–even target. In the case of a transition from $(j_1^{n_1} j_1'^{n_1'})$ to $(j_1^{n_1-1} j_1'^{n_1'+1})$, one should multiply by the following factor [7] which was used for the evaluation of the $B(El)$ values for partly filled shells:

$$\left(\frac{n_1}{2j_1 + 1} \frac{2j_1' + 1 - n_1'}{2j_1' + 1}\right)^{1/2}.$$

Therefore $d_{j_1 j_1';L}$ is given by

$$d_{j_1 j_1';L} = [n_1(2j_1' + 1 - n_1')/4\pi]^{1/2} \times (j_1 \tfrac{1}{2} j_1' - \tfrac{1}{2} | L\, 0).$$

Such an extension from a completely filled shell to a partly filled shell was made in the present calculation.

The pairing correlation was taken into account in the present calculation as follows. The BCS pairing which arises from the short range part of the nuclear force produces the characteristic energy gap. The single particle energies are replaced by their counterparts, the single quasiparticle energies which are defined as,

$$E_j = [(\epsilon_j - \lambda)^2 + \Delta^2]^{1/2},$$

where $\lambda$ is the Fermi energy and $\Delta$ is the pairing energy.

The pairing energy is adapted [8] from the general trend in the observed pairing energy.

$$\Delta \approx 12/A^{1/2}\ \text{MeV}.$$

With respect to the difference between n–p and n–n effective interactions, free n–p and n–n cross sections at a neutron energy around the Fermi energy of 50 MeV are referred [9] as

$$\xi_{np} = (\sigma_{np}/\sigma_{nn})^{1/2} = 1.7.$$

Therefore, taking this effect into account, the present MSDR calculations were carried out for the proton shell and for the neutron shell separately and then both cross sections were summed.

The $\beta$ amplitude in the model of Tamura et al. is

$$(\beta_L R)/(2L + 1)^{1/2} = a d_L V_L/U, \qquad (2)$$

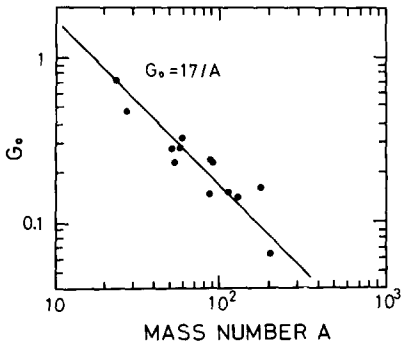


Fig. 1. The values of $G_0$ versus the mass number $A$. The straight line is a fit to the values of $G_0$.

with $d_L = \Sigma_{j_1 j_1'} d_{j_1 j_1';L}$, where the optical-model parameters $a$, $U$ and $R$ are taken from Menet et al. [10]. The values taken for $V_L$ with $L = 0$–3 were 7.4 and 1.4, respectively for $^{27}$Al and $^{209}$Bi; for $L \geqslant 4$ these values were divided by 1.5. However this normalization procedure did not satisfy [11] the sum rule predictions, and the mass-number dependence of $V_L$ is not so clear. Therefore we carried out a normalization using the transition strengths of low-lying $2^+$ states which are obtained experimentally for various nuclei.

We define $G_0$ as

$$G_0 = a V_L/U. \qquad (3)$$

Thus from eqs. (2) and (3) we get for $L = 2$,

$$G_0 = (\beta_2^2 R^2/5 d_2^2)^{1/2}.$$

Using the experimental values [12] of $\beta_2$ for low-lying $2^+$ states and the corresponding values of $d$ ($E_x = 0$–7 MeV) calculated from eq. (1), we obtained the values of $G_0$ for various nuclei. The values of $G_0$ versus the mass number $A$ are plotted in fig. 1. The values of $G_0$ can be fitted by a straight line $G_0 = 17/A$. The normalization of the absolute cross sections was carried out using $G_0 = 17/A$.

We carried out the MSDR analysis for the 14.1 MeV (n, n′) reactions on Fe, Cu, As, Nb, Ag and Au with the modifications mentioned above. The experimental data have been measured by two of the authors and others [13]. We first consider the one-step process only. As an example a comparison of the experimental with the calculated energy spectrum for Ag is shown in fig. 2. The histogram shows the energy spectrum calculated by the MSDR process. The results of the pre-equilibrium calculation are also shown in this figure. A

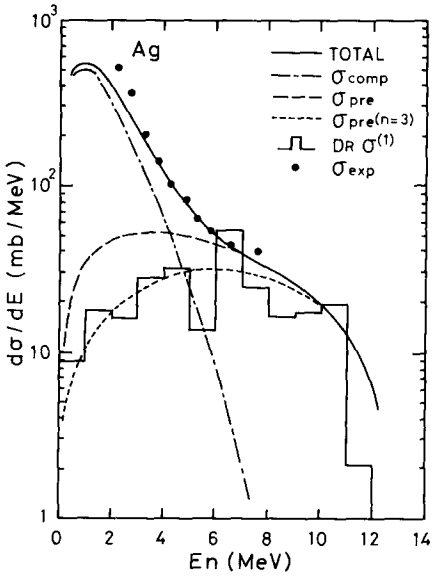


Fig. 2. Comparison of the experimental with the calculated energy spectrum. The histogram shows the present MSDR calculation.

considerable fluctuation of the histogram is seen. This fluctuation seems to be due to the statistical fluctuation of the number of corresponding levels in the energy interval of 1 MeV, because, for example, the average number of levels for $E'_n$ = 5.0–6.0 MeV is about five. Since the one-step direct reaction is considered to correspond to the pre-equilibrium process for $n$ = 3, the histogram should be compared with the pre-equilibrium calculation for $n$ = 3. The histogram is in fairly good agreement with the pre-equilibrium calculation for $n$ = 3 for both the shape and the magnitude of the cross sections.

