Dynamical generation of fermion mass hierarchy in an extra dimension

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Kobe University
References and collaborators

- Phase Structure of Gauge Theories on an Interval, Y. Fujimoto, T. Nagasawa, S. Ohya, M.S., PTP 126 (2011) 841

- Quark mass hierarchy and mixing via geometry of extra dimension with point interactions, Y. Fujimoto, T. Nagasawa, K. Nishiwaki, M.S., PTEP 023B07 (2013)


- Realization of lepton masses and mixing angles from point interactions in an extra dimension, Y. Fujimoto, K. Nishiwaki, M.S., R. Takahashi, JHEP 10 (2014) 191

- Dynamical generation of fermion mass hierarchy in an extra dimension, Y. Fujimoto, T. Miura, M.S., work in progress
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- Phase Structure of Gauge Theories on an Interval,
  Y. Fujimoto, T. Nagasawa, S. Ohya, M.S.,
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- Quark mass hierarchy and mixing via geometry of extra dimension with point interactions,
  Y. Fujimoto, T. Nagasawa, K. Nishiwaki, M.S.,
  PTEP 023B07 (2013)

- CP phase from twisted Higgs vacuum expectation value in extra dimension,
  Y. Fujimoto, K. Nishiwaki, M.S.,

- Realization of lepton masses and mixing angles from point interactions in an extra dimension,
  Y. Fujimoto, K. Nishiwaki, M.S., R. Takahashi,
  JHEP 10 (2014) 191

- Dynamical generation of fermion mass hierarchy in an extra dimension,
  Y. Fujimoto, T. Miura, M.S.,
  work in progress
We would like to show that
the quark & lepton flavor structure is naturally
generated from extra dimensions.
We would like to show that the quark & lepton flavor structure is naturally generated from extra dimensions.

from an extra dimensional point of view

closely related

# of generation

mass hierarchy

flavor mixing
Plan of my talk

- Motivation to considering extra dimensions
- Mysteries of the Standard Model
- General features of extra dimensions
- Setup
- Point interactions
- Dynamical generation of fermion mass hierarchy
- Summary
Plan of my talk

- Motivation to considering extra dimensions
- Mysteries of the Standard Model
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- Dynamical generation of fermion mass hierarchy
- Summary
Minimum length

\[ \Delta x \]

photon
Minimum length

\[ \Delta x \gtrsim \frac{\hbar}{\Delta p} \]

quantum effect
Minimum length

\[ \Delta x \gtrsim \frac{\hbar}{\Delta p} \oplus \frac{G}{c^3} \Delta p \]

quantum effect  gravity effect
Minimum length

\[ \Delta x \gtrsim \frac{\hbar}{\Delta p} \oplus \frac{G}{c^3} \Delta p \geq \sqrt{\frac{G \hbar}{c^3}} \equiv l_P \]

quantum effect

gravity effect

planck length

photon
Minimum length

\[ \Delta x \gtrsim \frac{\hbar}{\Delta p} \oplus \frac{G}{c^3} \Delta p \geq \sqrt{\frac{G\hbar}{c^3}} \equiv l_P \]

- Quantum effect
- Gravity effect

No space-time less than \( l_P \)
Disappearance of space-time

\[ E_p \frac{1}{l_p} \]

high energy short distance

low energy long distance

4d flat space-time
Disappearance of space-time

\[ E_p \sim \frac{1}{l_p} \]

high energy
short distance

low energy
long distance

Space-time is curved around heavy objects

4d flat space-time
Disappearance of space-time

Space-time is curved around heavy objects.

4d flat space-time

high energy
short distance

low energy
long distance

$E_P - \frac{1}{l_p}$

hardly distorted!
Disappearance of space-time

- High energy, short distance: Space-time is curved around heavy objects.
- Low energy, long distance: 4d flat space-time.

No space-time is hardly distorted!

Space-time is curved around heavy objects.

4d flat space-time
Disappearance of space-time

No space-time

appearance of space-time

4d flat space-time

\[ E_p \ll 1/l_p \]

high energy
short distance

low energy
long distance
Implications of the minimum length

- The Standard Model should be regarded as a low energy effective theory.
Implications of the minimum length

- There is no distinction between scalar, spinor, vector and tensor at high energies because these cannot be defined without space-time.
There is no distinction between scalar, spinor, vector and tensor at high energies because these cannot be defined without space-time.

unification of scalar, spinor, vector and tensor at high energies
Implications of the minimum length

There is no distinction between scalar, spinor, vector and tensor at high energies because these cannot be defined without space-time.

