Solitons in 1+1 dimensions

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Solitons in 1+1 dimension

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- 古典論におけるキンク
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Introduction

代表的なSoliton

- ４＋１次元：　インスタントン
- ３＋１次元：　モノポール
  スカーシオン
- ２＋１次元：　ボーテックス
  Σ模型lump
- １＋１次元：　キンク

Classical kinks - ϕ^4 theory-

Set up

$\phi(x^\mu)$:実スカラー場 $(\mu = 0, 1)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$\begin{aligned} V(\phi) &= -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda v^4}{4} \\ &= \frac{\lambda}{4}(\phi^2 - v^2)^2 \qquad \left(v = \sqrt{\frac{m^2}{\lambda}}\right) \end{aligned}$$

Classical kinks - ϕ^4 theory-

場の方程式: $$\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial x^2}+\lambda(\phi^2-v^2)\phi=0$$

時間依存しない場合: $$\frac{\partial^2\phi}{\partial x^2}=\lambda(\phi^2-v^2)\phi$$

境界条件

$$\phi(+\infty)=\pm v$$

$$\phi(-\infty)=\mp v$$

厳密解

$$\phi(x)=v\tanh\left[\frac{m}{\sqrt{2}}(x-x_0)\right]$$

Classical kinks - ϕ^4 theory-

質量: $M_{kink} = M_{anti-kink} = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}$

トポロジカル電荷: $Q = \int dx j^0$ $\left(j^\mu = \frac{1}{2v}\epsilon^{\mu\nu}\partial_\nu\phi \right)$

kink: $Q = +1$

anti-kink: $Q = -1$

Quantum kinks - ϕ^4 theory-

Set up

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$V(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda v^4}{4} = \frac{\lambda}{4}(\phi^2 - v^2)^2 \qquad \left(v = \sqrt{\frac{m^2}{\lambda}}\right)$$

厳密解

$$\phi_{kink} = v \tanh\frac{m}{\sqrt{2}}(x - x_0)$$

Quantum kinks - ϕ^4 theory-

$\psi = \phi - \phi_{kink}$　としてポテンシャルを展開する

$$V(\psi) = V_{kink} + \int dx \frac{1}{2}\psi\underbrace{(-\Delta^2 - m^2 + 3\lambda\phi_{kink}^2)}_{\text{対角化する}}\psi + \lambda \int dx \left(\phi_{kink}\psi^3 + \frac{1}{4}\psi^4\right)$$

$$(-\nabla^2 - m^2 + 3\lambda\phi_{kink}^2)\psi_n = \omega_n^2\psi_n$$

Quantum kinks - ϕ^4 theory-

$$(-\nabla^2 - m^2 + 3\lambda\phi_{kink}^2)\psi_n = \omega_n^2\psi_n$$

$$\omega_0^2 = 0 \qquad \psi_0 = \frac{1}{\cosh^2 x}$$

$$\omega_1^2 = \frac{3}{2}m^2 \qquad \psi_1 = \frac{\sinh z}{\cosh^2 z}$$

$$\omega_q^2 = \left(\frac{q^2}{2} + 2\right)m^2 \qquad \psi_q = e^{iqz}(3\tanh^2 z - 1 - q^2 - 3iq\tanh z)$$

$$\left(z = \frac{m}{\sqrt{2}}x, q = \frac{\sqrt{2}}{m}p\right)$$

Quantum kinks - ϕ^4 theory-

$$E_{kink[N_n]} = M_{kink} + \frac{\sqrt{3}}{2} m\hbar \left(N_1 + \frac{1}{2} \right) + m\hbar \sum_{q_n} \left(N_{q_n} + \frac{1}{2} \right) \sqrt{\frac{1}{2} q_n^2 + 2} + \mathcal{O}(\lambda)$$

$$E_{kink[0]} = M_{kink} + \frac{1}{2} \frac{\sqrt{3}}{2} m\hbar + \frac{m\hbar}{2} \sum_{q_n} \sqrt{\frac{1}{2} q_n^2 + 2} + \mathcal{O}(\lambda)$$

$$E_{vac} = E_0 = 0 + \sum_{n} \frac{\hbar}{2} \sqrt{k_n^2 + 2m^2} + \mathcal{O}(\lambda)$$

$$\rightarrow \boxed{M^q_{kink} = E_{kink[0]} - E_{vac}}$$

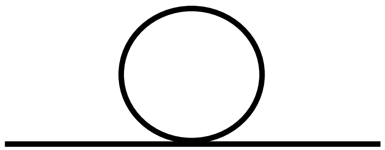
Quantum kinks - ϕ^4 theory-

発散

1. 零点振動の差

$$\Delta E_0 = \left(\frac{\sqrt{6}}{12} - \frac{3\sqrt{2}}{2\pi} \right) - m\hbar \int_0^\infty \frac{dp}{\sqrt{p^2 + 2m^2}}$$

2. ループからの紫外発散



$$= \frac{3\lambda}{2\pi} \int_0^\infty \frac{dk}{\sqrt{k^2 + 2m^2}}$$

Quantum kinks - ϕ^4 theory-

2.ループからの紫外発散の問題を解決するために、

$$\mathcal{L} = \frac{1}{2}\delta m^2 \phi^2 \qquad \left(\delta m^2 \phi^2 = \frac{3\lambda}{2\pi} \int_0^\infty \frac{dk}{\sqrt{k^2 + 2m^2}} \right)$$

をラグランジアンに付け加える。その結果、

$$E_{kink[0]} = M_{kink} + \sum_n \frac{\hbar}{2}\sqrt{k_n^2 + 2m^2} - \frac{\delta m^2}{2}\int dx \phi_{kink}^2 + \mathcal{O}(\lambda)$$

$$E_{vac} = 0 + \sum_n \frac{\hbar}{2}\sqrt{k_n^2 + 2m^2} - \frac{\delta m^2}{2}\int dx \phi_{vac}^2 + \mathcal{O}(\lambda)$$

を得る。

Quantum kinks - ϕ^4 theory-

kinkの質量は、

$$
\begin{aligned}
M^q_{kink} &= E_{kink[0]} - E_{vac} \\
&= M_{kink} + \underline{\underline{\Delta E_0}} - \frac{\delta m^2}{2}\int dx(\phi^2_{kink} - \phi^2_{vac}) + \mathcal{O}(\lambda) \\
&= M_{kink} + \underline{\underline{\left(\frac{\sqrt{6}}{12} - \frac{3\sqrt{2}}{2\pi}\right) m\hbar - \frac{3\sqrt{2}}{2\pi} m\hbar \int_0^\infty \frac{dp}{\sqrt{p^2 + 2m^2}}}} \\
&\qquad - \frac{\delta m^2}{2}\underbrace{\int dx(\phi^2_{kink} - \phi^2_{vac})}_{-2\sqrt{2}\frac{m}{\lambda}} + \mathcal{O}(\lambda) \\
&= M_{kink} + \left(\frac{\sqrt{6}}{12} - \frac{3\sqrt{2}}{2\pi}\right) m\hbar
\end{aligned}
$$

Conclusion

$$M^q_{kink} = M_{kink} + \underbrace{\left(\frac{\sqrt{6}}{12} - \frac{3\sqrt{2}}{2\pi}\right)}_{\simeq -0.47} m\hbar + \mathcal{O}(\lambda)$$

→量子補正でkinkの質量は $-0.47m\hbar$ しか変化しない

References

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