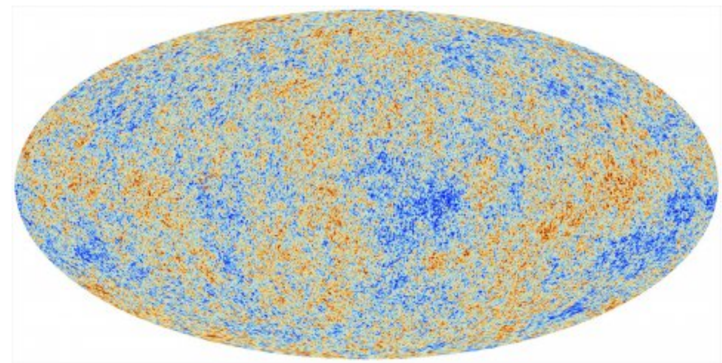
Revisiting primordial fluctuations in the curvaton scenario



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3rd, June, 2015 @Kobe University

Ref. arXiv: 1407.5148 JCAP10(2014)032
and in preparation

Contents

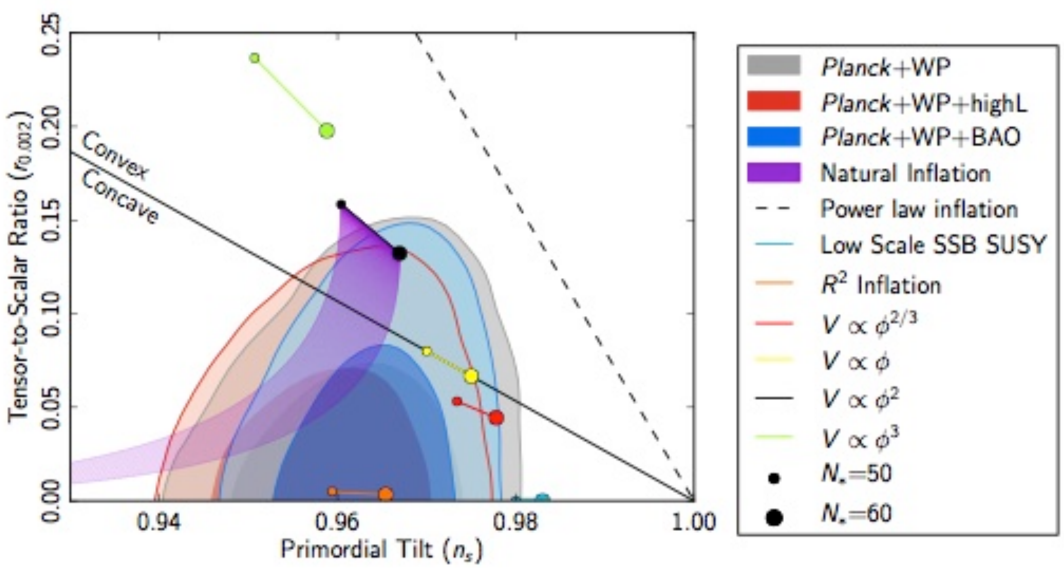
- Introduction - curvaton scenario -
- Curvature perturbations with temperature-dependent decay rate
- CDM isocurvature perturbations in curvaton
- Summary

Introduction I

- Planck data release in March 2013 & 2015

➔ Precise information about the physics of the early Universe

e.g., constraint on single-field inflation models



arXiv:1303.5082

constraint on non-Gaussian models

SMICA	KSW	Binned	Modal
Local	**2.7** ± **5.8**	2.2 ± 5.9	1.6 ± 6.0
Equilateral	**−42** ± **75**	−25 ± 73	−20 ± 77
Orthogonal	**−25** ± **39**	−17 ± 41	−14 ± 42

Flattened model (Eq. number)	Raw f_{NL}	Clean f_{NL}	Δf_{NL}
Flat model (13)	70	37	77
Non-Bunch-Davies (NBD)	178	155	78
Single-field NBD1 flattened (14)	31	19	13
Single-field NBD2 squeezed (14)	0.8	0.2	0.4
Non-canonical NBD3 (15)	13	9.6	9.7
Vector model $L = 1$ (19)	−18	−4.6	47
Vector model $L = 2$ (19)	2.8	−0.4	2.9

arXiv:1303.5084

make us discuss a variety of models in more detail..

Introduction II

- Primordial non-Gaussianity (local type)

Komatsu and Spergel (2001), and a lots of ref.

$$\Phi = \Phi_{\rm G} + f_{\rm NL}^{\rm local}\left(\Phi_{\rm G}^2 - \langle \Phi_{\rm G}^2 \rangle\right) + \cdots$$

$$\boxed{f_{\rm NL}^{\rm local} = 2.7 \pm 11.6}$$ 95% CL

small local-type non-Gaussianity is consistent with single inflation.

$f_{\rm NL}^{\rm local} \ll 1$

But, of course, the models which could predict the large nG are still viable and Planck result just gives tight constraint on the parameters in such models.

Curvaton, modulated reheating, and so on.

Introduction II

- Primordial non-Gaussianity (local type)

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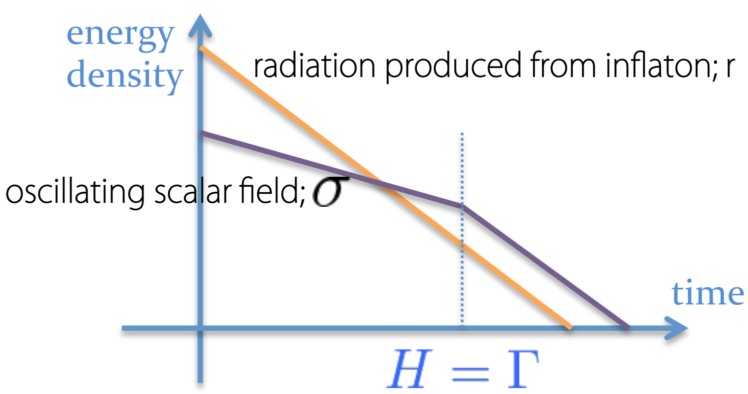
Introduction III

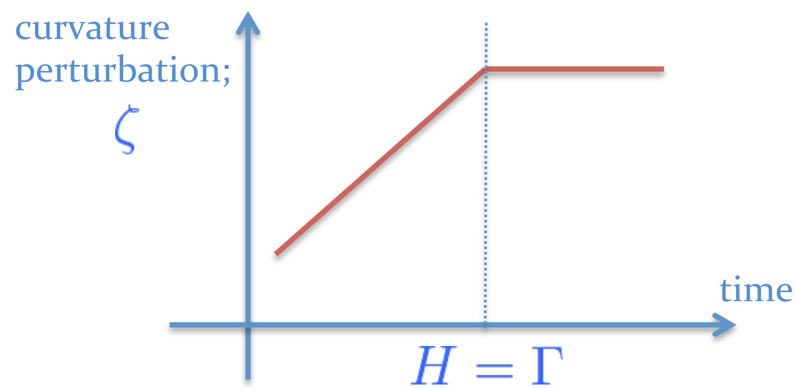
- Curvaton scenario

Lyth and Wands, Moroi and Takahashi,
Enqvist and Sloth (2002)

A mechanism of generating primordial adiabatic curvature perturbations through the decay of a scalar field (curvaton) other than inflaton



background dynamics



(or $\rho_r = \rho_\sigma$)

Γ ; decay rate of curvaton
(constant in time in simple standard case)

$$r_{\rm dec} := \left. \frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r} \right|_{H=\Gamma}$$

m; mass of curvaton
(for quadratic oscillation)

$$\boxed{\zeta = \left. \frac{r_{\rm dec}}{3} \frac{\delta\rho_\sigma}{\rho_\sigma} \right|_{H=m}}$$

neglecting the contribution from the inflaton fluctuations

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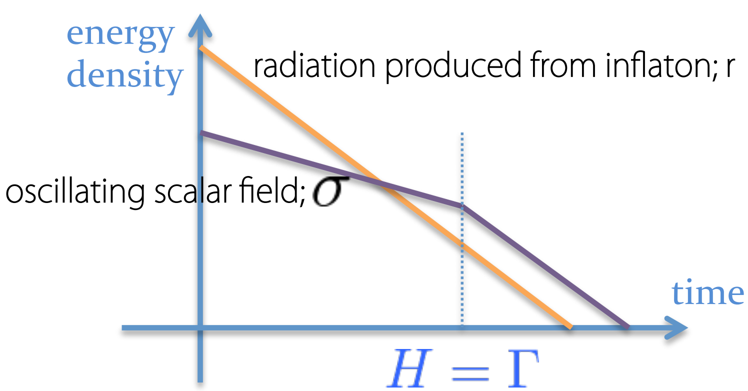

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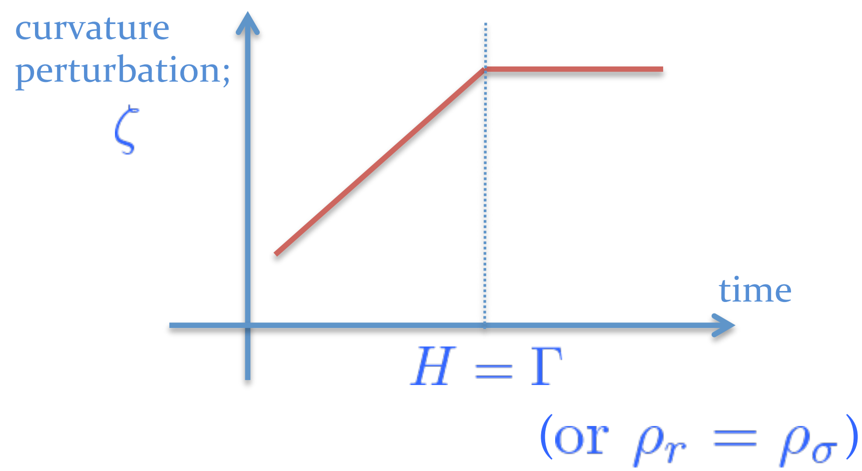


non-Gaussianity?

