Black Hole dynamics at large D

Roberto Emparan
ICREA & U. Barcelona & YITP Kyoto

w/ Kentaro Tanabe, Ryotaku Suzuki

Einstein's eqns – even in simplest cases

$$R_{\mu\nu} = 0$$

$$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

encode a vast amount of physics

Black holes colliding,

radiating gravitational waves ...

Einstein’s eqns – even in simplest cases

$$R_{\mu\nu} = 0$$

$$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

encode a vast amount of physics

Expanding, inflationary universe

Einstein's eqns – even in simplest cases

$$R_{\mu\nu} = 0$$

$$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

encode a vast amount of physics

Holography: quark-gluon plasma,

holographic superconductors...

Einstein's eqns – even in simplest cases

$$R_{\mu\nu} = 0$$

$$R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

encode a vast amount of physics

but they are **very hard** to solve (really!)

Numerical GR is a mature, indispensable tool

but insights are often hard to come by

A small parameter can take you a long way

- QED around $e^2 = 0$
- $SU(N)$ Yang-Mills around $N = \infty$

What parameter in

$$R_{\mu\nu} = 0?$$

Varying D in GR

D is the natural parameter in $R_{\mu\nu} = 0$

$\rightarrow$ natural to study GR in $D \neq 4$

Low *D*

$D < 4$: widely studied

UV regulator

Changes IR behavior

$D \leq 3$:

- no propagating dof's – no gravity force
- no asympflatness
- but keep diffeo invce, so may learn *something* about Quantum Gravity

Low *D*

Plus:

$2D$ and $3D$ black holes ***are*** relevant for more 'realistic' ($D = 4, 5$ ...) bhs as near-horizon geometries

Relevant also for $D \to \infty$!

Soda

Large D

$D > 4$: IR regulator / UV wrecker

$D \to \infty$: lots of local gravitational dynamics

strongly localized close to horizons

Maybe not good for Quantum Grav

but OK for classical GR

Kol+Miyamoto et al

Large *D*

Large *D* expansion may help for

– deeper understanding of the theory (reformulation?)

– calculations: new perturbative expansion

Universality (due to strong localization) is good for both

How do we take

$$D \to \infty$$

in

$$R_{\mu\nu} = 0?$$

Regard $R_{\mu\nu} = 0$ as a theory of

Black Holes

interacting with/via

gravitational waves

Black Hole
dynamics at large D

General Relativity @ large D

Two main effects:

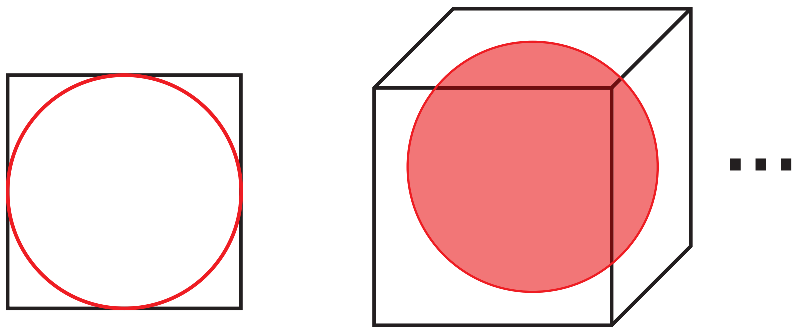
1. Small cross sections
    - elementary geometry effect

2. Interactions localized near horizon
    - gravitational effect

Elementary geometry @ large D

Area of spheres becomes small

compared to hypercubes that enclose them



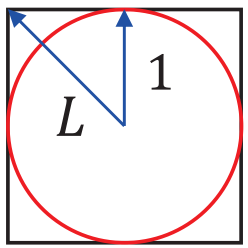
$$\frac{\mathrm{Area}_{(\mathrm{sphere})}}{\mathrm{Area}_{(\mathrm{cube})}} = \frac{\Omega_{D-2}}{(D-1)2^{D-1}}$$

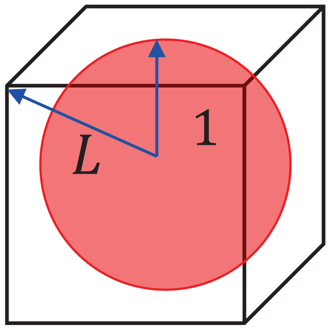
$$\sim D^{-D/2} \to 0$$

$$\Omega_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)} \sim D^{-D/2}$$

Elementary geometry @ large D

Lots of space in diagonal directions





$$L^2 = x_1^2 + \cdots + x_{D-1}^2 = D - 1$$

$$L \to D^{1/2} \gg 1$$

Sphere of *finite radius* but *zero area*

$\Rightarrow$ vanishing cross sections

Large *D* black holes

Basic solution

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

length scale r_0

Large *D* black holes

r_0 not the only scale

Small *parameter* $1/D \Longrightarrow$ scale hierarchy

$$r_0/D \ll r_0$$

Localization of interactions

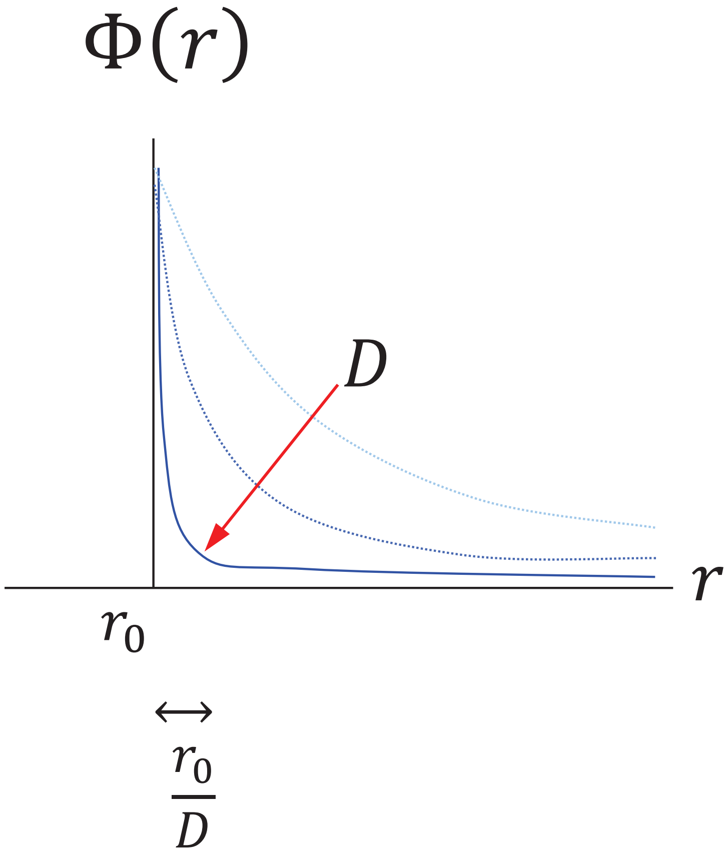
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\frac{r_0}{D} \ll r_0$$



Far zone

Fixed $r > r_0$ $\quad D \to \infty$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