Supersymmetry unifies fields whose spins differ by 1/2.
Implications of the minimum length

- There is no distinction between scalar, spinor, vector and tensor at high energies because these cannot be defined without space-time.

- Unification of scalar, spinor, vector and tensor at high energies

**Supersymmetry**
\[
\Phi = (\phi, \psi) \\
W = (\lambda, A_\mu)
\]
unifies fields whose spins differ by 1/2

**Kaluza-Klein theory**
\[
A_M = (A_\mu, \phi) \\
g_{MN} = \begin{pmatrix}
g_{\mu\nu} & A_\mu \\
A_\mu & \phi
\end{pmatrix}
\]
unifies fields whose spins differ by 1
Implications of the minimum length

- There is no distinction between space-time and matter at high energies because of vanishing space-time.
There is no distinction between space-time and matter at high energies because of vanishing space-time.

“degrees” of space-time \(\longleftrightarrow\) “degrees” of matter
Implications of the minimum length

- There is no distinction between space-time and matter at high energies because of vanishing space-time.

"degrees" of space-time ↔ convert ↔ "degrees" of matter

There is no reason to persist in 4-dimensions because extra-dimensions can be converted into matter, and vice versa.
Implications of the minimum length

- There is no distinction between space-time and matter at high energies because of vanishing space-time.

- There is no reason to persist in 4-dimensions because extra-dimensions can be converted into matter, and vice versa.

- This Fourier expansion may tell us that the degrees of $S^1$ can be converted into infinitely many 4d fields $\phi_n(x)$.

\[
\Phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x)f_n(y)
\]

\[f_n(y) = \frac{1}{\sqrt{2\pi R}} e^{iy/R}\]
Implications of the minimum length

- There is no distinction between space-time and matter at high energies because of vanishing space-time.

- High energy short distance $E_P = \frac{1}{l_P}$

- Low energy long distance

Fundamental theory without space-time

Field theories with extra dimensions

4d Standard Model
• Motivation to considering extra dimensions

• Mysteries of the Standard Model

• General features of extra dimensions

• Setup

• Point interactions

• Dynamical generation of fermion mass hierarchy

• Summary
Mystery of gauge group
Why isn't the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?
Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

I have the impression that small gauge groups are chozen !?
Mystery of matter

Mystery of matter
Why are the matter representations chosen such that

<table>
<thead>
<tr>
<th></th>
<th>$SU(3)_c$ singlet</th>
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### Mystery of matter

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I have the impression that small representations are chosen!?
Mystery of chiral structure of SM
Why is the SM a chiral gauge theory?

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*a left-right asymmetric gauge theory!*
Mystery of scalar quadratic divergence
$\Delta m^2 = \phi \cdots \phi \sim \Lambda^2$

*It seems unnatural that fundamental scalars (Higgs?) appear in low energies!*
Mystery of fermion generations
Mystery of fermion generations

Who ordered exactly the same three sets of quarks and leptons?
Mystery of mass hierarchy
Mystery of mass hierarchy

\[
\frac{m_t}{m_u} \sim 10^5
\]
Why is there the hierarchical mass difference between different generations of quarks and leptons?
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?
Motivation to considering extra dimensions
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Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

Answer from our point of view
Mystery of gauge group

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We have to pay much cost for large gauge group!?
Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

**Answer from our point of view**

We have to pay much cost for large gauge group!?

“degrees” of space-time \[\longleftrightarrow\] “degrees” of gauge group

**KK theory on $M^4 \times S^1$:** \[
\begin{pmatrix}
g_{\mu\nu} & A_\mu \\
A_\mu & \phi
\end{pmatrix}
\]
Mystery of gauge group

Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

Answer from our point of view

We have to pay much cost for large gauge group!?

“degrees” of space-time

convert

“degrees” of gauge group

Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

KK theory on $M^4 \times S^1$:

$$g_{MN} = \begin{pmatrix} g_{\mu \nu} & A_\mu \\ A_\mu & \phi \end{pmatrix}$$

$\leftrightarrow$ the rank of gauge groups $\leq$ # of extra dimensions
Mystery of gauge group

Why isn’t the SM gauge group $SU(1000000)$ but $SU(3) \times SU(2) \times U(1)$?

Answer from our point of view

We have to pay much cost for large gauge group!?