$$f_{\rm NL} = \frac{5}{4r_{\rm dec}} - \frac{5}{3} - \frac{5r_{\rm dec}}{6}$$

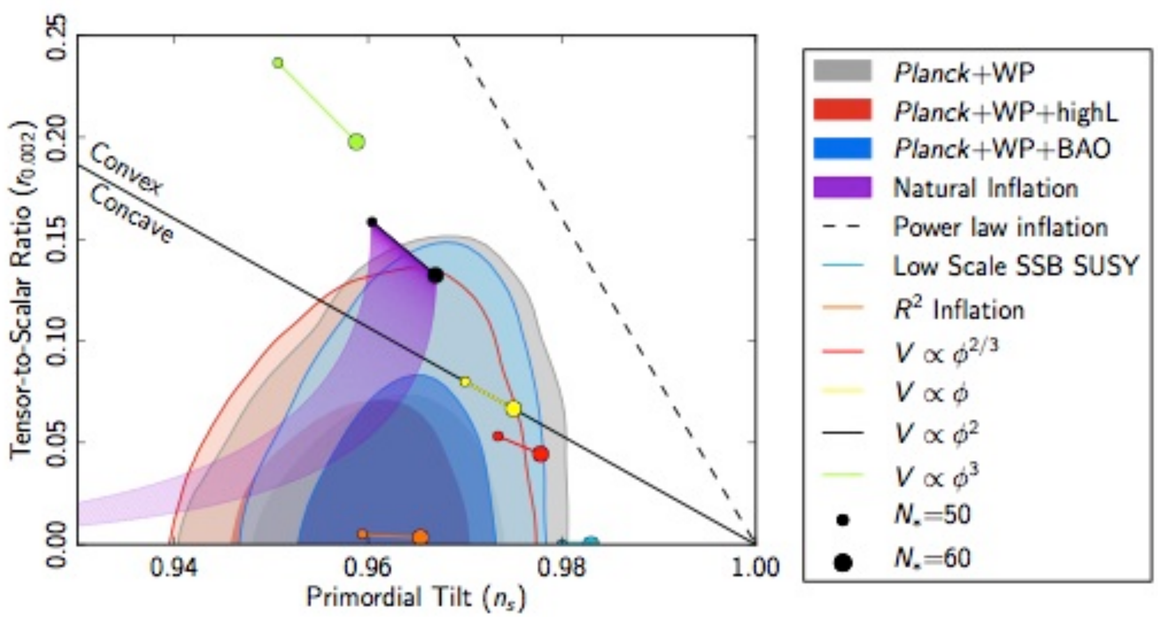
Planck constraint;

$$0.15 < r_{\rm dec} \quad \text{95\% CL}$$

Curvaton should decay after it becomes relatively dominant component in the Universe.

Introduction IV

A motivation of the curvaton scenario..



Ref. Kobayashi et al. (2013), .

ϕ^4 -inflation + curvaton $(m_\sigma^2/H_{\rm inf}^2 \ll 1)$

$n_s - 1 = 2\epsilon \sim 0.96$ & $r < 0.1$ can be realized

favored ??

Introduction V

There are several works which discuss the curvaton scenarios in more detail, in order to construct realistic curvaton scenario.

Potential ?

Self-interacting curvaton,
Hill-top curvaton, ...

Enqvist and Nurmi (2005),
Enqvist et al.(2009, 2010),
Enqvist, Lerner and Taanila (2011)...

Kawasaki et al. (2009),
Kawasaki et al.(2011,2013), ...

Initial condition ?

Stochastic approach,
Attractor behavior, ...

Demozzi et al.(2010), Enqvist et al. (2012),
Nurmi et al.(2013), Lerner and Melville (2014), ...

Decay process ?

Resonant decay, ..

Enqvist, Figueroa and Lerner (2012),
Enqvist et al.(2013), ...

Introduction VI

Among such detailed discussions, we focus on the fact that the curvaton lives in thermal bath.

We would like to revisit the primordial fluctuations;

- Curvature perturbations with temperature-dependent decay rate
- CDM isocurvature perturbations in curvaton scenario

By using sudden decay approximation and also numerical calculation

Curvature perturbations in the curvaton scenario with temperature-dependent decay rate

arXiv: 1407.5148

Curvaton in thermal bath

- Thermal effects?

curvaton; oscillating scalar field in the radiation dominated Universe

➔ It is expected that some thermal effects should exist.

e.g.,

$$\mathcal{L}_{\rm int} = -y\sigma\bar{\psi}\psi - gA_\mu\bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L}_{\rm int} = -M\sigma\chi^2 - \lambda\chi\xi^2,$$

For background dynamics of the curvaton decay,

e.g., temperature-dependent mass/decay rate

- modulation of the evolution of the curvaton energy density $\rho_\sigma \propto a^{-3}$?
- life time of the curvaton (related to the decay rate)

There are several works
about the dynamics of oscillating scalar field in thermal bath;

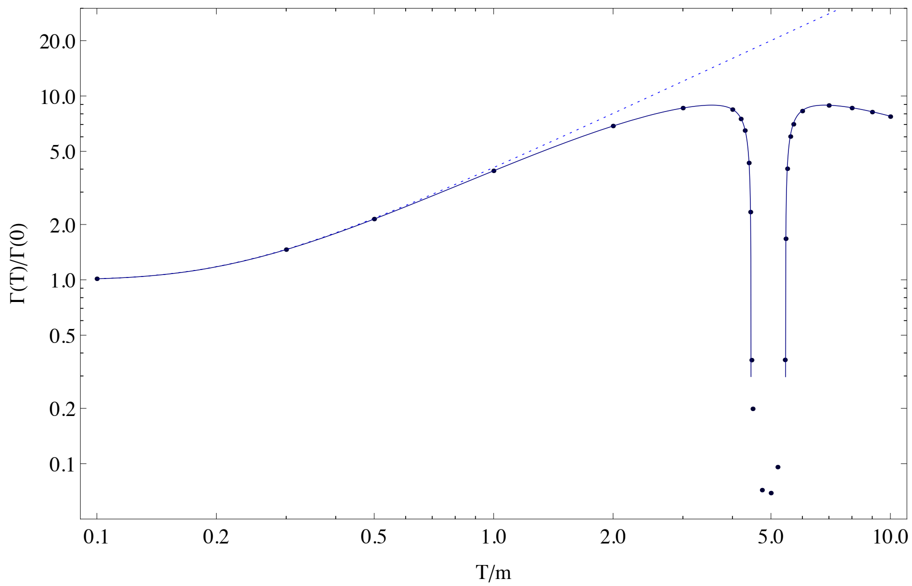
Parwani (1992), J. Yokoyama (2004,2005), Enqvist et al. (2011), Mukaida and Nakayama (2012),
Drewes and Kang (2013), Mukaida, Nakayama and Takimoto (2013,2014), Enqvist, Lerner and Takahashi (2013)....

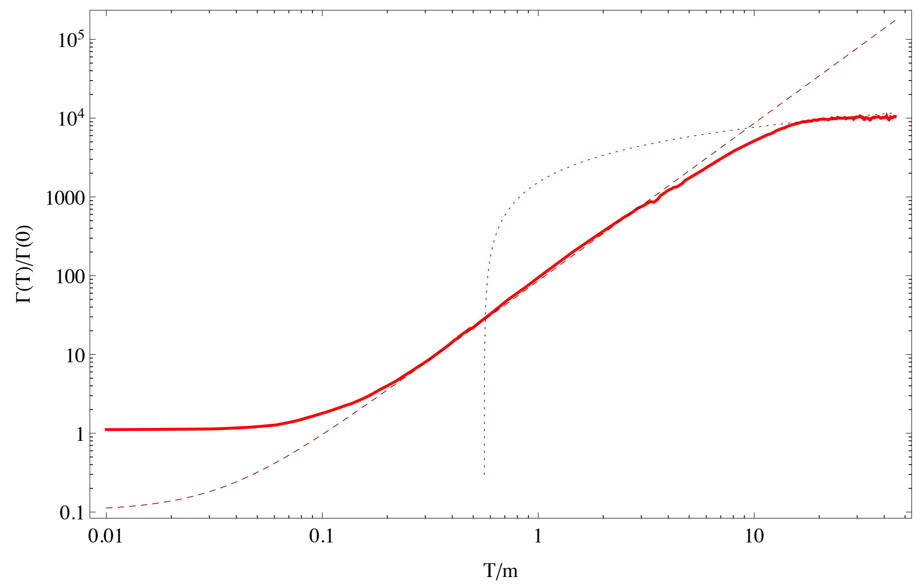
We focus on the effect of the temperature dependent decay rate on the primordial curvature fluctuations in the curvaton scenario.

Temperature dependent decay rate

Through the dissipation, thermal mass blocking, …

Drewes and Kang, 1305.0267





$$\mathcal{L} \quad = -g\phi\chi_1\chi_2$$

$$\mathcal{L} \quad = -\sum_{i=1}^{2}\frac{h_i}{4!}\phi\chi_i^3$$

Here, ϕ is curvaton, χ; fields in thermal bath

Temperature dependence seems to be very model-dependent..