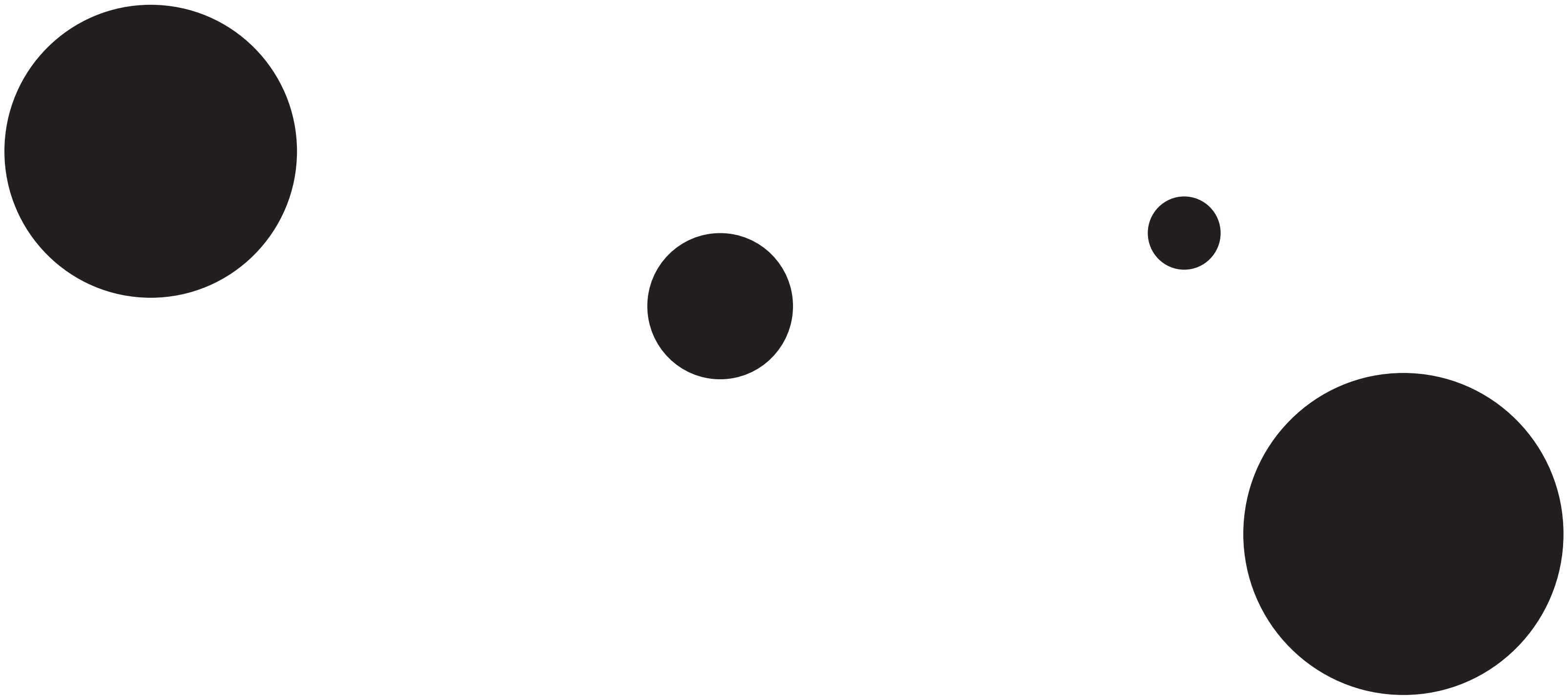
Flat, empty space at $r > r_0$

no gravitational field

Far zone geometry

scale $\mathcal{O}(r_0 D^0)$

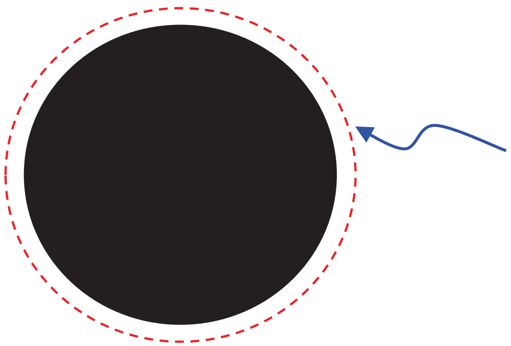
Holes cut out in Minkowski space



Near-horizon

Gravitational field appreciable only in *thin near-horizon region*

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 < \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

Near-horizon

$$ds^2 = -\left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0}t \end{aligned}\right\} \text{ finite as } D \to \infty$$

Near-horizon

$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\underbrace{\left(-\tanh^2\rho\ dt^2_{near} + d\rho^2\right)}_{\textbf{2d string black hole}} + r_0^2 d\Omega^2_{D-2}$$

Elitzur et al
Mandal et al
Witten 1991

Soda 1993
Grumiller et al 2002

$$\ell_{string} \sim \frac{r_0}{D}, \qquad \alpha' \sim \left(\frac{r_0}{D}\right)^2$$

Near-horizon universality

2d string bh is near-horizon geometry of **all neutral non-extremal bhs**

- rotation appears as a local boost

  (in a direction along horizon)

- cosmo const shifts 2d bh mass

Near-horizon geometries

Limiting well-defined geometry

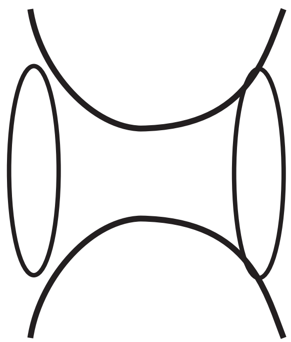
Well known: **(near-)extremal black holes**

Charge $Q \lesssim M$

Rotation $J \lesssim M^2$

Increasing charge of black hole

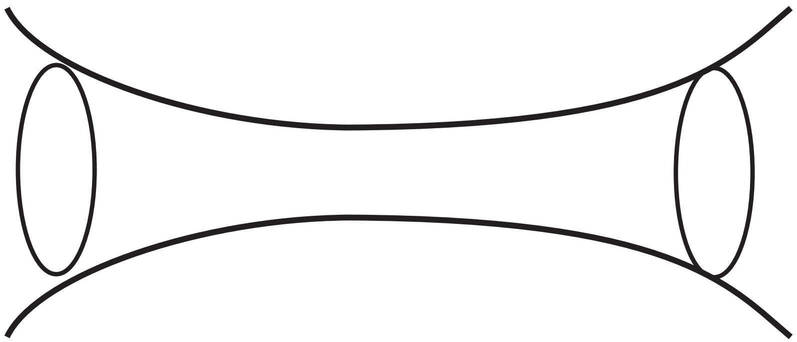
Spatial geometry of neutral black holes



$M \gg Q$

Increasing charge of black hole

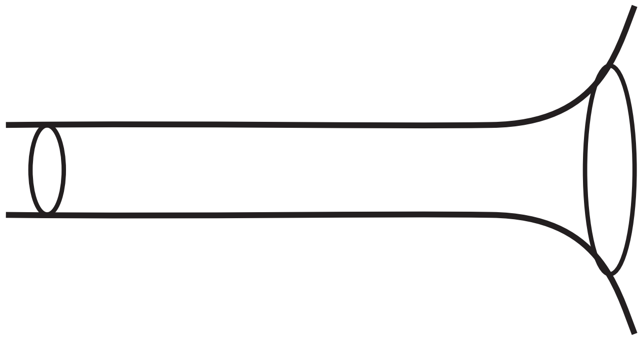
Spatial geometry of charged black holes



$M > Q$

Increasing charge of black hole

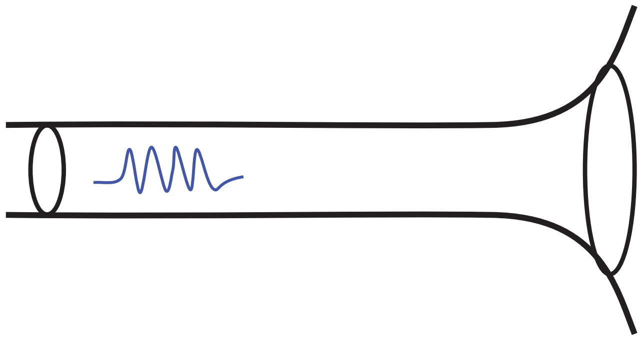
Spatial geometry of **(near-)extremal** bh



$$M \gtrsim Q$$

(Near-)Extremal black holes

Throat geometries near-horizon

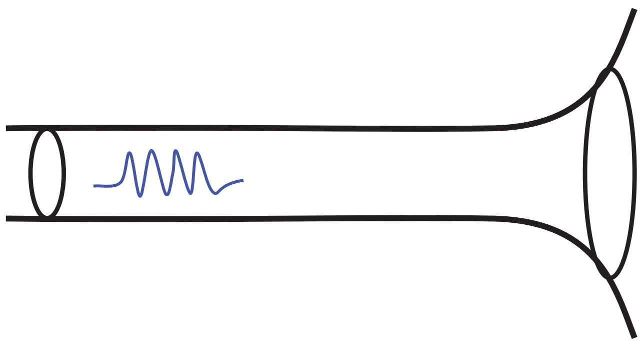


throat supports
"decoupled" dynamics

e.g. AdS/CFT decoupling limit

(Near-)Extremal black holes

Decoupled dynamics:

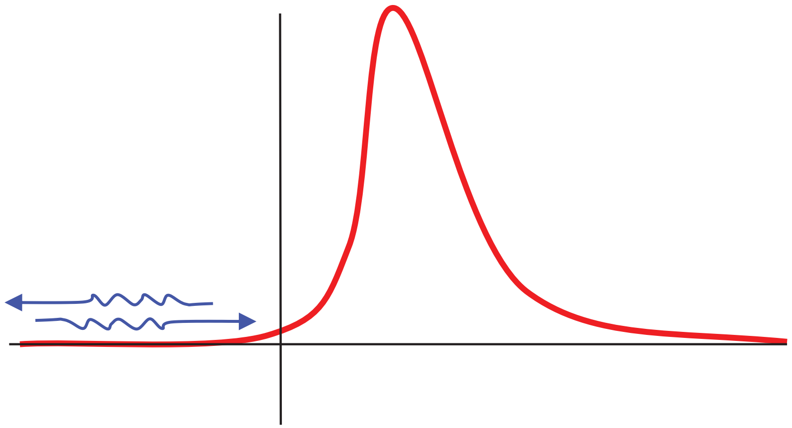


finite-frequency excitations that are normalizable in n-h geometry

Don't propagate into far-region

(Near-)Extremal black holes

Decoupled dynamics:



effective radial
potential

finite-frequency excitations that are normalizable in n-h geometry

Don’t propagate into far-region

Is the large D limit
a decoupling limit?