"degrees" of space-time $\leftrightarrow$ convert $\leftrightarrow$ "degrees" of gauge group

$\alpha$ the rank of gauge groups $\%$ # of extra dimensions

KK theory on $M^4 \times S^1$:

$$ g_{MN} = \begin{pmatrix} g_{\mu\nu} & A_\mu \\ A_\mu & \phi \end{pmatrix} $$

$\Rightarrow$ the rank of gauge groups $\leq$ # of extra dimensions

$\Rightarrow E_6, SO(10), SU(5), SU(3) \times SU(2) \times U(1), \cdots$ for 6 dim.

I will not discuss this subject in my talk.
**Mystery of matter**

Why are small representations for the quarks & leptons are chosen?

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Mystery of matter

Answer from our point of view

We have to pay much cost for higher dimensional representations!?
Mystery of matter

We have to pay much cost for higher dimensional representations!? 

- 5d field on $M^4 \times S^2$
  
  $\Psi(x; \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_{l,m}(x) Y_{l,m}(\theta, \phi)$

  - spherical harmonics on $S^2$
  - 4d fields belonging to spin $l$ representation of SU(2)
Mystery of matter

We have to pay much cost for higher dimensional representations!? 

- 5d field on $M^4 \times S^2$
  \[\Psi(x; \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_{l,m}(x) Y_{l,m}(\theta, \phi)\]

- mass$^2$ of $\psi_{l,m}$
  \[m_{l}^2 = \frac{l(l + 1)}{R^2}\]
We have to pay much cost for higher dimensional representations!? 

- 5d field on $M^4 \times S^2$
  $$\Psi(x; \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_{l,m}(x) Y_{l,m}(\theta, \phi)$$

- mass$^2$ of $\psi_{l,m}$
  $$m_{l}^2 = \frac{l(l + 1)}{R^2}$$

At low energies, only 4d fields belonging to small dimensional representations can appear!

I will not discuss this subject in my talk.
Why is the SM a chiral gauge theory?

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*a left-right asymmetric gauge theory!*
Why is the SM a chiral gauge theory?

Answer from our point of view

The SM should be regarded as a low energy effective theory, which will be described by \textit{massless} particles.

- massless spinors
- massless vectors
Mystery of chiral structure of SM

Why is the SM a chiral gauge theory?

Answer from our point of view

The SM should be regarded as a low energy effective theory, which will be described by massless particles.

massless spinors $\rightarrow$ chiral fermions
massless vectors
Why is the SM a chiral gauge theory?

Answer from our point of view

The SM should be regarded as a low energy effective theory, which will be described by massless particles.

- Massless spinors → Chiral fermions
- Massless vectors → Gauge fields
Why is the SM a chiral gauge theory?

Answer from our point of view

The SM should be regarded as a low energy effective theory, which will be described by massless particles.

- massless spinors
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\[\text{chiral fermions}\]
\[\text{gauge fields}\]

\textit{chiral gauge theory!}
\[ \Delta m^2 = \phi \cdots \cdots \phi \sim \Lambda^2 \]
Mystery of scalar quadratic divergence

\[ \Delta m^2 = \phi \sim \Lambda^2 \]

Answer from our point of view

There is no distinction between scalars, spinors, vectors at high energies.
Mystery of scalar quadratic divergence

\[ \Delta m^2 = \phi \quad \sim \quad \Lambda^2 \]

Answer from our point of view

There is no distinction between scalars, spinors, vectors at high energies.

 Scalars should belong to some multiplets with spinors and/or vectors at high energies.

e.g.

\[ \Phi = (\phi, \psi) \quad \text{supersymmetry} \]

\[ A_M = (A_\mu, \phi) \quad \text{extra dimensions} \]
Mystery of scalar quadratic divergence

\[ \Delta m^2 = \phi \sim \Lambda^2 \]

Answer from our point of view

There is no distinction between scalars, spinors, vectors at high energies.

\[ \downarrow \]

Scalars should belong to some multiplets with spinors and/or vectors at high energies.

\[ \Phi = (\phi, \psi) \quad \text{supersymmetry} \]

\[ A_M = (A_\mu, \phi) \quad \text{extra dimensions} \]

No quadratic divergences!
Mystery of fermion generations

What is the origin of generations?
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

\[ \Psi(x, y) = \sum_{n} \left\{ \psi^{(n)}_R(x) f_n(y) + \psi^{(n)}_L(x) g_n(y) \right\} \]

- Higher dim. spinor
- 4d right-handed spinors
- 4d left-handed spinors
- Wavefunctions on extra dim.

\[
\begin{align*}
\psi_R^{(n)} & \\
\psi_L^{(n)} & \\
\psi_R^{(1)} & \\
\psi_L^{(1)} & \\
\psi_R^{(2)} & \\
\psi_L^{(2)} & \\
\psi_R^{(3)} & \\
\psi_L^{(3)} &
\end{align*}
\]

\[ m_0 = 0 \]

\[ m_1 \]

\[ m_2 \]

\[ m_3 \]
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

\[ \Psi(x, y) = \sum_n \left\{ \psi_R^{(n)}(x)f_n(y) + \psi_L^{(n)}(x)g_n(y) \right\} \]

- Higher dim. spinor
- 4d left-handed spinors
- 4d right-handed spinors
- Wavefunctions on extra dim.