Simple toy model

$$\zeta = \frac{r_{\rm dec}}{3}\frac{\delta\sigma_*}{\sigma_*}$$

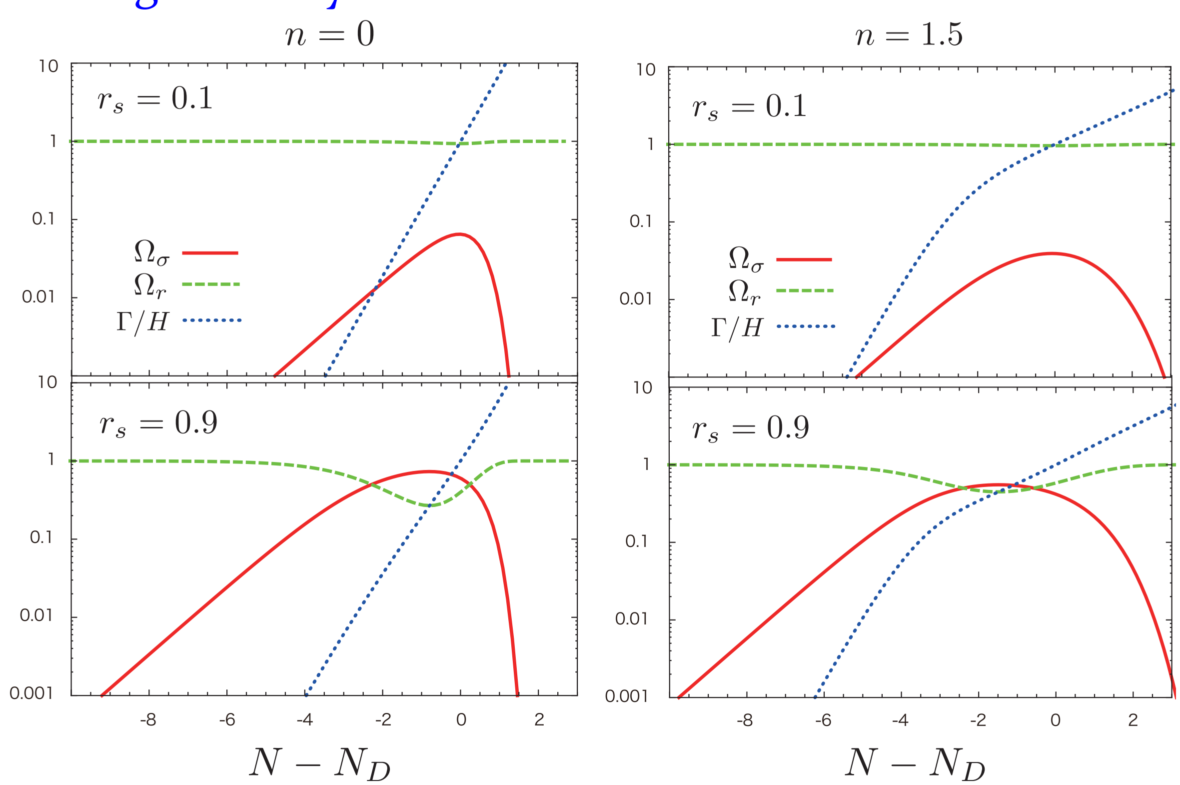
- Temperature dependent decay rate

$$\Gamma(T) = \Gamma_0\left[A + \frac{C\,(T/m_\sigma)^n}{1 + C\,(T/m_\sigma)^n}\right]$$

A,C; constant parameters

m_σ ;curvaton mass

Background dynamics



$$m_\sigma = 10^{-16}M_{\rm P}$$

$$\Gamma_0 = 10^{-12}m_\sigma$$

for the case with $n=0$,

$A=1$ and $C=0$

for the case with $n=1.5$,

$A=10^{-5}$ and $C=10^{-3}$

$$r_s \leftrightarrow r_{\rm dec}$$

N; e-folding number
as a time coordinate

N_D ; time when $\Gamma = H$

- Let us consider the primordial curvature perturbation in case with such temperature-dependent decay rate.

- First we consider the analytic estimation by using sudden decay approximation.

Enhancement of primordial adiabatic fluctuations? I

- Additional curvature perturbations ??

In case with the temperature-dependent decay rate,

decay hypersurface $H = \Gamma$ $\neq$ constant Hubble hypersurface $H = \mathrm{const.}$

due to the existence of the iso-curvature fluctuations

Cf. modulated reheating

➔ Additional primordial fluctuations from the fluctuations of the decay hypersurface??

cf. In the reheating era (inflaton decay), because of no iso-curvature fluctuation, the enhancement does not occur..

ref. Armend´ariz-Pic´on; astro-ph/0312389, Weinberg; astro-ph/0401313, 0405397

Delta N formalism

Starobinsky (1985), Sasaki and Stewart (1996), Sasaki and Tanaka (1998), ...

Curvature perturbation can be related with the fluctuation of the e-folding number measured between initial time (flat hypersurface) and final time (uniform energy density);

Naively,

$$ds^2 = -dt^2 + \underline{a^2 e^{2\zeta}} \delta_{ij} dx^i dx^j$$

$$\Updownarrow$$

$$e^{2(N+\delta N)}$$

$$N(t_F, t_*) \equiv \int_{t_*}^{t_F} H dt$$

; e-folding number

$$\boxed{\zeta_i = \delta N + \frac{1}{3(1+w_i)} \ln\left[1 + \frac{\delta\rho_i}{\bar{\rho}_i}\right]}$$

$$w_i \equiv p_i/\rho_i$$

; equation of state of i-

``bar''; background quantity

Curvature perturbation on uniform energy density of the i-component hypersurface

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$$N(t_F, t_*) \equiv \int_{t_*}^{t_F} H dt$$

; e-folding number

$$\boxed{\bar{\rho}_i + \delta\rho_i = \bar{\rho}_i e^{3(1+w_i)(\zeta_i - \delta N)}}$$

$$w_i \equiv p_i/\rho_i$$

; equation of state of i-

``bar"; background quantity

Energy density (including fluctuation component) of i-component

sudden decay approximation I

In the sudden decay approximation,
the curvaton instantaneously decays into radiation when H=Γ.

$$\bar{\rho}_r e^{4(\zeta_r - \delta N)} + \bar{\rho}_\sigma e^{3(\zeta_\sigma - \delta N)} = \rho_{\rm total}$$; just before decay

$$\bar{\rho}_{r({\rm total})} e^{4(\zeta_{r({\rm total})} - \delta N)} = \rho_{\rm total}$$; just after decay

$$\rho_{\rm total} = 3M_{\rm Pl}^2 H^2 = 3M_{\rm Pl}^2 \Gamma^2$$; at the decay

``bar” means the background quantity

Introducing curvaton iso-curvature fluctuation as

$$\mathcal{S} = 3(\zeta_\sigma - \zeta_r)$$; const. in time before curvaton decay in sudden decay approx.

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Introducing curvaton iso-curvature fluctuation as

$$\mathcal{S} = 3(\zeta_\sigma - \zeta_r)$$; const. in time before curvaton decay in sudden decay approx.

$$= \mathcal{S}_i = 3(\zeta_{\sigma,i} - \zeta_{\mathrm{inf}})$$ we use $\zeta_r = \zeta_{\mathrm{inf}}$ before curvaton decay

$$= \frac{2\delta\sigma_i}{\bar{\sigma}_i} - \frac{\delta\sigma_i^2}{\bar{\sigma}_i^2}$$ related to the fluctuation of the curvaton

sudden decay approximation II

- Standard case = decay/dissipation rate; Γ = constant case;

uniform total energy density hypersurface
= decay hypersufrace (H=Γ hypersurface)

- Then we have

$$\zeta_{\rm total} = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i \qquad r_{\rm dec} := \left.\frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r}\right|_{H=\Gamma}$$

and non-linearity parameter fNL is

$$f_{\rm NL} = \frac{5}{4r_{\rm dec}} - \frac{5}{3} - \frac{5}{6}r_{\rm dec}$$

Large fNL can be realized
for the small r_dec.

sudden decay approximation III

- Thermal effect ➔ decay/dissipation rate; Γ = Γ(T) can fluctuate;

  Γ depends on the temperature, i.e., only radiation component.
  On the other hand, total energy density is determined by the radiation component and curvaton component.

  ➔ decay hypersurface $H = \Gamma$ $\neq$ constant Hubble hypersurface $H = \text{const.}$

- Then we have

$$\zeta_{\text{total}} = \zeta_{\text{inf}} + \frac{r_{\text{dec}}}{3}\mathcal{S}_i - \frac{r_{\text{dec}}}{6}\frac{\delta\Gamma}{\Gamma}\bigg|_{H=\Gamma}$$

cf. modulated decay of the curvaton, with another field

Langlois and Takahashi (2013)

Assadullahi et al. (2013)
Enomoto et al. (2013)

and in our setting,

$$\frac{\delta\Gamma}{\Gamma} = \frac{\partial \ln \Gamma}{\partial \ln T} \times \frac{\delta T}{T} = \frac{\partial \ln \Gamma}{\partial \ln T} \times (\zeta_r - \delta N)$$

$T \propto \rho_r^{1/4}$; temperature

(``self-modulated" decay of the curvaton) cf. Mukaida et al. (2014)

sudden decay approximation IV

- Finally we obtain,

$$\zeta_{\rm total} = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i - \frac{r_{\rm dec}}{6}\left.\frac{\delta\Gamma}{\Gamma}\right|_{H=\Gamma}$$

$$\left.\frac{\delta\Gamma}{\Gamma}\right|_{H=\Gamma} = \frac{\partial \ln\Gamma}{\partial \ln T} \times \frac{-r_{\rm dec}/3}{1-\left(\frac{r_{\rm dec}}{6}+\frac{1}{2}\right)\frac{\partial\ln\Gamma}{\partial\ln T}} \times \mathcal{S}_i$$

and also, the non-linearity parameter is given by

$$f_{NL} = \frac{5}{4r}\left(\frac{1-\frac{1}{2}\tilde{\Gamma}'}{1-\left(\frac{r}{6}+\frac{1}{2}\right)\tilde{\Gamma}'}\right) - \frac{5}{3}\left(\frac{1-\frac{3}{4}\tilde{\Gamma}'}{1-\left(\frac{r}{6}+\frac{1}{2}\right)\tilde{\Gamma}'}\right) - \frac{5r}{6}\left(\frac{1+\frac{1}{6}\tilde{\Gamma}''-\frac{4}{3}\tilde{\Gamma}'+\frac{5}{8}\tilde{\Gamma}'^2-\frac{7}{48}\tilde{\Gamma}'^3}{\left[1-\frac{1}{2}\tilde{\Gamma}'\right]^2\left[1-\left(\frac{r}{6}+\frac{1}{2}\right)\tilde{\Gamma}'\right]}\right)$$

where $\tilde{\Gamma}' := \dfrac{\partial\ln\Gamma}{\partial\ln T}$

$$r = r_{\rm dec}$$

sudden decay approximation IV

- Finally we obtain,

$$\zeta_{\rm total} = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i - \frac{r_{\rm dec}}{6}\left.\frac{\delta\Gamma}{\Gamma}\right|_{H=\Gamma}$$

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For example, $\Gamma \propto T^n$

$$\frac{\partial\ln\Gamma}{\partial\ln T} = n$$

for $n = 1.5,\ r_{\rm dec} \to 1,$

Diverge??

where $\tilde{\Gamma}' := \dfrac{\partial\ln\Gamma}{\partial\ln T}$

$r = r_{\rm dec}$

However,...