Is the large D limit
a decoupling limit?

No

Perturbative BH dynamics @ large D

is concentrated close to the horizon

States can be characterized in terms of their properties within N-H geometry

but N-H geometry is **not long** throat

$$ds_{nh}^2 = \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \; dt_{near}^2 + d\rho^2\right) + r_0^2 d\Omega_{D-2}^2$$

small extent $\propto r_0/D$

crossed very quickly $t_{near} = \frac{D}{2r_0} t$

Can't expect to support excitations fully trapped within

Black Hole dynamics:
Quasinormal modes

Quasinormal modes

Black holes *vibrate* when perturbed:

metric perturbation $\delta g_{\mu\nu} \sim e^{-i\omega t}$

But they don't have *normal modes:*

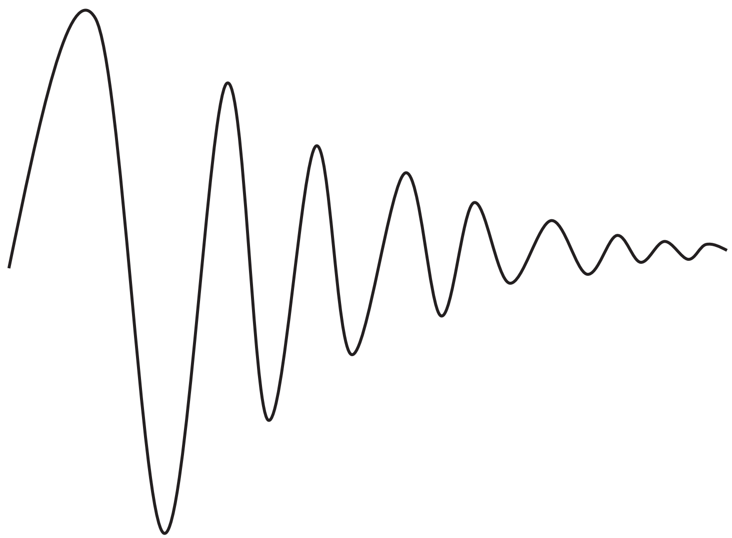
$$\mathrm{Im}\,\omega \neq 0$$

due to absorption of the vibration by the horizon

Quasinormal modes

$\delta g_{\mu\nu} \sim e^{-i\mathrm{Re}\omega t+\mathrm{Im}\omega t}$

$\mathrm{Im}\omega < 0$: stable



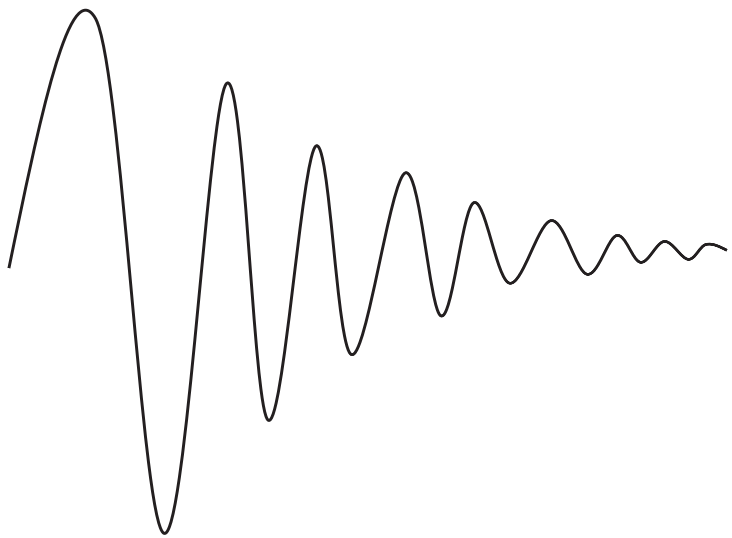
damped oscillations

$\sim$ ringing of a bell

Quasinormal modes

$$\delta g_{\mu\nu} \sim e^{-i\mathrm{Re}\omega t+\mathrm{Im}\omega t}$$

$\mathrm{Im}\omega < 0$: stable



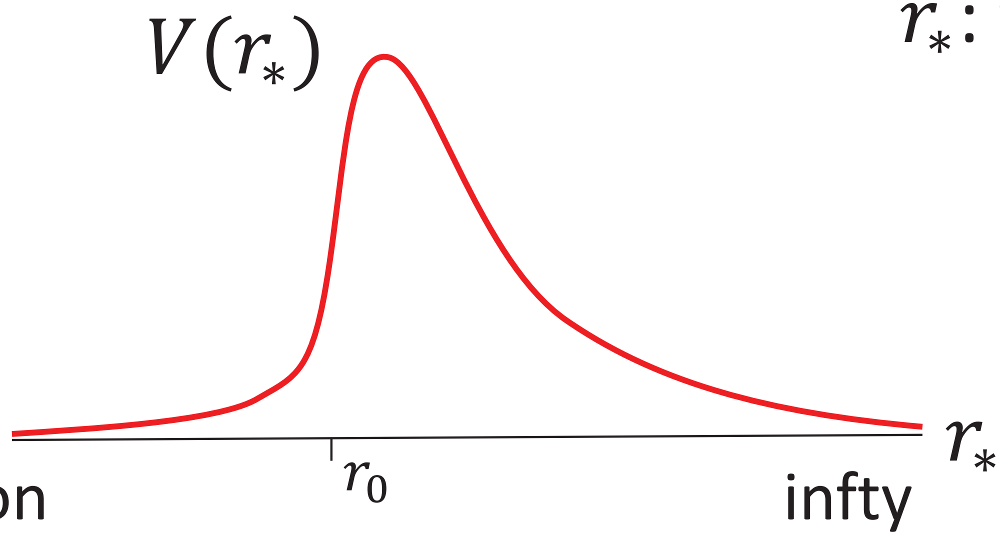
QNM spectrum characterizes a black hole, much like normal modes characterize other systems

Massless scalar field dynamics

$\Box\Phi = 0$ $\qquad\qquad$ $\Phi = r^{-\frac{D-2}{2}}\,\phi(r)\,e^{-i\omega t}\,Y_\ell(\Omega)$