# of generations \( \equiv \left| \# \text{ of } \psi_R^{(0)} - \# \text{ of } \psi_L^{(0)} \right| \)

= a topological # of extra dimensions
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

Change the parameters \((m, g, L, \bar{h}, \cdots)\) of the theory.
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

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Change the parameters \((m, g, L, \bar{h}, \cdots)\) of the theory.
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

Change the parameters \((m, g, L, \hbar, \cdots)\) of the theory.

\(\psi^{(n)}_R\)

\(\psi^{(n)}_L\)

Each number of \(\psi^{(0)}_R\) and \(\psi^{(0)}_L\) can change but NOT their difference!
Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

Change the parameters \((m, g, L, \bar{h}, \cdots)\) of the theory.

Each number of \(\psi^{\scriptscriptstyle (n)}_R\) and \(\psi^{\scriptscriptstyle (n)}_L\) can change but \textbf{NOT} their difference!

\[ \Rightarrow \text{# of generators is a topological number!} \]
<table>
<thead>
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<tr>
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<td>1</td>
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<tr>
<td>monopole on $S^2$</td>
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<tr>
<td>magnetic flux on $T^2$</td>
<td>$M$ (magnetic flux charge)</td>
</tr>
<tr>
<td>point interactions</td>
<td>$M$ ($#$ of point interactions $-1$)</td>
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**Mystery of fermion generations**

**What is the origin of generations?**

**Answer from our point of view**

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<td>$M$ (# of point interactions – 1)</td>
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Mystery of mass hierarchy

What is the origin of the hierarchical masses?

What is the origin of the hierarchical masses?

- $m_t$ to $10^5 t$
- $\frac{m_t}{m_u} \sim 10^5$
- $\sim 1 t$

USS Enterprise aircraft

Mass [MeV]

generations
Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

Yukawa interactions

\[ \int d^4x \lambda \bar{\psi}_L(x) \phi(x) \psi_R(x) \]

\[ \downarrow \]

\[ \int d^4x \ m \ \bar{\psi}_L(x) \psi_R(x) \]

\[ m = \lambda \langle \phi \rangle \]
What is the origin of the hierarchical masses?

Answer from our point of view

Yukawa interactions

4 dim.

\[
\int d^4 x \, \lambda \bar{\psi}_L(x) \phi(x) \psi_R(x) \downarrow \\
\int d^4 x \, m \bar{\psi}_L(x) \psi_R(x) \\
m = \lambda \langle \phi \rangle
\]

4 dim. + extra dim.

\[
\int d^4 x \int dy \, \lambda \bar{\Psi}(x, y) \Phi(x, y) \psi_r(x, y) \downarrow \\
\int d^4 x \, m \bar{\psi}_L^{(0)}(x) \psi_R^{(0)}(x)
\]
Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

Yukawa interactions

4 dim. 

\[ \int d^4 x \, \lambda \, \bar{\psi}_L(x) \phi(x) \psi_R(x) \]

\[ \int d^4 x \, m \, \bar{\psi}_L(x) \psi_R(x) \]

\[ m = \lambda \langle \phi \rangle \]

4 dim. + extra dim.

\[ \int d^4 x \int dy \, \lambda \, \bar{\Psi}(x, y) \Phi(x, y) \psi_r(x, y) \]

\[ \int d^4 x \, m \, \bar{\psi}_L^{(0)}(x) \psi_R^{(0)}(x) \]

\[ m = \lambda \int dy \, (g_0(y))^* \langle \Phi(y) \rangle f_0(y) \]

\[ \begin{cases} 
\Psi(x, y) = \psi_L^{(0)}(x)g_0(y) + \text{(massive modes)} \\
\Psi'(x, y) = \psi_R^{(0)}(x)f_0(y) + \text{(massive modes)} \\
\Phi(x, y) = \langle \Phi(y) \rangle + \text{(massive modes)} 
\end{cases} \]
Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

\[ m = \lambda \int dy \ (g_0(y))^* \langle \Phi(y) \rangle f_0(y) \]

Two ways to produce mass hierarchy:
Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

Two ways to produce mass hierarchy:

\[ m = \lambda \int dy \, (g_0(y))^* \langle \Phi(y) \rangle f_0(y) \]

Localization naturally leads to mass hierarchy!
Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

Two ways to produce mass hierarchy:

\[ m = \lambda \int dy \ (g_0(y))^* \langle \Phi(y) \rangle f_0(y) \]

Localization naturally leads to mass hierarchy!