How about in numerical calculation??

Numerical study by delta N I

- r-parameter related with entropy production rate

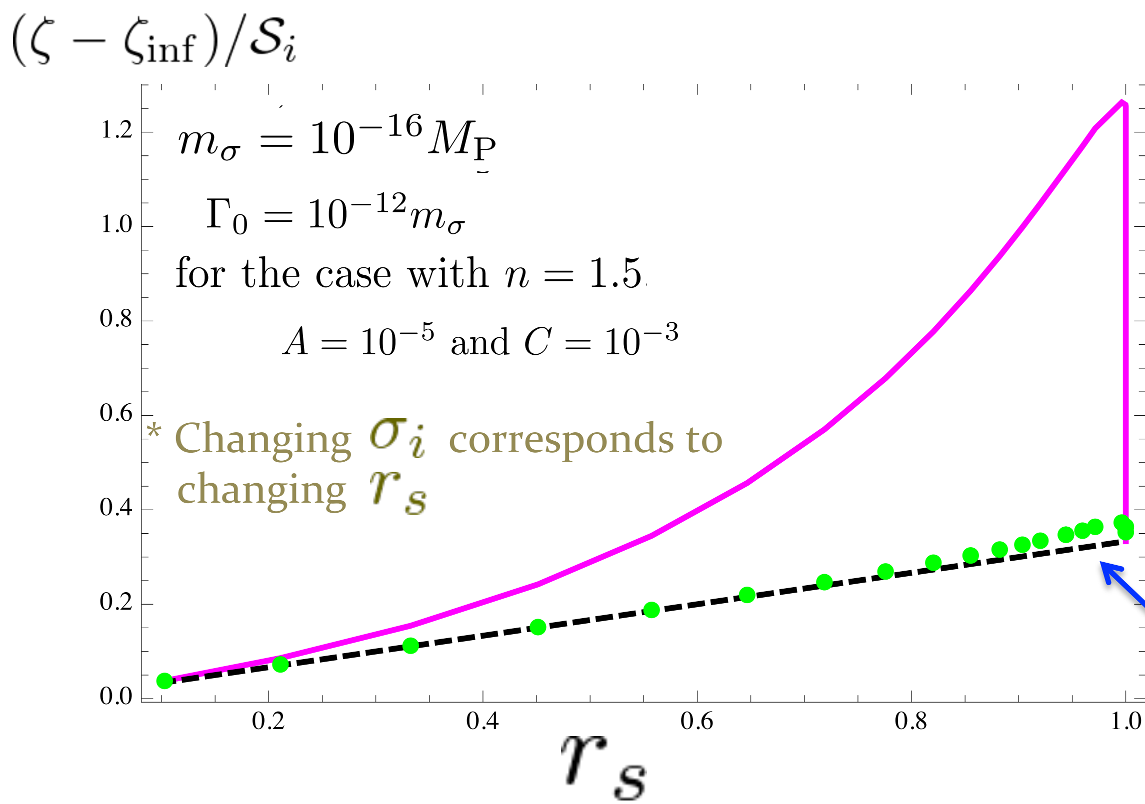
$$r_s(q) = 1 - \left(1 + \frac{3}{4}q\right)^{-1} \quad \text{with } q := \left(\frac{S_f}{S_i}\right)^{4/3} - 1$$

S_i ; initial entropy

$$\Gamma(T) = \Gamma_0 \left[A + \frac{C\,(T/m_\sigma)^n}{1 + C\,(T/m_\sigma)^n}\right]$$

$$r_s \leftrightarrow r_{\rm dec}$$

S_f ; total entropy after the complete curvaton decay



magenta; sudden decay formula

green; numerical result

black dashed; $r/3$

➔ sudden decay approximation does not work well..

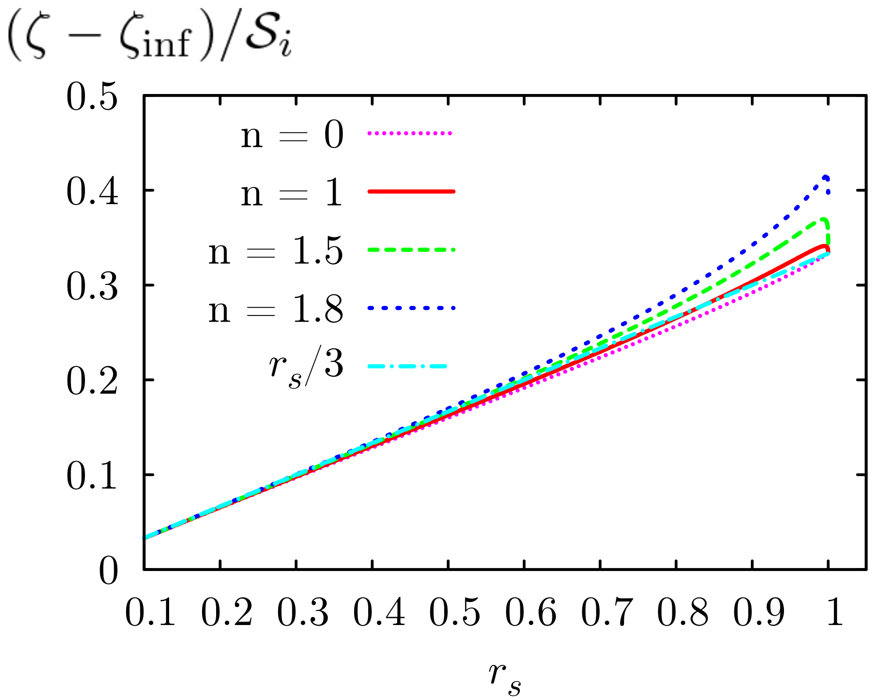
thermal effect is not so large

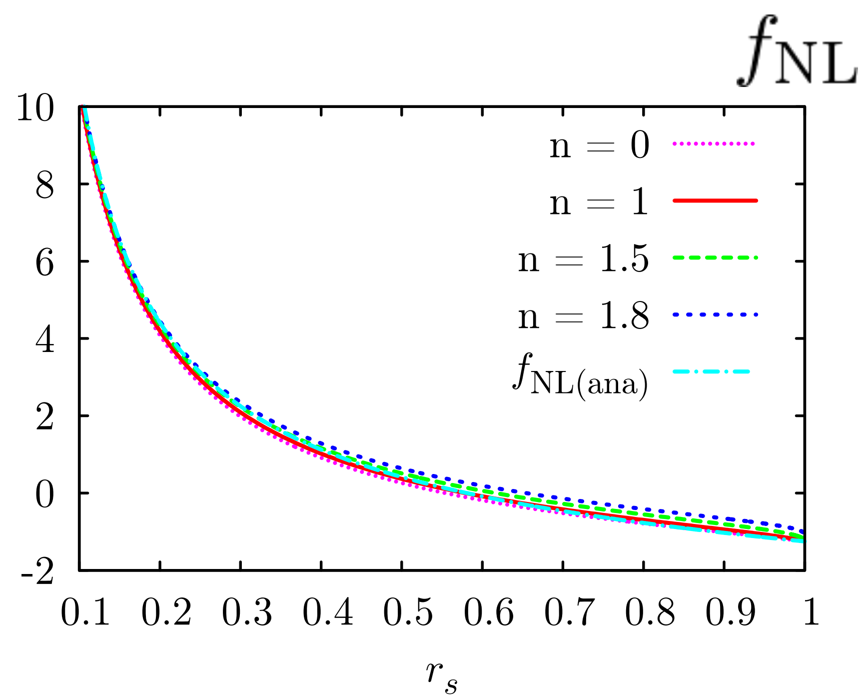
small deviation appear around r~1 ?

Numerical study by delta N II

- n-dependence

$$\Gamma(T) = \Gamma_0 \left[A + \frac{C\,(T/m_\sigma)^n}{1 + C\,(T/m_\sigma)^n} \right]$$





In sudden decay approximation, it could be possible to realize the large enhancement!
However, we cannot see such effect in the numerical calculations..

Discussion

- Validity of sudden decay approximation
- Focus on the evo. of the curvaton iso-curv.