$$\frac{d^2\phi}{dr_*^2} + (\omega^2 - V(r_*))\phi = 0$$

r_*: tortoise coord



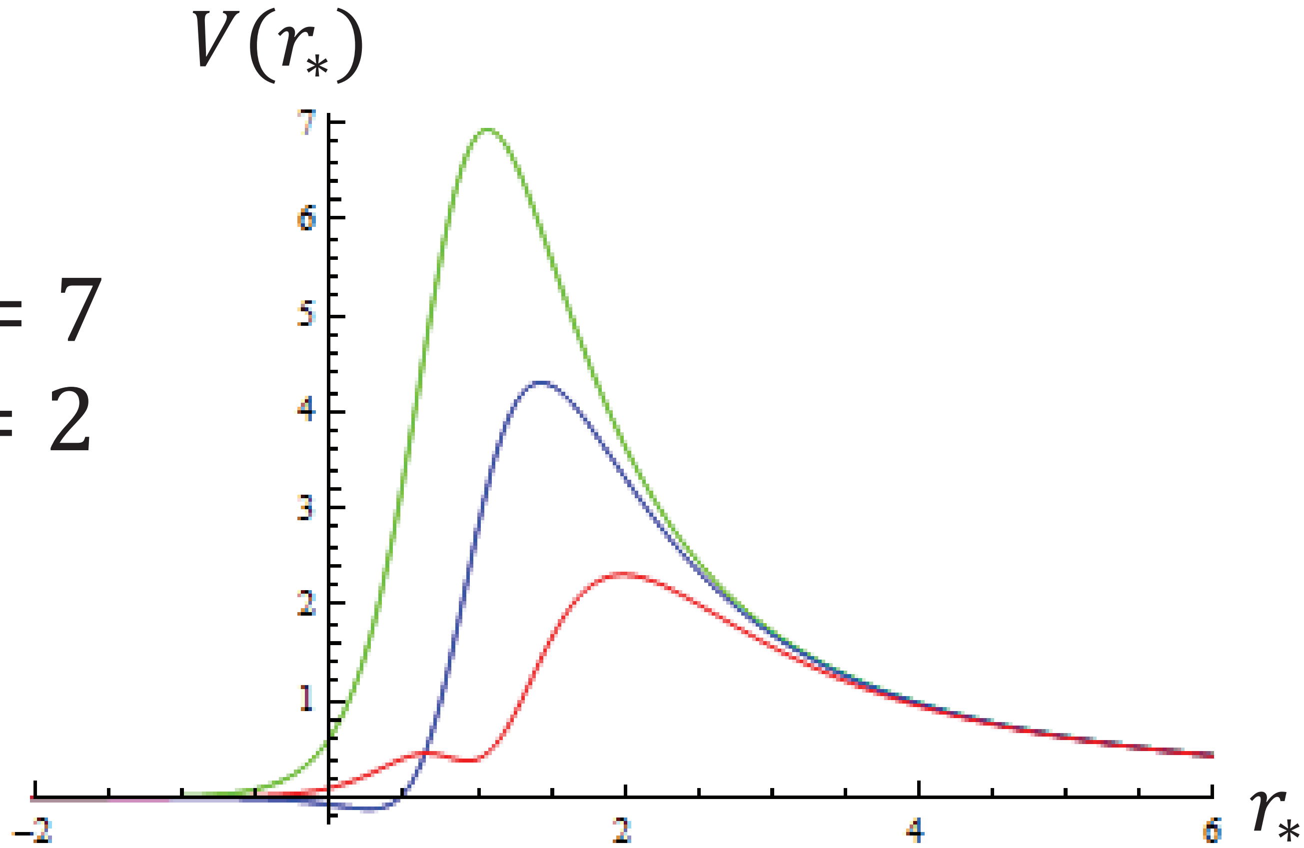
Schwarzschild bh grav perturbations

Kodama+Ishibashi

Gravitational scalar, vector, tensor modes

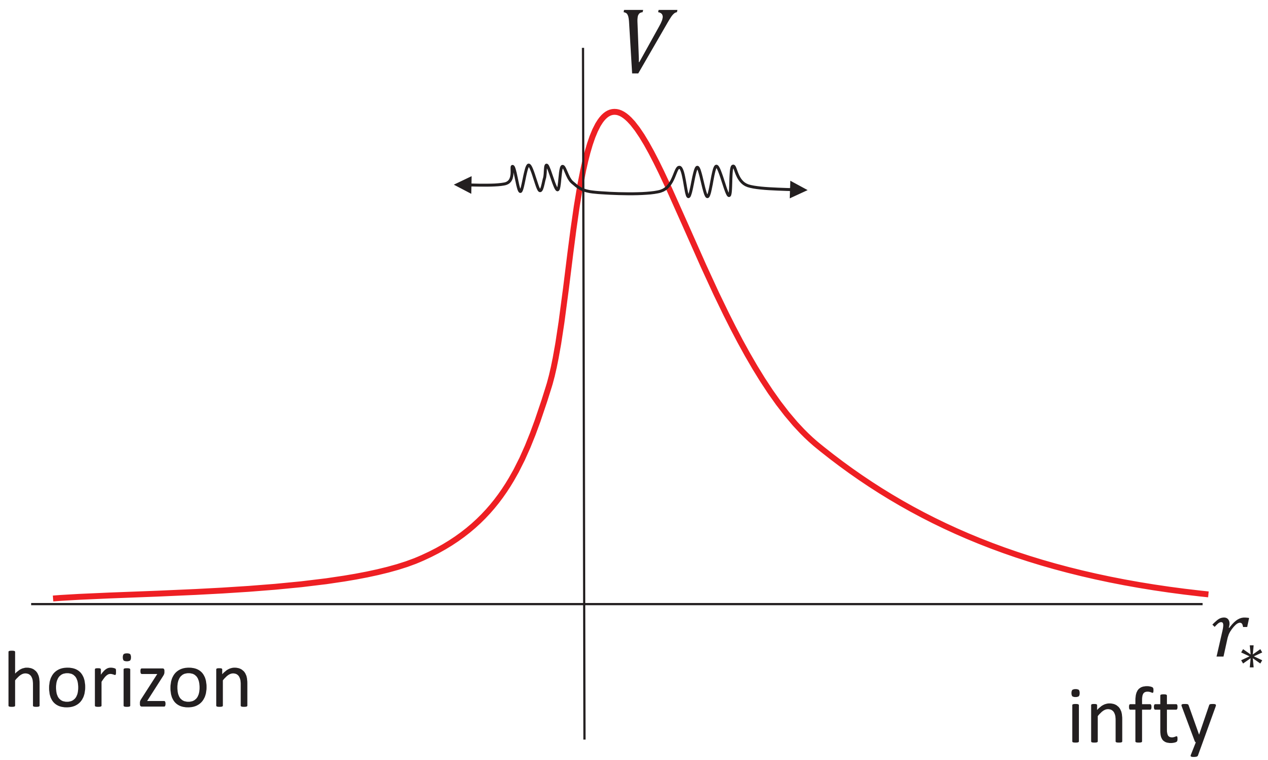
$SO(D-1)$ reps

$D = 7$
$\ell = 2$



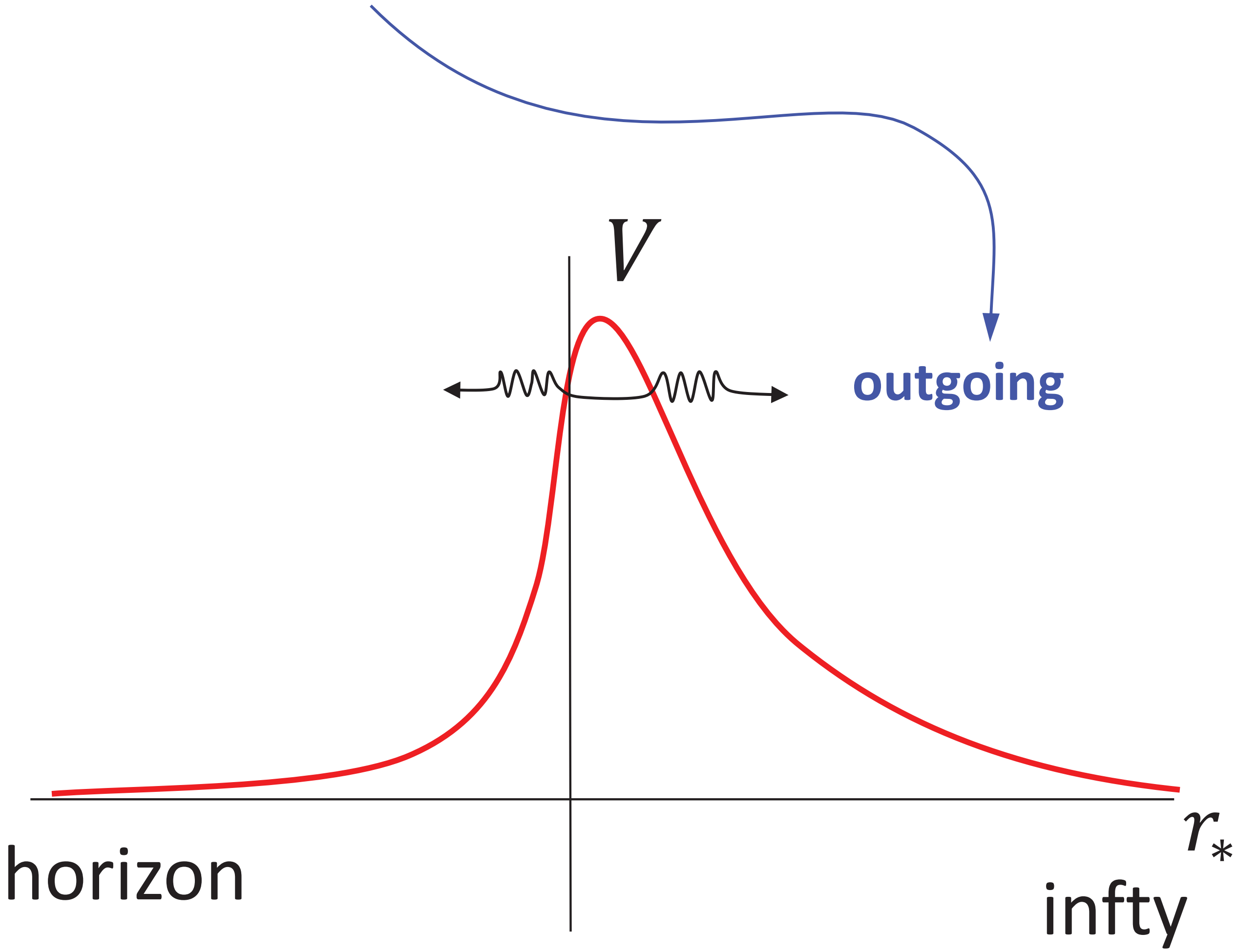
Quasinormal modes

Free, **damped** oscillations of bh



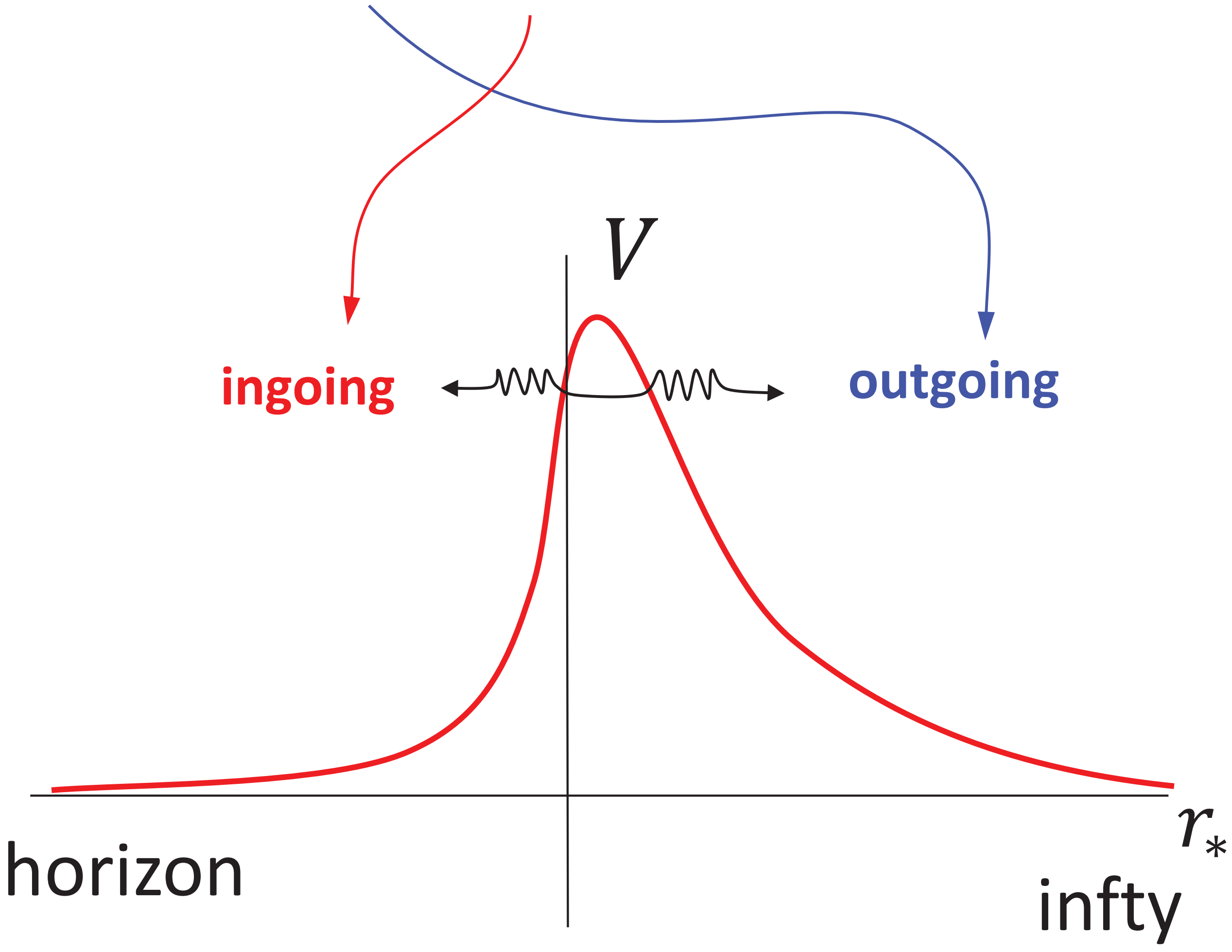
Quasinormal modes

Free, **damped** oscillations of bh



Quasinormal modes

Free, **damped** oscillations of bh



Quasinormal modes @ large D