The y-dependent vacuum expectation value can happen in extra dimensions!
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view

- mass $\leftrightarrow$ large
- large $\rightarrow$ mass
- small $\rightarrow$ mixing
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?
Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view

large mass → small mixing
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view

- large mass $\rightarrow$ small mixing
- quarks $\rightarrow$ small mixing
Mystery of flavor mixing

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- Large mass $\rightarrow$ small mixing
- Quarks $\rightarrow$ small mixing
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Answer from our point of view

mass $\leftarrow$ large $\rightarrow$ mass

small $\rightarrow$ mixing

large mass $\rightarrow$ small mixing

quarks $\rightarrow$ small mixing

mass $\leftarrow$ small $\rightarrow$ mass

large $\rightarrow$ mixing

small mass $\rightarrow$ large mixing
Mystery of flavor mixing

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Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view

mass $\leftarrow$ large $\rightarrow$ mass

mass $\leftarrow$ small $\rightarrow$ mass

small $\rightarrow$ mixing

large mass $\rightarrow$ small mixing

small mass $\rightarrow$ large mixing

quarks $\rightarrow$ small mixing

leptons $\rightarrow$ large mixing
(neutrinos)
Motivation to considering extra dimensions
Mysteries of the Standard Model
General features of extra dimensions
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Point interactions
Dynamical generation of fermion mass hierarchy
Summary
We want to find an extra-dimensional model which realizes the ideas discussed so far!
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We propose an extra-dimensional model such that

5-dimensional gauge theory on an interval with *point interactions*.

\[
\begin{array}{c}
\begin{pmatrix}
U(x, y) & D(x, y) \\
U'(x, y) & D'(x, y)
\end{pmatrix} & \begin{pmatrix}
N(x, y) \\
N'(x, y)
\end{pmatrix} & \Phi(x, y)
\end{array}
\]

quarks  leptons  Higgs
We want to find an extra-dimensional model which realizes the ideas discussed so far!

We propose an extra-dimensional model such that

5-dimensional gauge theory on an interval with point interactions.

\[
\begin{pmatrix}
U(x, y) \\
D(x, y)
\end{pmatrix}
\]
\[
\begin{pmatrix}
U'(x, y) \\
D'(x, y)
\end{pmatrix}
\]
\[
\begin{pmatrix}
N(x, y) \\
E(x, y)
\end{pmatrix}
\]
\[
\begin{pmatrix}
N'(x, y) \\
E'(x, y)
\end{pmatrix}
\]

\[\Phi(x, y)\]

We prepare only one generation of quarks & leptons!
Point interactions

- delta-function potential

\[ \text{can be regarded as a point interaction} \]
Point interactions

- delta-function potential
  - can be regarded as a point interaction

- infinite square well
  - $V = \infty$
Point interactions

- delta-function potential
  - can be regarded as a point interaction

- infinite square well
  - Dirichlet b.c. $\psi(0) = \psi(L) = 0$
Point interactions

- delta-function potential
  - can be regarded as a point interaction

- infinite square well
  - Dirichlet b.c. \( \psi(0) = \psi(L) = 0 \)
  - can be regarded as point interactions
Point interactions

- fixed points on orbifolds

\[ S^1/\mathbb{Z}_2 \]
Point interactions

- fixed points on orbifolds

\[ S^1/Z_2 = \text{interval} \]

fixed point

fixed point
Point interactions

- fixed points on orbifolds

\[ \frac{S^1}{\mathbb{Z}_2} = \text{interval} \]

can be regarded as point interactions
Point interactions

- fixed points on orbifolds

\[ S^1/Z_2 \]

- zero thickness brane

- interval
  - fixed point
  - can be regarded as point interactions

- fixed point
Point interactions

- Fixed points on orbifolds
  
  \[ S^1/\mathbb{Z}_2 = \]

- Zero thickness brane
  
  The zero thickness brane may be regarded as a point interaction in field theory!
5d scalar on an interval
5d scalar on an interval

most general b.c. (Robin b.c.)

\[ \Phi'(0) + L_+ \Phi(0) = 0 \quad \Phi'(L) - L_- \Phi(L) = 0 \]
5d scalar on an interval

Interesting observations are that
★ non-vanishing vacuum expectation value $\langle \Phi(y) \rangle$ can occur even for $M^2 > 0$!
★ $\langle \Phi(y) \rangle$ can depend on $y$!