Perturbation equation

$$\frac{d\zeta}{dN} = \mathcal{T}_\zeta \, \frac{\mathcal{S}}{3}, \qquad \frac{d\mathcal{S}}{dN} = \mathcal{T}_\mathcal{S} \, \mathcal{S},$$

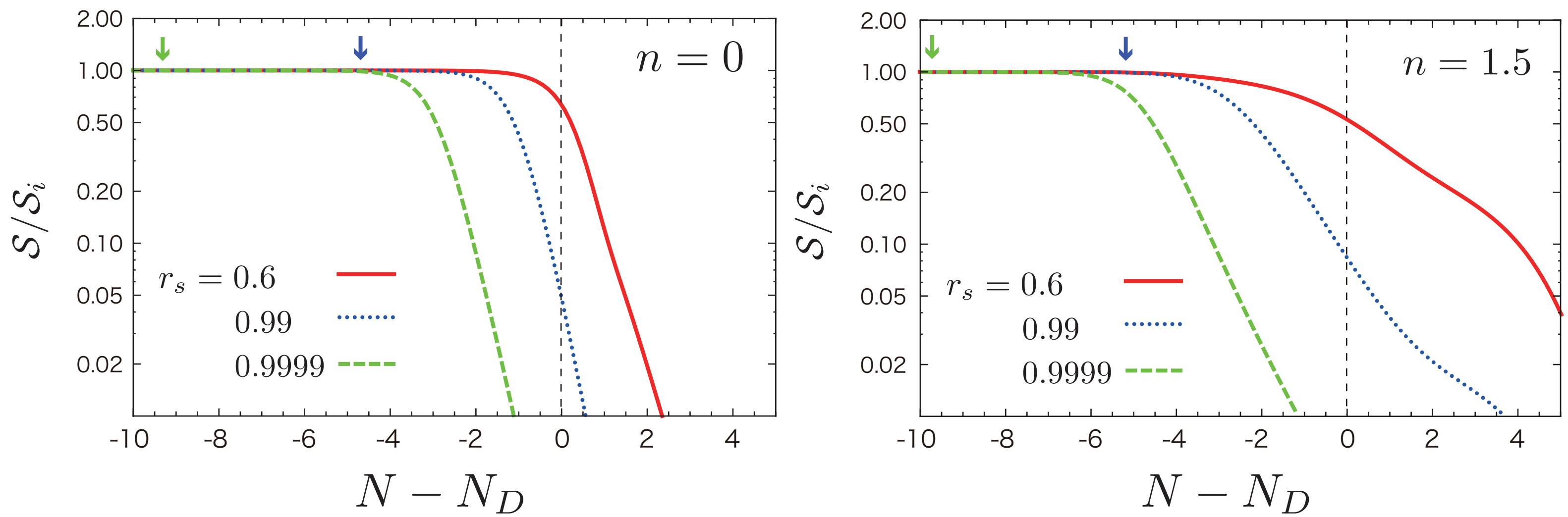
$$\begin{aligned}
\mathcal{T}_\zeta \; &\equiv \; \left(\frac{3-2g}{3(1-g)}\right)\left(\frac{4-\frac{4-3g}{1-g}\Omega_\sigma}{4-\Omega_\sigma}\right)\left(\frac{3\Omega_\sigma}{4-\Omega_\sigma}\right), \\
\mathcal{T}_\mathcal{S} \; &\equiv \; -\frac{g}{2(1-g)}\frac{4(1-g)-(4-3g)\Omega_\sigma}{3-2g} \\
&\quad \times \left[1+\left(\frac{3-2g}{4(1-g)-(4-3g)\Omega_\sigma}\right)^2 \Omega_\sigma(2-\Omega_\sigma) - \frac{\alpha}{2(1-\Omega_\sigma)}\right],
\end{aligned}$$

$$g(N) \equiv \Gamma/(\Gamma + H)$$

$$\Omega_a \; \equiv \; \bar{\rho}_a/\bar{\rho}_{\rm tot}$$

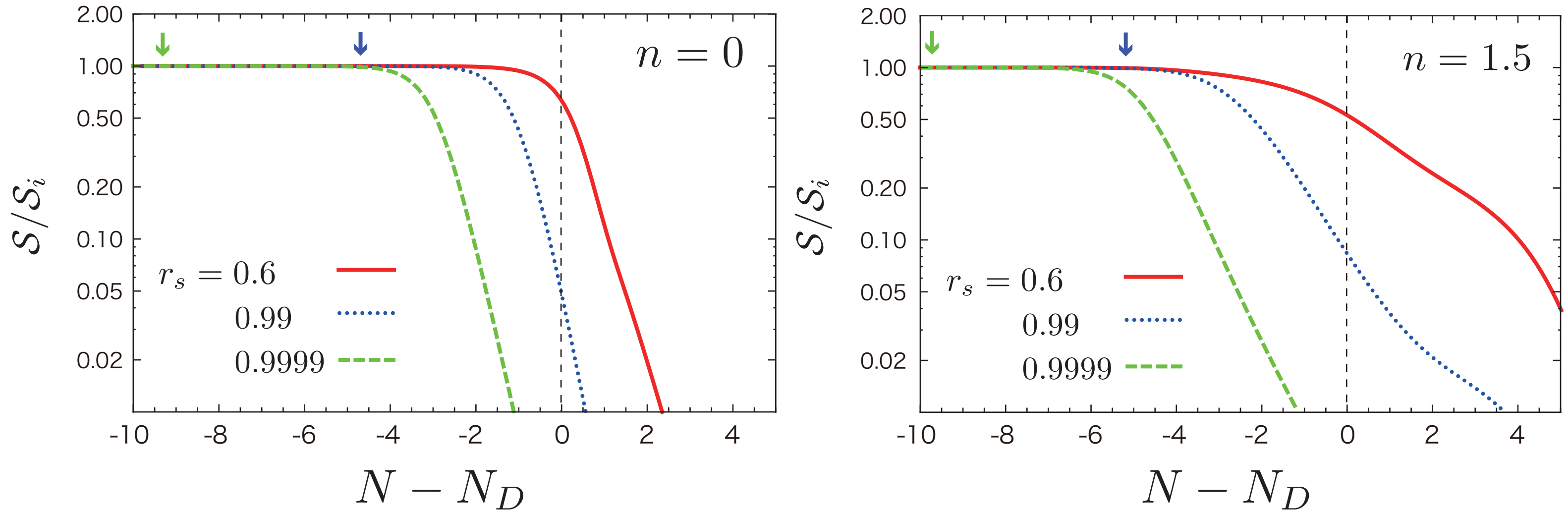
$$\boxed{\begin{aligned}
\zeta &= \zeta_{\rm inf} + \int dN \mathcal{F}(N) \, \frac{\mathcal{S}_i}{3}, \\
\mathcal{F}(N) &\equiv \mathcal{T}_\zeta(N) \exp\left[\int^N dN' \, \mathcal{T}_\mathcal{S}(N')\right] .
\end{aligned}}$$

Evolution of the iso-curvature



In curvaton scenario, for relatively large r_s case,
The iso-curvature perturbations have been already suppressed at the decay!!

Evolution of the iso-curvature



Sudden decay formula in case with temperature dependent decay rate,

$$\zeta_{\rm total} = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i - \frac{r_{\rm dec}}{6}\left.\frac{\delta\Gamma}{\Gamma}\right|_{H=\Gamma}$$

$$\left.\frac{\delta\Gamma}{\Gamma}\right|_{H=\Gamma} = \frac{\partial\ln\Gamma}{\partial\ln T} \times \frac{-r_{\rm dec}/3}{1 - \left(\frac{r_{\rm dec}}{6} + \frac{1}{2}\right)\frac{\partial\ln\Gamma}{\partial\ln T}} \times \mathcal{S}_i$$

We assumed that the isocurvature perturbations remain constant ➔ not correct!!

Summary 1

- Thermal effect really appears in primordial curvature perturbations?

By using simple sudden decay approximation, large thermal effect seems to appear.

But, in the numerical result obtained by using delta N formalism, such large effect does not appear.
➔ Sudden decay approximation is not always valid!

Temperature dependence of the decay rate seems not to give large effects in the adiabatic curvature perturbations. (but small deviations appear..)

- New questions;

Sudden decay approximation is good or not?

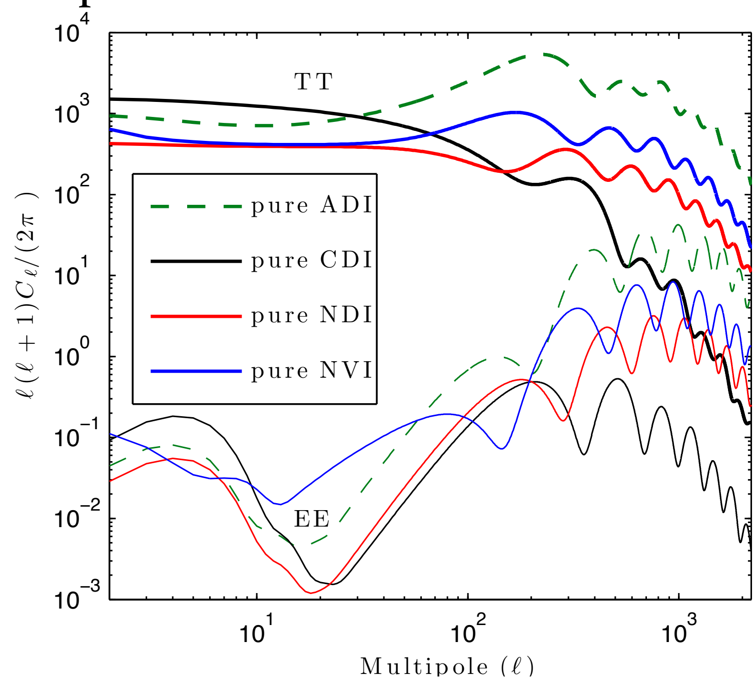
- How about the estimation for the CDM isocurvature perturbations?

Kitajima, Langlois, Tahakashi, SY in prep.

CDM isocurvature perturbations

CMB temperature anisotropies
and E-polarization

Planck 2015



$$S_{\rm CDM} \equiv 3(\zeta_{\rm CDM} - \zeta),$$

CMB constraints;

$$\beta_{\rm iso}(k) = \frac{\mathcal{P}_{\mathcal{I}\mathcal{I}}(k)}{\mathcal{P}_{\zeta\zeta}(k) + \mathcal{P}_{\mathcal{I}\mathcal{I}}(k)} \,. \qquad \mathcal{I} = \mathrm{CDM}$$

$\beta_{\rm iso} < 0.033$;uncorrelated

$\beta_{\rm iso} < 0.04$;correlated

95%CL.

Basically,

$$\boxed{S_{\rm CDM} \ll \zeta}$$

ADI; adiabatic pert.
CDI; CDM isocurvature pert.
NDI; neutrino density iso-.
NVI; neutrino velocity iso-.