Most QNMs are **not decoupled** states

not normalizable N-H states

But $\exists$ **a few decoupled** QNMs

normalizable N-H states

Non-decoupling and decoupling sectors are very different

Non-decoupling QNMs

High frequencies $\omega \sim D/r_0$

Small damping ratios $\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \to 0$

Control interaction between bh and environment

Little information about black hole

Universal spectrum

Decoupling QNMs

Low frequencies $\omega \sim D^0/r_0$

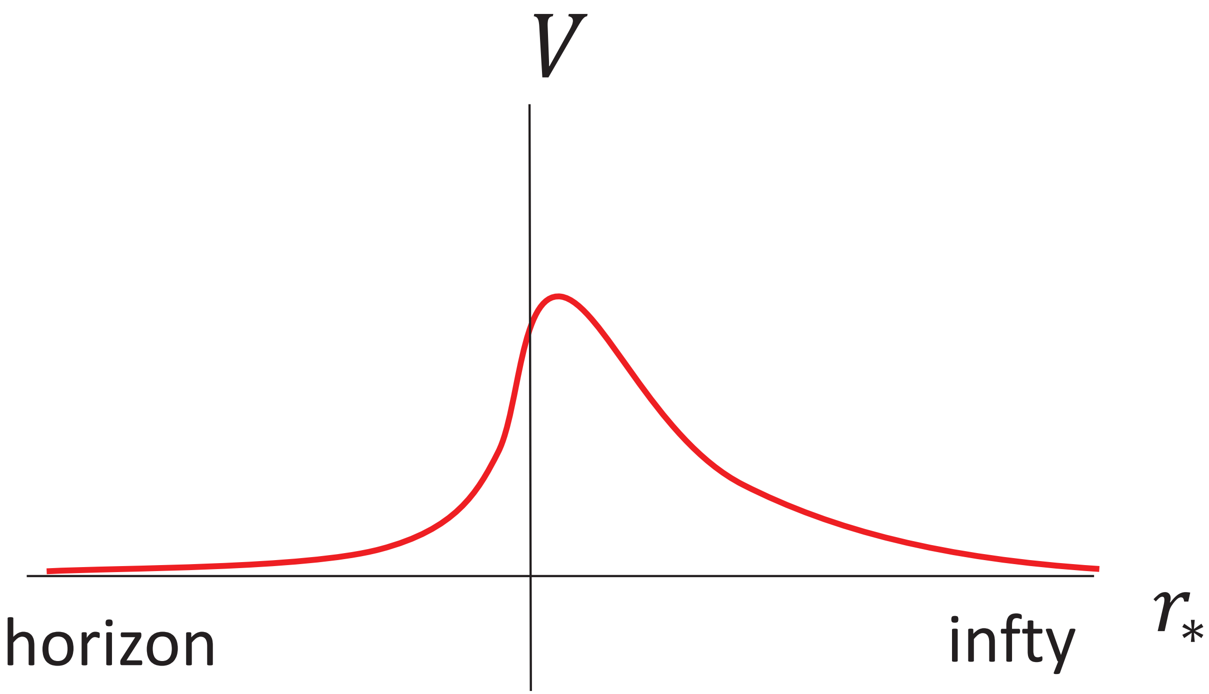
Damping ratio $\dfrac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \sim 1$