$\Phi'(0) + L_+ \Phi(0) = 0$
$\Phi'(L) - L_- \Phi(L) = 0$

most general b.c. (Robin b.c.)

Y. Fujimoto, T. Nagasawa, S. Ohya, M.S., PTP 126 (2011) 841
$5d$ spinor on an interval

\[ S = \int d^4x \int_0^L d\,y \, \bar{\Psi}(y)(i\Gamma^\mu \partial_\mu + i\Gamma^y \partial_y - M)\Psi(y) \]
5d spinor on an interval

\[ S = \int d^4x \int_0^L dy \bar{\Psi}(y) (i \Gamma^\mu \partial_\mu + i \Gamma^y \partial_y - M) \Psi(y) \]

The action principle \( \delta S = 0 \) gives

\[ \begin{align*}
\text{eq. of motion} \\
\text{b.c. } \bar{\Psi}_R(y) \Psi_L(y) = 0 \quad \text{at } y = 0, L
\end{align*} \]
5d spinor on an interval

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\[ \begin{align*}
\text{eq. of motion} & \Rightarrow \\
\text{b.c.} & \bar{\Psi}_R(y) \Psi_L(y) = 0 \quad \text{at } y = 0, L 
\end{align*} \]

boundary conditions

\[ \Psi_R(y) = 0 \quad \text{or} \quad \Psi_L(y) = 0 \quad \text{at } y = 0, L \]
boundary conditions

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5d spinor on an interval

**Boundary Conditions**

\[ \Psi_R(y) = 0 \text{ or } \Psi_L(y) = 0 \text{ at } y = 0, L \]

\[ \Psi_L(0) = \Psi_L(L) = 0 \]

\[ \Psi_R(0) = \Psi_R(L) = 0 \]

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5d spinor on an interval

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bulk mass
5d spinor on an interval

boundary conditions

\[ \Psi_R(y) = 0 \quad \text{or} \quad \Psi_L(y) = 0 \quad \text{at} \quad y = 0, L \]

- \( \Psi_L(0) = \Psi_L(L) = 0 \)
- \( \Psi_R(0) = \Psi_R(L) = 0 \)
- \( \Psi_R(0) = \Psi_L(L) = 0 \)

**good news**

- *a chiral fermion*
- *localization*
5d spinor on an interval

boundary conditions

\[ \Psi_R(y) = 0 \text{ or } \Psi_L(y) = 0 \text{ at } y = 0, L \]

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good news
- a chiral fermion
- localization

bad news
- one generation
To realize *three generations*, we introduce point interactions, which are specified by BC’s.

\[
\Psi_L(0) = 0 \quad \Psi_L(L_1) = 0 \quad \Psi_L(L_2) = 0 \quad \Psi_L(L) = 0
\]
To realize *three generations*, we introduce point interactions, which are specified by BC’s.

\[ \Psi(x, y) = \sum_{i=1}^{3} \psi^{(0)}_{R,i}(x) f_{0,i}(y) + \text{(massive modes)} \]
To realize *three generations*, we introduce point interactions, which are specified by BC’s.

\[ \Psi(x, y) = \sum_{i=1}^{3} \psi_{R,i}^{(0)}(x) f_{0,i}(y) + \text{(massive modes)} \]

*localized at point interactions*

*three generations*
Quark flavor structure in our model

Quark flavor mixing is small!

$\mu \ll m_c \ll m_t$
$md \ll ms \ll mb$
$mb \ll mt$

Y. Fujimoto, T. Nagasawa, K. Nishiwaki, M. S.
PTEP 2013(2013)023B07
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$m_u \ll m_c \ll m_t$
$m_d \ll m_s \ll m_b$

$m_b \ll m_t$

Y.Fujimoto, T.Nagasawa, K.Nishiwaki, M.S.
PTEP 2013(2013)023B07
Characteristic features of our model

★ The mass hierarchy is naturally realized.

★ Our model naturally produces small quark flavor mixings and large lepton flavor ones.