CDM isocurv. in curvaton scenario I

$$S_{\rm CDM} \equiv 3(\zeta_{\rm CDM} - \zeta),$$

Final diabatic curvature perturbations in curvaton scenario;

$$\zeta = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i$$

: $\mathcal{S}_i = 3(\zeta_{\sigma,i} - \zeta_{\rm inf})$

;intrinsic curvaton fluctuations

CDM; in thermal equilibrium in the early Universe (e.g. thermal WIMP)

Naively,

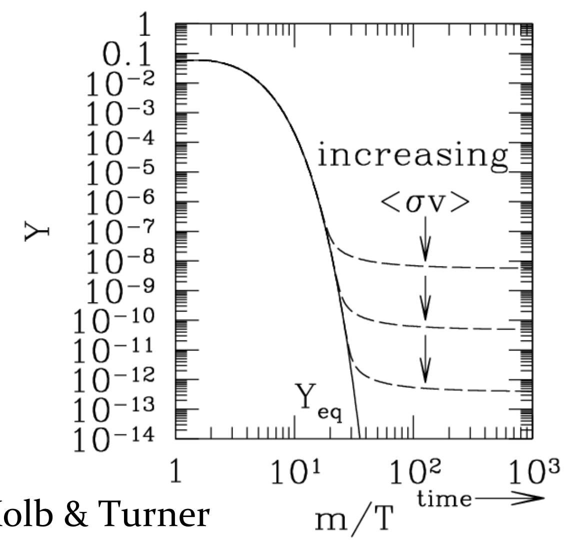
- Decoupling before curvaton decay

$$\zeta_{\rm CDM} = \zeta_{\rm inf}$$

$$S_{\rm CDM} = -3\frac{r_{\rm dec}}{3}\mathcal{S}_i \sim -3\zeta$$

ruled out !

if $\zeta_{\rm inf}$ is negligible



Ref. The early Universe, Kolb & Turner

CDM isocurv. in curvaton scenario I

$$S_{\rm CDM} \equiv 3(\zeta_{\rm CDM} - \zeta),$$

Final diabatic curvature perturbations in curvaton scenario;

$$\zeta = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i$$

$$\mathcal{S}_i = 3(\zeta_{\sigma,i} - \zeta_{\rm inf})$$

;intrinsic curvaton fluctuations

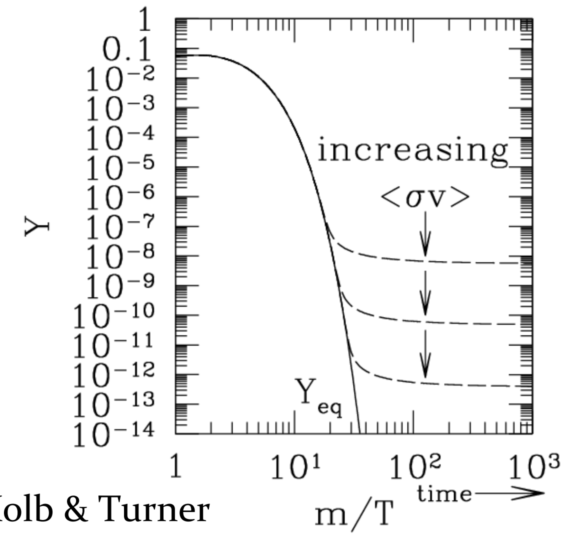
CDM; in thermal equilibrium in the early Universe (e.g. thermal WIMP)

Naively,

- Decoupling after curvaton decay

$$\zeta_{\rm CDM} = \zeta$$

$$S_{\rm CDM} = 0$$



Ref. The early Universe, Kolb & Turner

CDM isocurv. in curvaton scenario II

$$S_{\rm CDM} \equiv 3(\zeta_{\rm CDM} - \zeta),$$

Final diabatic curvature perturbations in curvaton scenario;

$$\zeta = \zeta_{\rm inf} + \frac{r_{\rm dec}}{3}\mathcal{S}_i$$

: $\mathcal{S}_i = 3(\zeta_{\sigma,i} - \zeta_{\rm inf})$

;intrinsic curvaton fluctuations

CDM; generated directly from the curvaton decay

Naively, $\zeta_{\rm CDM} = \zeta_{\sigma,i}$

→ $S_{\rm CDM} = (1 - r_{\rm dec})\mathcal{S}_i$

→ $S_{\rm CDM}/\zeta = 3\dfrac{1 - r_{\rm dec}}{r_{\rm dec}}$

if $\zeta_{\rm inf}$ is negligible

From the observational constraint,

$$0.98 < r_{\rm dec}$$

➔ $f_{\rm NL} \sim 1.2$

- Isocurvature perturbations could be a powerful tool to see the early Universe.

Other example, axion DM (Planck 2015)

and, Hayakawa, Harigaya, Kawasaki, SY (2014)
discusses a specific model,
so-called ``sneutrino curvaton scenario"

- Tight constraint obtained from current observations

➔ More precise discussion seems to be useful.

Sudden freeze out approx. I

Lyth and Wands, astro-ph/0306500, and more…

- Let us focus on the case;

CDM; in thermal equilibrium in the early Universe (e.g. thermal WIMP)

System;

$$n_{\rm CDM}^{\rm (eq)} = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

for non-rela.

Radiation; $$\frac{d\rho_r}{dt} + 4H\rho_r = \Gamma\rho_\sigma,$$

Curvaton; $$\frac{d\rho_\sigma}{dt} + 3H\rho_\sigma = -\Gamma\rho_\sigma,$$

CDM; $$\frac{dn_{\rm CDM}}{dt} + 3Hn_{\rm CDM} = -\lambda\left(n_{\rm CDM}^2 - \left(n_{\rm CDM}^{\rm (eq)}\right)^2\right)$$

Friedmann eq.; $$H^2 = \frac{1}{3M_{\rm pl}^2}\left(\rho_r + \rho_\sigma + \rho_{\rm CDM}\right) \simeq \frac{1}{3M_{\rm pl}^2}\left(\rho_r + \rho_\sigma\right),$$

$\lambda = \langle\sigma v\rangle$; thermally-averaged cross section

Sudden freeze out approx. I

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$\lambda = \langle\sigma v\rangle$; thermally-averaged cross section

Sudden freeze out approx. II

Lyth and Wands, astro-ph/0306500, and more...

- evolution of n_CDM

$$\frac{dn_{\rm CDM}}{dt}+3Hn_{\rm CDM}=-\lambda\left(n_{\rm CDM}^2-\left(n_{\rm CDM}^{\rm (eq)}\right)^2\right)$$

- Large RHS; $n_{\rm CDM}\simeq n_{\rm CDM}^{\rm (eq)}$
- Large LHS; $n_{\rm CDM}\propto a^{-3}$

Criterion;

$$H \gtrless \Gamma_{\rm CDM} := n_{\rm CDM}^{\rm (eq)}\lambda$$

$$H=\Gamma_{\rm CDM}$$

; time of freeze-out

$$n_{\rm CDM}(t)=n_{\rm CDM}^{\rm (eq)}(t_{\rm fr})$$

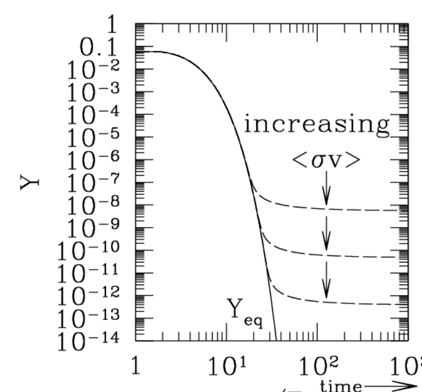
for $t>t_{\rm fr}$

; sudden freeze-out

$$n_{\rm CDM}^{\rm (eq)}=g_*\left(\frac{mT}{2\pi}\right)^{3/2}e^{-m/T}$$

$$(n_{\rm CDM}^{\rm (eq)}\propto T^3)$$

for relativistic m << T



$$Y:=\frac{n_{\rm CDM}}{s}$$

Sudden freeze out approx. III

Lyth and Wands, astro-ph/0306500,
and more…

- Sudden decay approx.

➔ Adiabatic curvature perturbations in curvaton scenario

➔ In case with temperature dependent decay rate, it seems not to be good…

- Sudden freeze-out approx.

➔ Matter isocurvature perturbations for thermal relics

$$H = \Gamma_{\rm CDM} = n^{\rm (eq)}_{\rm CDM}(T)\lambda$$

➔ Effective ``annihilation rate" depends on temperature !!
➔ Naively, by taking into account this ``modulation of freeze-out hypersurface" seriously, isocurvature perturbations would be reduced..

Sudden freeze-out is good or not?

CDM isocurv. in curvaton scenario I

$$S_{\mathrm{CDM}} \equiv 3(\zeta_{\mathrm{CDM}} - \zeta),$$

Final adiabatic curvature perturbations in curvaton scenario;

$$\zeta = \zeta_{\mathrm{inf}} + \frac{r_{\mathrm{dec}}}{3}\mathcal{S}_i$$

$$\mathcal{S}_i = 3(\zeta_{\sigma,i} - \zeta_{\mathrm{inf}})$$

;intrinsic curvaton fluctuations

CDM; in thermal equilibrium in the early Universe (e.g. thermal WIMP)

Naively,

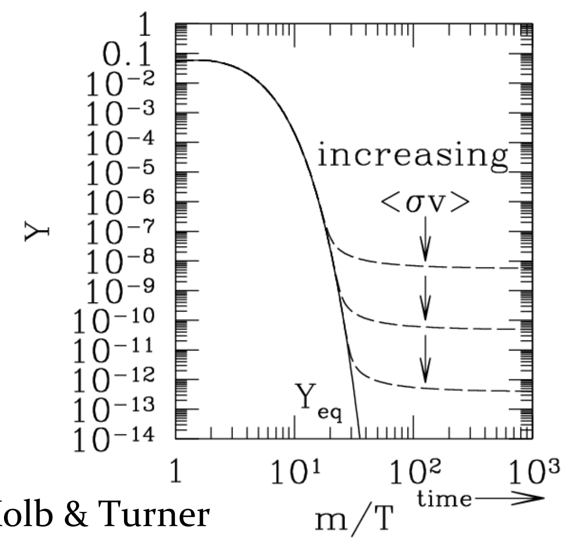
- Decoupling before curvaton decay

$$\zeta_{\mathrm{CDM}} = \zeta_{\mathrm{inf}}$$

$$S_{\mathrm{CDM}} = -3\frac{r_{\mathrm{dec}}}{3}\mathcal{S}_i \sim -3\zeta$$

ruled out !

if ζ_{inf} is negligible



Ref. The early Universe, Kolb & Turner

Precise expression

Kitajima, Langlois, Takahashi, SY in prep.