Insulated from asymptotic zone

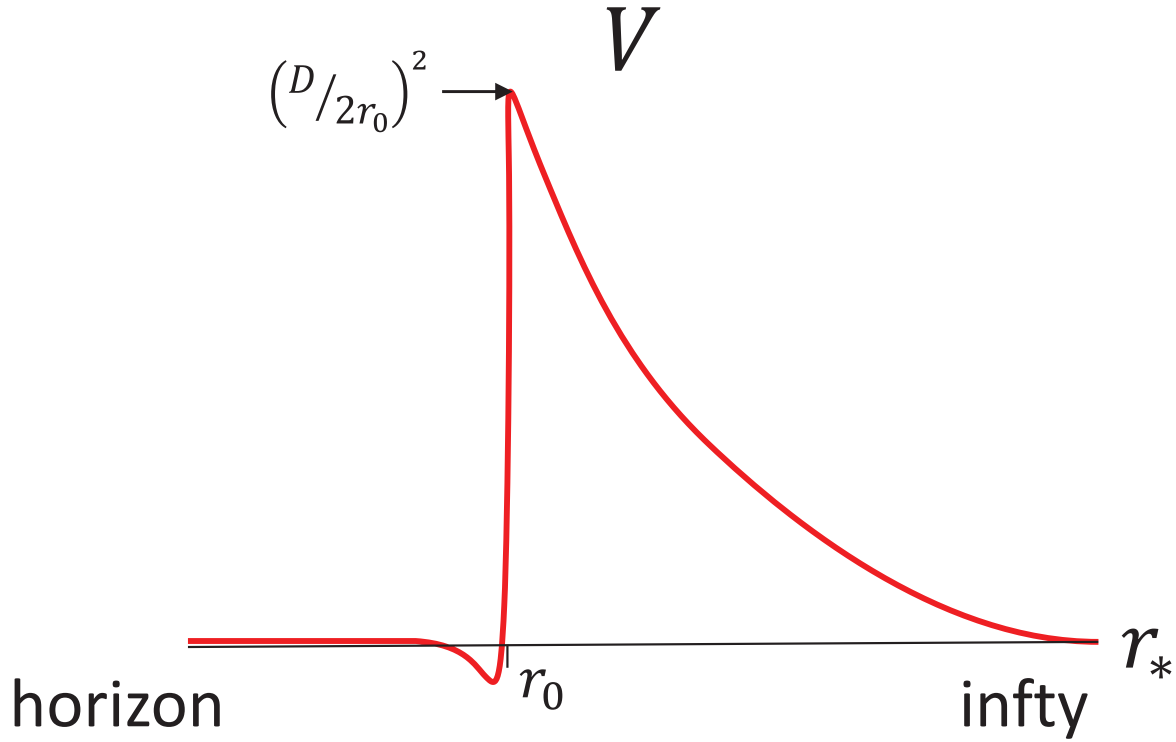
Specific dynamics of each black hole

instabilities, hydrodynamic modes etc

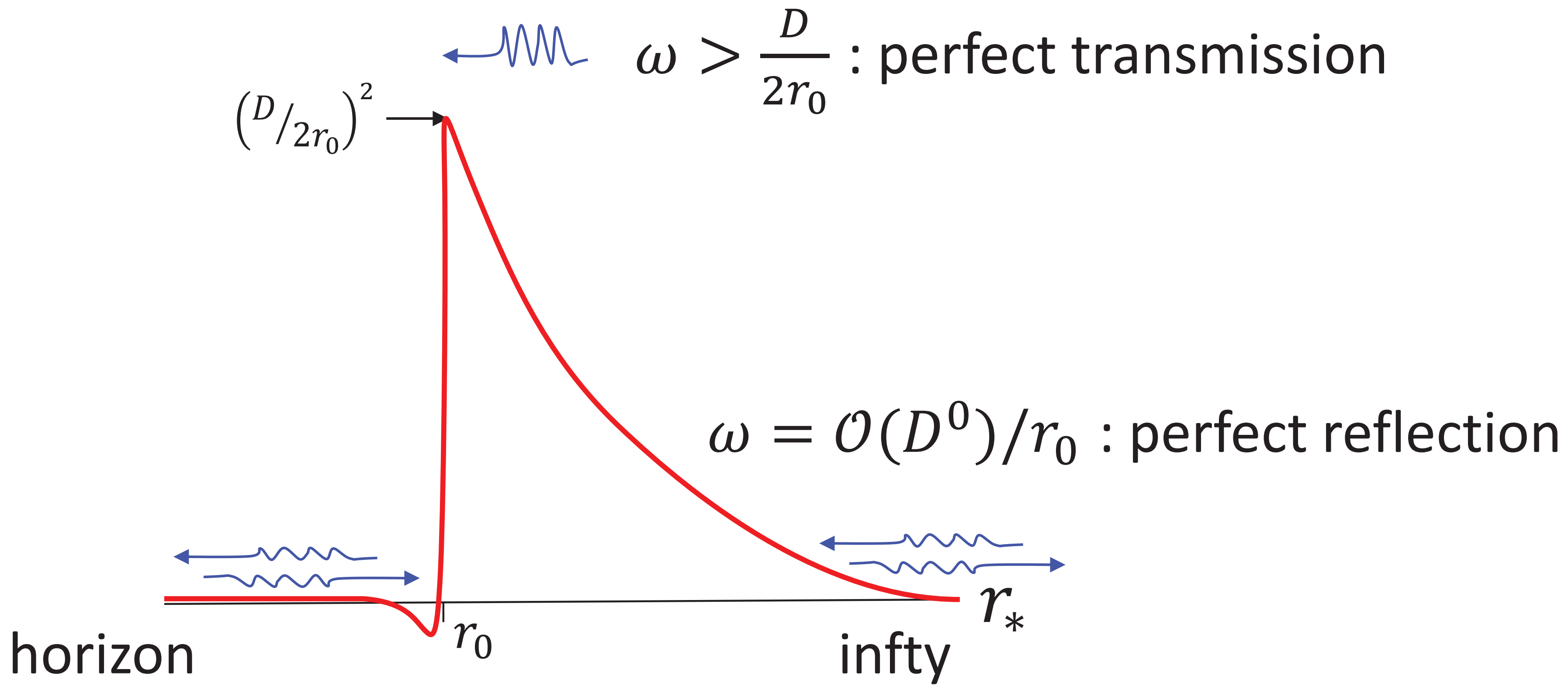
Non-universal


V
r_*
horizon
infty

$$D \to \infty$$



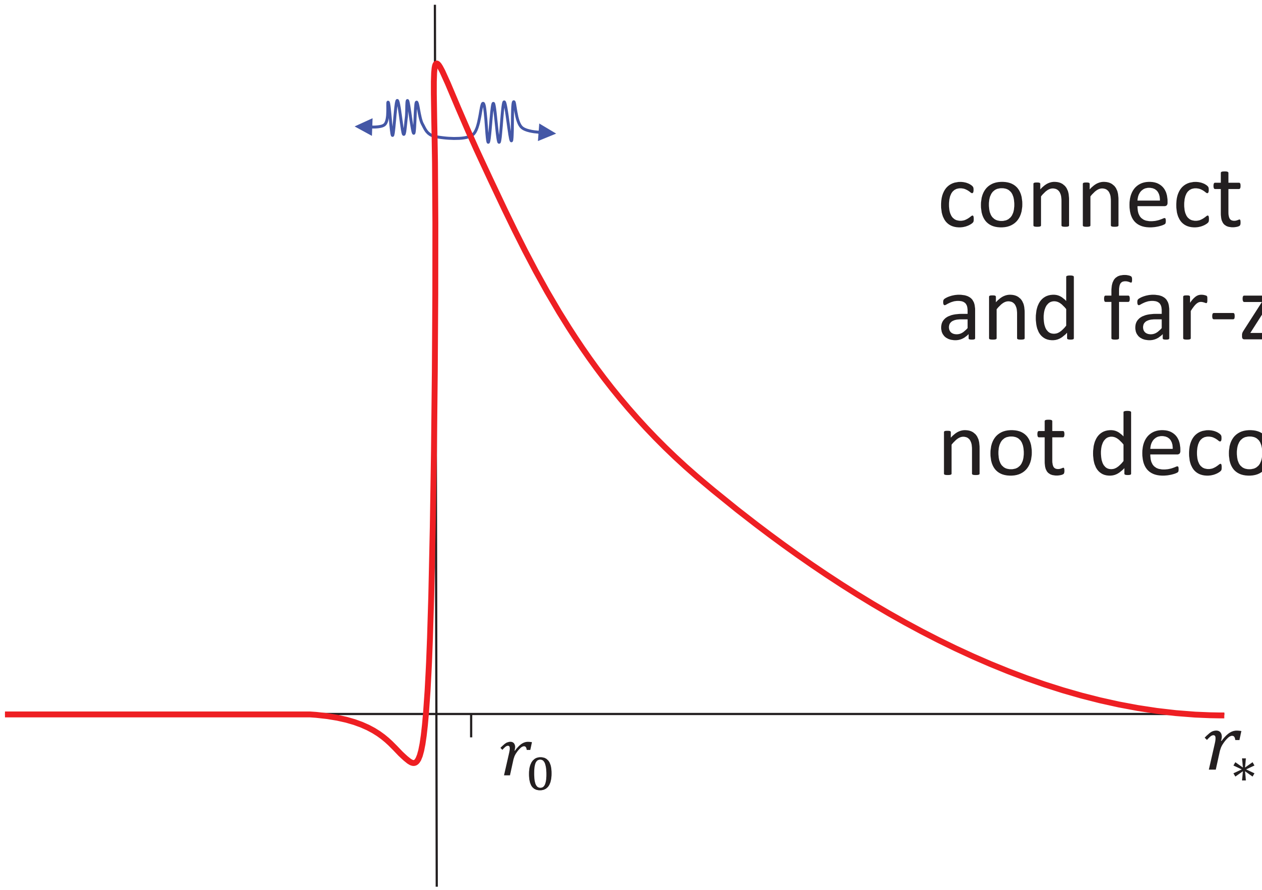
$$D \to \infty$$



$\omega > \frac{D}{2r_0}$: perfect transmission
$\left({}^{D}/_{2r_0}\right)^2$
$\omega = \mathcal{O}(D^0)/r_0$: perfect reflection
r_*
r_0
horizon
infty