★ The mass matrices are severely restricted from the geometry of the extra dimension.

\[
M_{ij} = \begin{pmatrix}
m_{11} & m_{12} & 0 \\
0 & m_{22} & m_{23} \\
0 & 0 & m_{33}
\end{pmatrix}
\]

★ It is impossible to obtain observed quark masses without quark flavor mixing in our model.

★ The observed values of quark & lepton masses and mixings can be realized within 10% errors.
- Motivation to considering extra dimensions
- Mysteries of the Standard Model
- General features of extra dimensions
- Setup
- Point interactions
- Dynamical generation of fermion mass hierarchy
- Summary
### Parameters of our models

#### Quark sector

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark bulk mass</td>
<td>3</td>
</tr>
<tr>
<td># of point interactions</td>
<td>6 (+3)</td>
</tr>
<tr>
<td>size of interval</td>
<td>1</td>
</tr>
<tr>
<td>Higgs parameters</td>
<td>2</td>
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The number of the physical parameters (=10) is less than our input parameters, although it does not mean, due to the geometrical restriction, that our model could reproduce any values of physical observables.
We have determined the positions of the point interactions to reproduce the observed values.
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Can the positions of the point interactions be determined dynamically?
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Can the positions of the point interactions be determined dynamically?

Yes!
We have determined the positions of the point interactions to reproduce the observed values.

Can the positions of the point interactions be determined dynamically?

Yes!

They can be determined by minimizing the vacuum energy (= Casimir energy), which is a function of the positions of the point interactions!
A preliminary result

A 5d fermion on an interval with 2 point interactions

\[ \Psi(x, y) \]

0 \quad L_1 \quad y \quad L_2 \quad L
A preliminary result

A 5d fermion on an interval with 2 point interactions

\[ \Psi(x, y) \]

Minimizing the Casimir energy \( E_0(L_1, L_2) \)
A preliminary result

A 5d fermion on an interval with 2 point interactions

\[ \Psi(x, y) \]

Minimizing the Casimir energy \( E_0(L_1, L_2) \)

The regular intervals are a vacuum configuration!
A preliminary result

A 5d fermion on an interval with 2 point interactions

\[ \Psi(x, y) \]

Minimizing the Casimir energy \( E_0(L_1, L_2) \)

The regular intervals are a vacuum configuration!

\[ \langle \Phi(y) \rangle \propto e^{\alpha y} \]
A preliminary result

Minimizing the Casimir energy $E_0(L_1, L_2)$

The regular intervals are a vacuum configuration!

The exponential mass hierarchy can be dynamically generated!
A problem

If the positions of the point interactions are exactly regular intervals, then there are no flavor mixings!

We need some discrepancy from the uniform distribution.

We should analyze our model in more realistic situation.

↓

work in progress!
We have shown that the quark & lepton flavor structure can naturally be explained from the geometry of an extra dimension with point interactions.
Numerical results

- **Quark sector**
  
  \[ m_{up} = 2.5 \text{ MeV} \]
  \[ m_{charm} = 1.34 \text{ GeV} \]
  \[ m_{top} = 173 \text{ GeV} \]
  \[ m_{down} = 4.8 \text{ MeV} \]
  \[ m_{strange} = 104 \text{ MeV} \]
  \[ m_{bottom} = 4.18 \text{ GeV} \]

\[
|V_{CKM}| = \begin{pmatrix}
0.971 & 0.238 & 0.00377 \\
0.237 & 0.971 & 0.0403 \\
0.00887 & 0.0395 & 0.999
\end{pmatrix}
\]

\[ J_{\text{quark}} = 3.23 \times 10^{-5} \]
Numerical results

- **Lepton sector**
  - $m_{\nu_1} = 0.0092 \text{ eV}$
  - $m_{\nu_2} = 0.013 \text{ eV}$
  - $m_{\nu_3} = 0.018 \text{ eV}$
  - $m_{\text{electron}} = 0.519 \text{ MeV}$
  - $m_{\mu \text{on}} = 106 \text{ MeV}$
  - $m_{\tau \text{au}} = 1.778 \text{ GeV}$
  - $\sin^2 \theta_{12} = 0.333$
  - $\sin^2 \theta_{23} = 0.435$
  - $\sin^2 \theta_{13} = 0.0239$
  - $J_{\text{lepton}} = 0.0214 \ (\sin \delta = 0.607)$
Neutrino masses

How can tiny neutrino masses be generated in our model?

Answer from our point of view
Neutrino masses

How can tiny neutrino masses be generated in our model?

Answer from our point of view

Large bulk neutron mass can generate tiny mass!
Neutrino masses

How can tiny neutrino masses be generated in our model?