Lyth and Wands (2003)

Basically derivation is the same as in the previous discussion.

- Freeze-out before curvaton decay;

$\zeta_{\rm inf}$ is neglected

$$\boxed{S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,\rm fr}}{r_{\rm dec}}\left(\frac{m}{T_{\rm fr}}-\frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,\rm fr}-2)+\Omega_{\sigma,\rm fr}}-1\right]}$$

$$\alpha_{\Lambda,\rm fr} := \left.\frac{\partial\ln\Gamma_{\rm CDM}}{\partial\ln T}\right|_{H=\Gamma_{\rm CDM}} = \frac{m}{T_{\rm fr}}+\frac{3}{2}$$

For $\Omega_{\sigma,\rm fr} \ll r_{\rm dec}$ ➔ $S_{\rm CDM}/\zeta \simeq -3$

For $m/T_{\rm fr} \simeq 20$ ➔ $S_{\rm CDM}/\zeta \simeq 3\left(\frac{\Omega_{\sigma,\rm fr}}{2r_{\rm dec}}-1\right)$

due to the modulation of freeze-out hypersurface

(to obtain $\Omega_{\rm CDM}h^2 = 0.115$)

Not so suppressed..

$$\Omega_{\sigma,\rm fr} := \left.\frac{\rho_\sigma}{3M_{\rm Pl}^2}\right|_{H=\Gamma_{\rm CDM}}$$

- Freeze-out after curvaton decay;

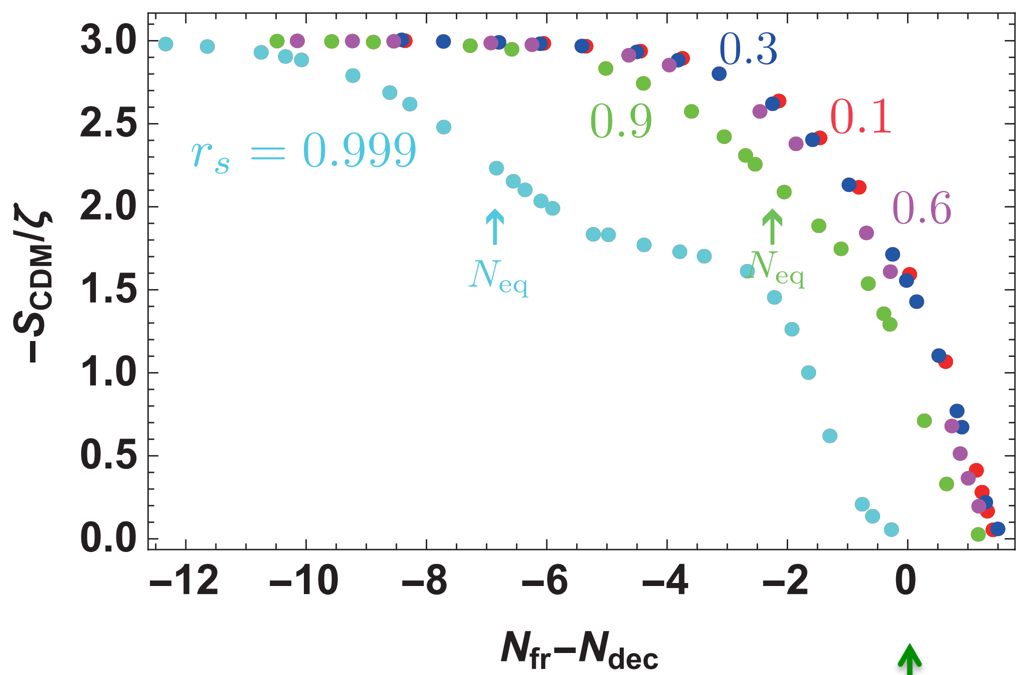
$$S_{\rm CDM} = 0$$

sigma; curvaton

How about in numerical calculation??

Numerical study (by delta N) I

non-relativistic case $n_{\rm CDM}^{\rm (eq)} \propto T^{3/2} e^{-m/T}$



$r_s \leftrightarrow r_{\rm dec}$

$m/T_{\rm fr} \simeq 20$

Neq; equality time

$\rho_\sigma = \rho_r$

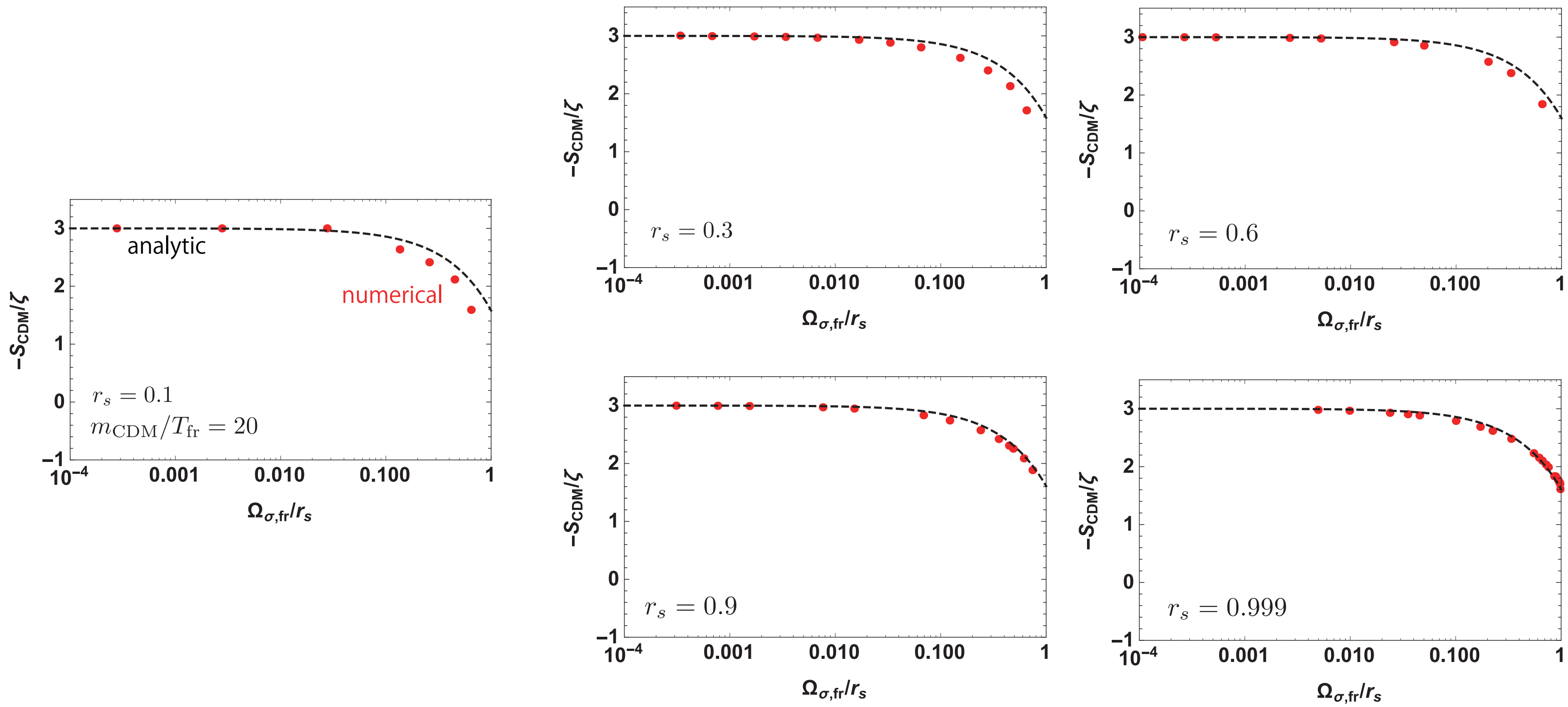
f.o. before curvaton decay

f.o. after ...

Basic behavior is almost consistent with analytic expression.

Kitajima, Langlois, Takahashi, SY in prep.

Numerical study (by delta N) II



Black dashed ;

$$S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,\rm fr}}{r_{\rm dec}}\left(\frac{m}{T_{\rm fr}}-\frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,\rm fr}-2)+\Omega_{\sigma,\rm fr}}-1\right]$$

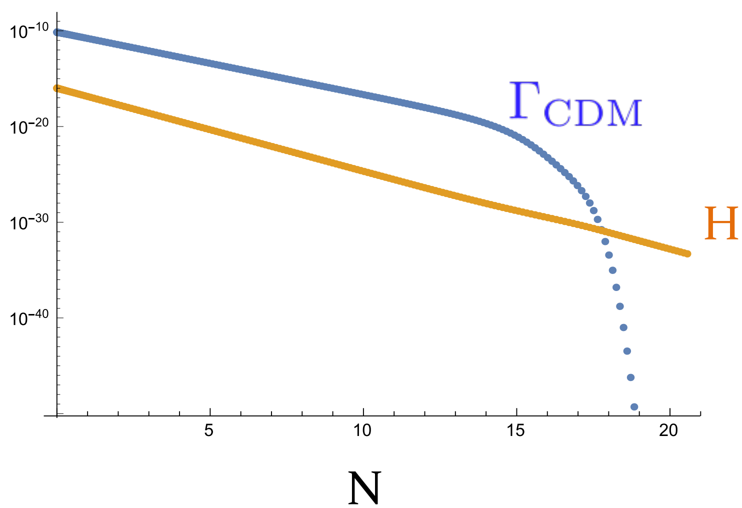
Kitajima, Langlois, Takahashi, SY in prep.