Comparisons of the experimental and calculated angular distributions are shown in fig. 3. In fig. 3 there are presented angular distributions of the cross sections integrated over 4 MeV energy bins in $E'_n$ = 4–8 MeV. The experimental angular distributions in this figure were obtained by subtraction of the compound contribution from the total experimental angular distributions. The calculated angular distributions are the results with the one-step process only. The solid curves present the absolute values, while the dashed curves present the fit to the experimental data for ready comparison. The disagreement between theoretical curves and experiment for vertical normalization is mainly due to the exclusion of the multi-step process and the

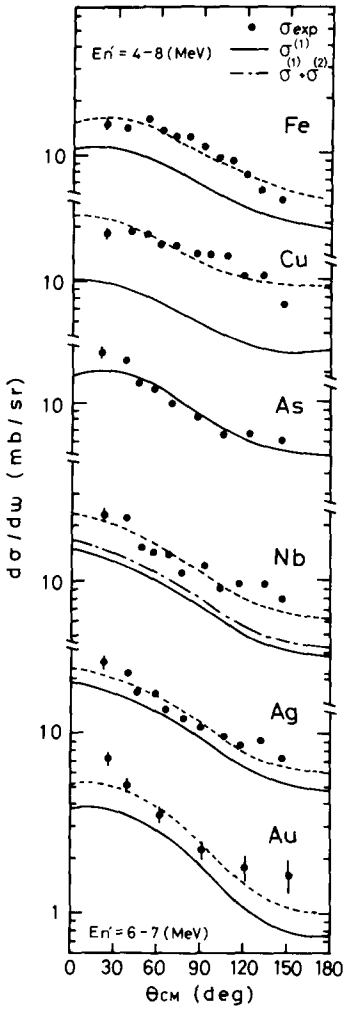


Fig. 3. Comparison of the calculated with the experimental angular distributions of the cross sections integrated over 4 MeV bins in $E_{n'}$ = 4–8 MeV except for Au ($E_{n'}$ = 6–7 MeV). The solid and dot–dashed curves present the one-step and two-step cross sections, respectively. The dashed curves present fits to the experimental data for ready comparison.

fluctuation of the calculated spectrum as mentioned above.

Numerical calculations for the two-step process consumed quite a long time. Therefore the calculations were limited only to the energy range for $E'_n$ = 4–8 MeV in $^{93}$Nb. The results are shown in fig. 3. The dot–dashed curve presents the sum of one- and two-step cross sections. The two-step cross sections are about 10% of the one-step cross sections as is seen in fig. 3, while the $n \geqslant 5$ cross sections for the pre-equilibrium

calculation are about 50% of the $n = 3$ cross sections as is seen in fig. 2.

Finally, it should be noted that the absolute normalization with the method of Tamura et al. does not satisfy the sum rule as criticized by Tsai and Bertsch [11]. This may be due to the neglect of a long range attractive force in the residual interaction. Although the present method does not satisfy the sum rule, it should be noted that this model reproduces well the experimental 14 MeV (n, n′) data for nuclei of a wide mass range for both the energy and angular distributions with a reasonable normalization which is carried out by using the experimental values of $\beta_2$ for low-lying $2^+$ states. Two-step cross sections calculated by the present model could not exhaust fully the higher exciton contributions of the pre-equilibrium calculation.

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*References*

[1] J.J. Griffin, Phys. Rev. Lett. 17 (1966) 478; Phys. Lett. 24B (1967) 5.
[2] M. Blann, Phys. Rev. Lett. 27 (1971) 337; Ann. Rev. Nucl. Sci. 25 (1975) 123.
[3] Y. Irie et al., J. Phys. Soc. Japan 39 (1975) 537; Phys. Lett. 62B (1976) 9.
[4] G. Mantzournis, Phys. Lett. 63B (1976) 25.
[5] T. Tamura, T. Udagawa, D.H. Feng and K.-K. Kan, Phys. Lett. 66B (1977) 109.
[6] T. Tamura and T. Udagawa, Phys. Lett. 71B (1977) 273.
[7] A. Bohr and B.R. Mottelson, Nuclear structure Vol. 2 (Benjamin, New York, 1975) ch. 6.
[8] N. Zeldes, A. Grill and A. Simievic, K. Dan. Vidensk. Selsk, Mat. Fys. Skr. 3 (1967) no. 5.
[9] K. Kikuchi and M. Kawai, Nuclear matter and nuclear reactions (North-Holland, Amsterdam, 1968) ch. 2.
[10] J.J.H. Menet et al., Phys. Rev. C4 (1971) 1114.
[11] S.F. Tsai and G.F. Bertsch, Phys. Lett. 73B (1978) 247.
[12] A.M. Bernstein, Advances in nuclear physics, Vol. 3 (Plenum P, New York, 1969) ch. 5.
[13] Y. Irie et al., Mem. Fac. Eng. Kyushu Univ. 37 (1977) 19.