Answer from our point of view

Large bulk neutrino mass can generate tiny mass!

\[ f_0(y) \quad g_0(y) \]

\[ \propto e^{-M_\nu y} \]

\[ M_\nu \rightarrow \text{large} \]

large overlapping

\[ f_0(y) \quad g_0(y) \]

\[ \propto e^{-M_\nu y} \]

tiny overlapping

large bulk neutrino mass \iff tiny neutrino mass
What is the origin of the CP phases in our model?

Answer from our point of view
What is the origin of the CP phases in our model?

Answer from our point of view

one scalar model $\rightarrow$ no source of CP phases
What is the origin of the CP phases in our model?

**Answer from our point of view**

- **one scalar model** $\rightarrow$ no source of CP phases
- **two scalar model** $\Rightarrow$

Robin b.c.

\[ \Phi(x, y) \rightarrow \langle \Phi(y) \rangle \propto e^{\alpha y} \]

\[ H(x, y) \rightarrow \langle H(y) \rangle = \nu e^{i\theta y/L} \]

Twisted b.c.: $H(y + L) = e^{i\theta} H(y)$
What is the origin of the CP phases in our model?

**Answer from our point of view**

One scalar model $\rightarrow$ no source of CP phases

Two scalar model $\iff$ Robin b.c.

\[
\Phi(x, y) \quad \rightarrow \quad \langle \Phi(y) \rangle \propto e^{\alpha y}
\]

\[
H(x, y) \quad \rightarrow \quad \langle H(y) \rangle = \nu e^{i\theta y/L}
\]

Twisted b.c.: $H(y + L) = e^{i\theta} H(y)$

The origin of CP phases of both quark and lepton sectors!
What is the origin of the CP phases in our model?

**Answer from our point of view**

One scalar model $\rightarrow$ no source of CP phases

Two scalar model $\rightarrow$ good news

Robin b.c.

$\Phi(x, y) \rightarrow \langle \Phi(y) \rangle \propto e^{\alpha y}$

$H(x, y) \rightarrow \langle H(y) \rangle = v e^{i \theta y / L}$

Twisted b.c.: $H(y + L) = e^{i \theta} H(y)$

**Good news**

We have found that the parameter fitting becomes better with the CP phase from 20% to 10%!
Profiles on an extra dimension

\[ 0 \quad L_1 \quad y \quad L_2 \quad L \]

\[ \Psi_L = 0 \quad \Psi_L = 0 \quad \Psi_L = 0 \quad \Psi_L = 0 \]
Profiles on an extra dimension

\[ \Psi(x,y) = \sum_{i=1}^{3} \psi^{(i)}_{R,0}(x) f^{(i)}_{0}(y) + \text{(massive modes)} \]

ゼロモード解: \((\partial_y - M) f_{0}(y) = 0 \quad (0 < y < L)\)

\[ f_{0}(y) \]

\[ f^{(1)}_{0}(y) \]

\[ f^{(2)}_{0}(y) \]

\[ f^{(3)}_{0}(y) \]

\[ M > 0 \]
Profiles on an extra dimension

\[ \Psi_L(x,y) = \sum_{i=1}^{3} \psi^{(i)}_{R,0}(x) f^{(i)}_0(y) + \text{(massive modes)} \]

ゼロモード解： \((\alpha - M) f_0(y) = 0 \quad (0 < y < L)\)
Profiles on an extra dimension

\[ \Psi(x,y) = \sum_{i=1}^{3} \psi^{(i)}_{R,0}(x) f^{(i)}_0(y) + \text{(massive modes)} \]

ゼロモード解: \((\partial_y - M) f_0(y) = 0 \quad (0 < y < L)\)
Profiles on an extra dimension

\[ \Psi(x,y) = \sum_{i=1}^{3} \psi^{(i)}_{R,0}(x) f^{(i)}_0(y) + \text{(massive modes)} \]

Zero mode solution: \((\partial_y - M)f_0(y) = 0 \quad (0 < y < L)\)
Profiles on an extra dimension

\[ \Psi(x,y) = \sum_{i=1}^{3} \psi_L^{(i)}(x) g^{(i)}_0(y) + \text{(massive modes)} \]

ゼロモード解: \((-\partial_y - M) g_0(y) = 0 \quad (0 < y < L)\)

\[ M > 0 \]

- \[ g^{(1)}_0(y) \]
- \[ g^{(2)}_0(y) \]
- \[ g^{(3)}_0(y) \]