Non-decoupled QNMs

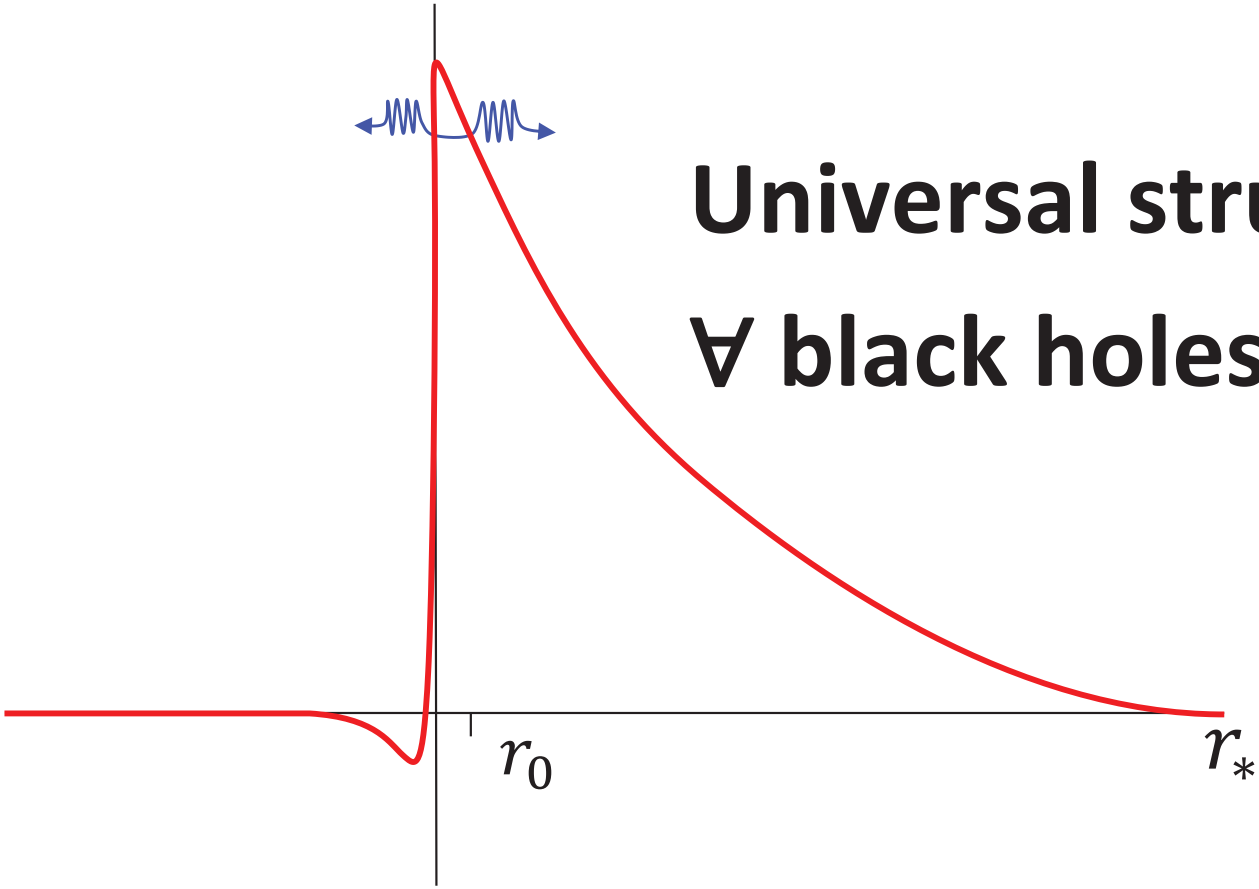
$$\omega \sim D/r_0$$



connect near-zone
and far-zone:
not decoupled

Non-decoupled QNMs

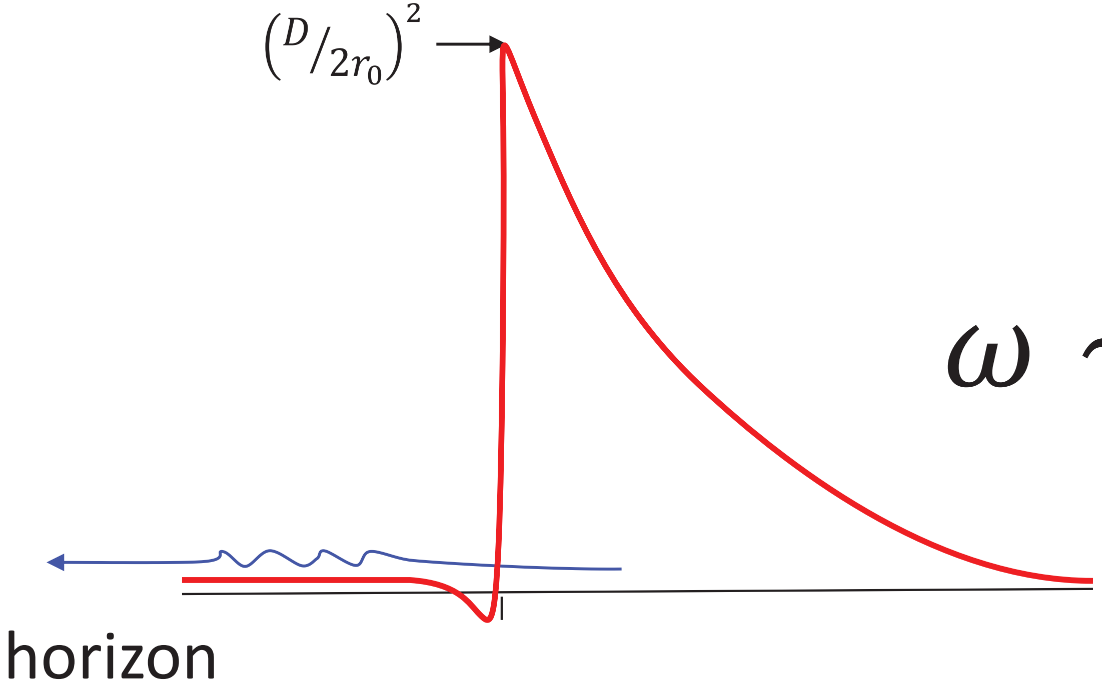
$\omega \sim D/r_0$



Universal structure

$\forall$ black holes @ $D \to \infty$

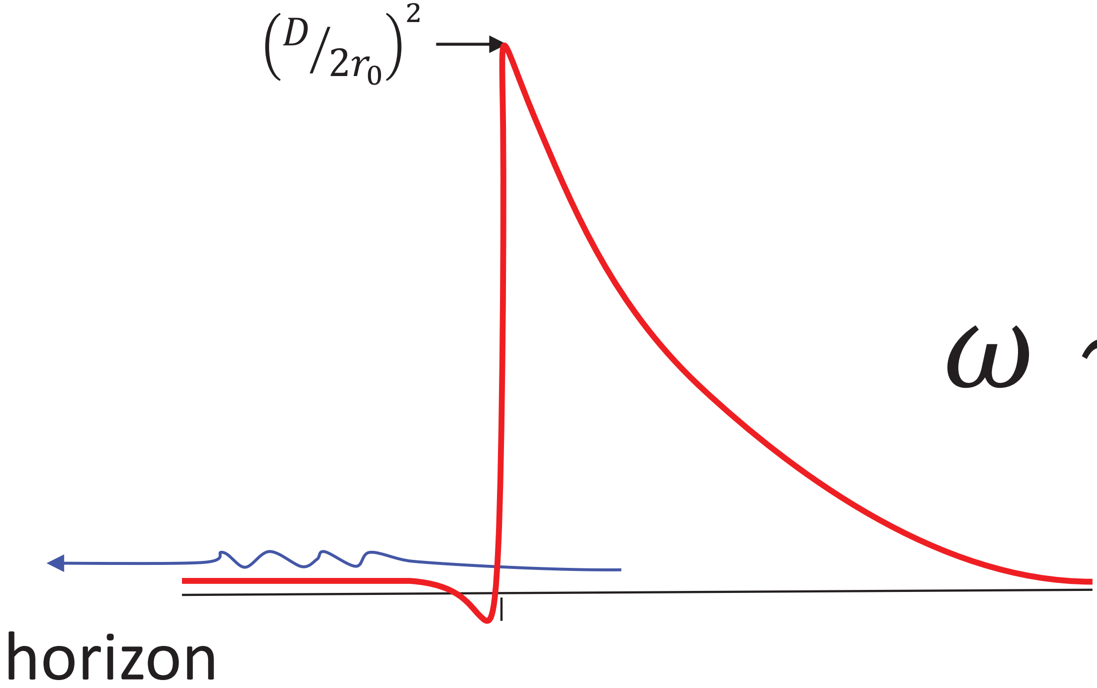
Decoupled QNMs



$$\omega \sim \frac{1}{r_0} \ll \frac{D}{r_0}$$

Strongly suppresed in far-zone:
decoupled

Decoupled QNMs



$$\omega \sim \frac{1}{r_0} \ll \frac{D}{r_0}$$

We have computed these in the $1/D$ expansion up to $1/D^3$

Quantitative accuracy

Decoupled modes $\omega r_0 = \mathcal{O}(1)$

Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\mathrm{Im}\,\omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

Quantitative accuracy

Decoupled modes $\omega r_0 = \mathcal{O}(1)$

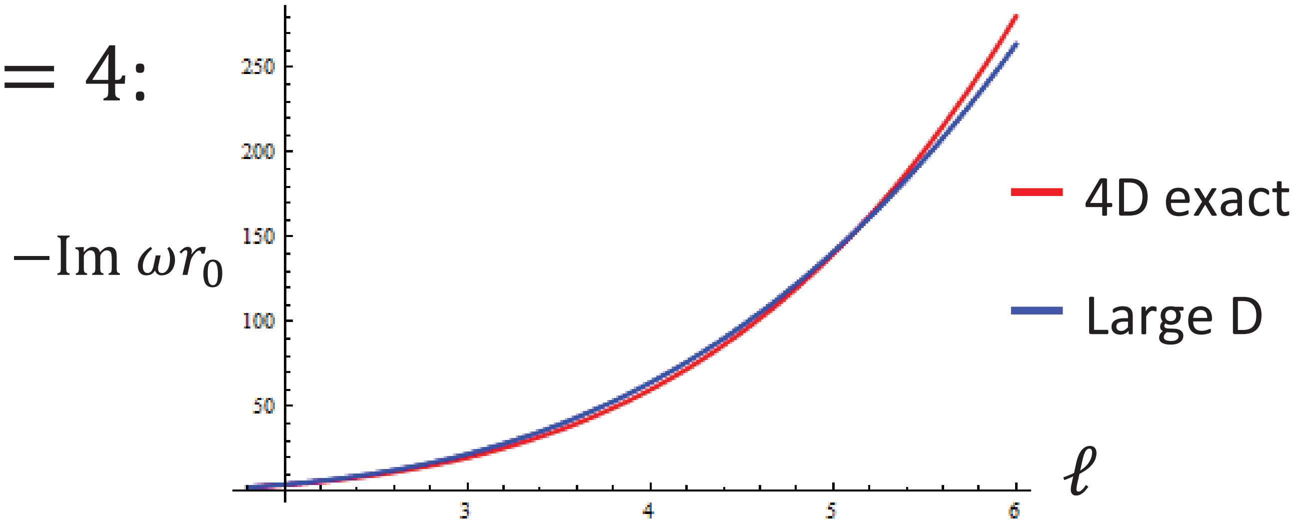
Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\quad \mathrm{Im}\,\omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

- At $D = 4$:

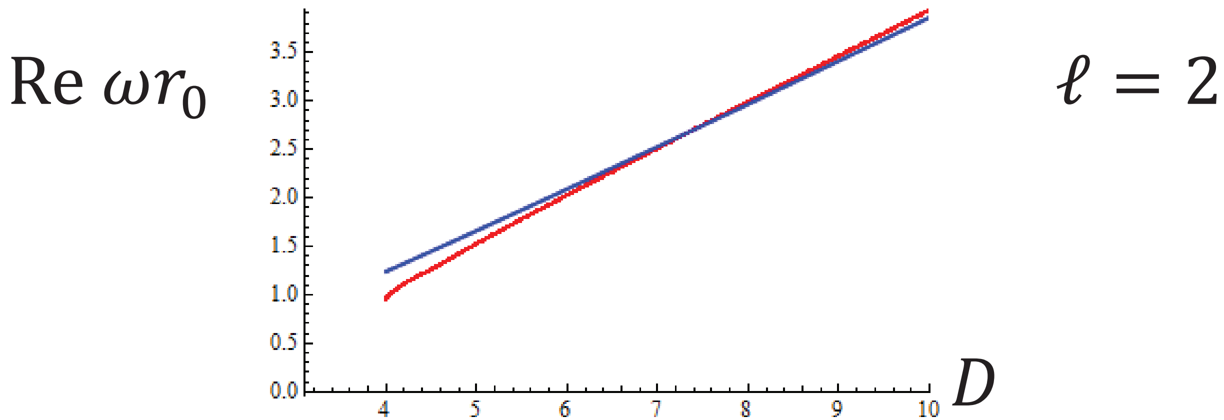


'algebraically special' mode

Quantitative accuracy

Non-decoupled modes $\boldsymbol{\omega r_0 = \mathcal{O}(D)}$

$\mathrm{Re}\,\omega r_0$: good at moderate D



$\mathrm{Im}\,\omega r_0 \sim D^{1/3}$: only good at *very* high D

Outlook

Universal features @ large D

Far region

∀bhs: empty space

Near-horizon region

∀neutral bhs: 2D string bh

BH dynamics splits into:

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

scalar field **oscillations of a hole** in space

universal normal modes

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

localized in near-horizon region

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

- **specific** of each bh
- less numerous
- ultraspinning instabilities in this sector
- hydro modes of black branes

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

- **universal** normal modes of hole in space
- much more **numerous**
- describe **interaction** of bh **w/ environment**
- connection to BH entropy?