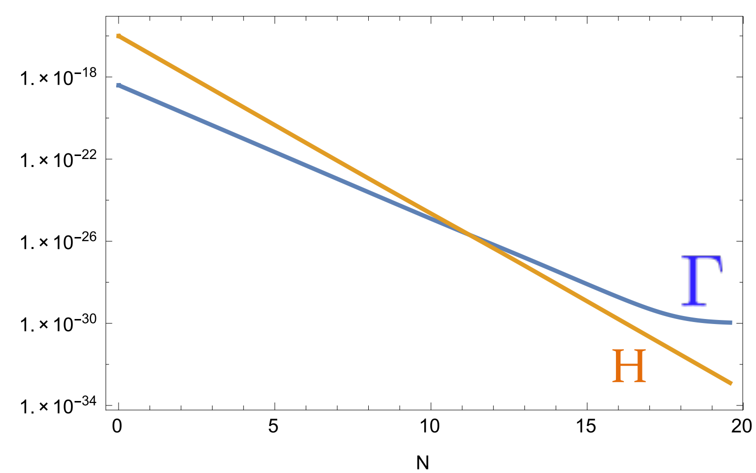
Discussion I

- For CDM isocurvature

Sudden freeze-out formalism seems to be valid.

Small change of Hubble
➔ large change of e-folds





does not give large modification..

in contrast with temperature-dependent decay rate of curvaton

Discussion II

- For CDM isocurvature

- Freeze-out before curvaton decay;

$$\boxed{S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,{\rm fr}}}{r_{\rm dec}}\left(\frac{m}{T_{\rm fr}} - \frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,{\rm fr}} - 2) + \Omega_{\sigma,{\rm fr}}} - 1\right]}$$

This seems to be correct.
Even if the modulation of the freeze-out hypersurface is taken into account, CDM isocurvature perturbations are still large in this case. .

- Freeze-out after curvaton decay;

$$S_{\rm CDM} = 0$$

Summary 2

- Revisiting CDM (thermal relics) isocurvature perturbations in curvaton scenario.

- Sudden freeze-out approximation seems to be good for such case.

$$\boxed{S_{\rm CDM}/\zeta = 3\left[\frac{\Omega_{\sigma,\rm fr}}{r_{\rm dec}}\left(\frac{m}{T_{\rm fr}} - \frac{3}{2}\right)\frac{1}{2(\alpha_{\Lambda,\rm fr} - 2) + \Omega_{\sigma,\rm fr}} - 1\right]}$$

The simplest case in this scenario is already ruled out..

If we include the fluctuations of the inflaton, we can rescue this scenario.
And from the observational constraint, we can obtain implications for the parameters.

Summary 1

- Thermal effect really appears in primordial curvature perturbations?

By using simple sudden decay approximation, large thermal effect seems to appear.

But, in the numerical result obtained by using delta N formalism, such large effect does not appear.
➔ Sudden decay approximation is not always valid!

Temperature dependence of the decay rate seems not to give large effects in the adiabatic curvature perturbations. (but small deviations appear..)

Other issues

- Axion case with temperature dependent mass

$$m \propto T^{-\beta}$$

➔ Modulation of the time of starting oscillation

- Any other interesting examples??

Hot relics

By sudden freeze-out approx.,

Before

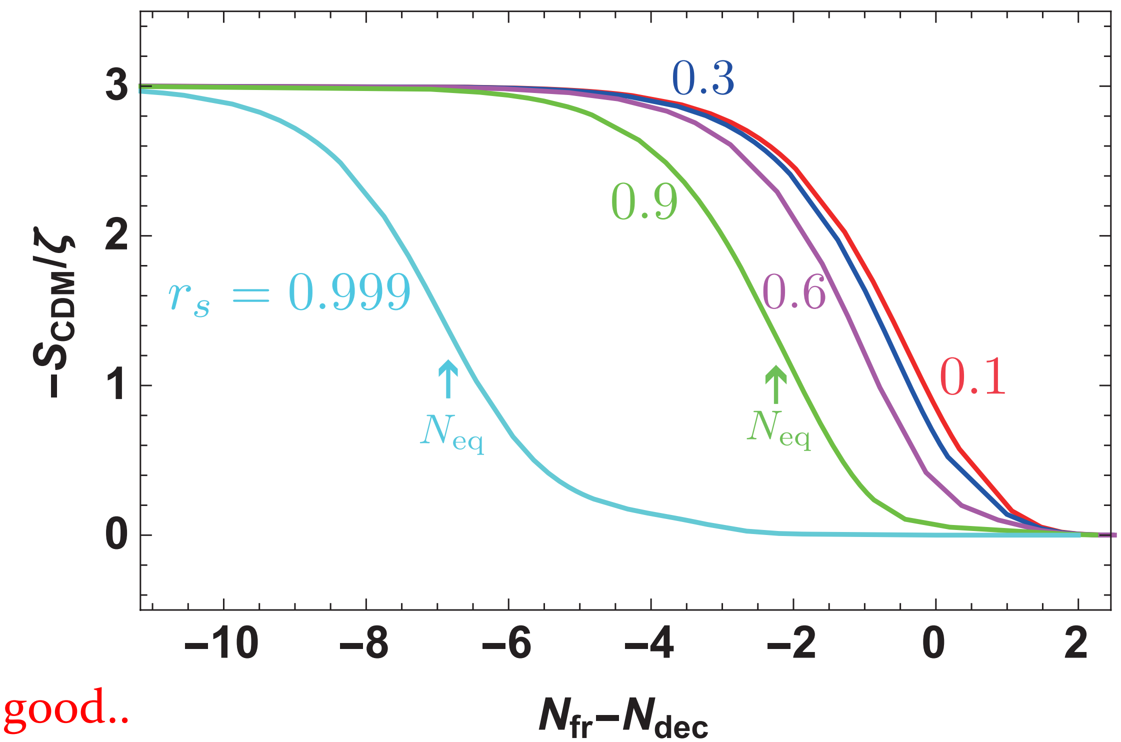
$$S_{\rm CDM}/\zeta = -3$$

after

$$S_{\rm CDM} = 0$$

Sudden decay seems not to be good..

relativistic case $n^{\rm (eq)}_{\rm CDM} \propto T^3$



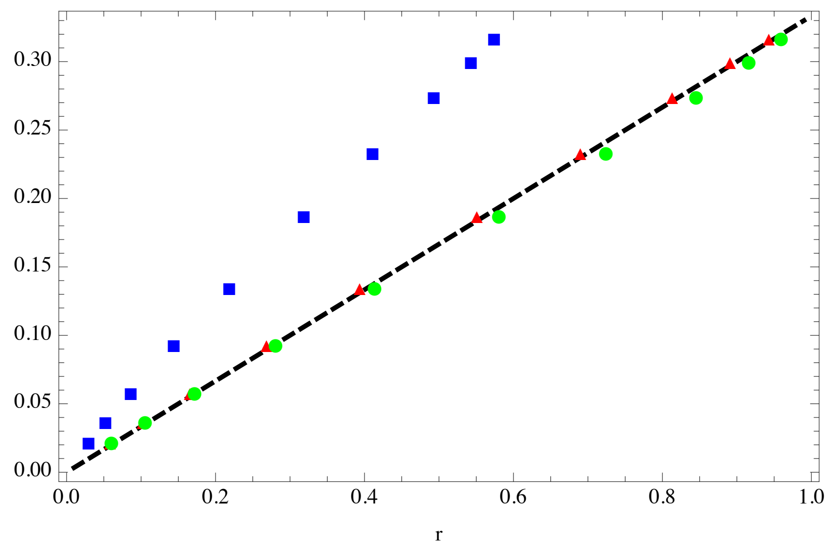
But for large r_s,
``before/after decay" doesn't matter.. ``before/after dominated" is important..

appendix

Numerical analysis by delta N I

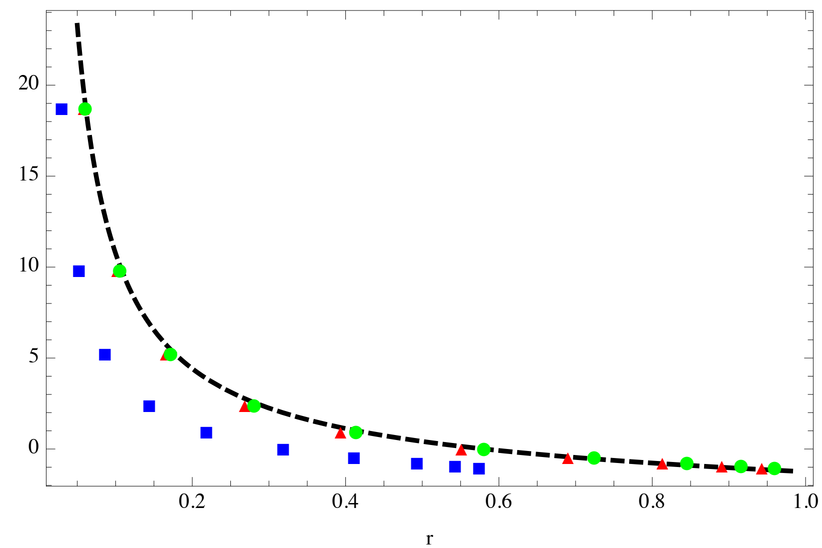
- Appropriate parameter; r $\qquad \Gamma = \text{constant}$

$(\zeta - \zeta_{\rm inf})/\mathcal{S}_i$



black dashed; $r/3$

$f_{\rm NL}$



black dashed; $\dfrac{5}{4r} - \dfrac{5}{3} - \dfrac{5r}{6}$

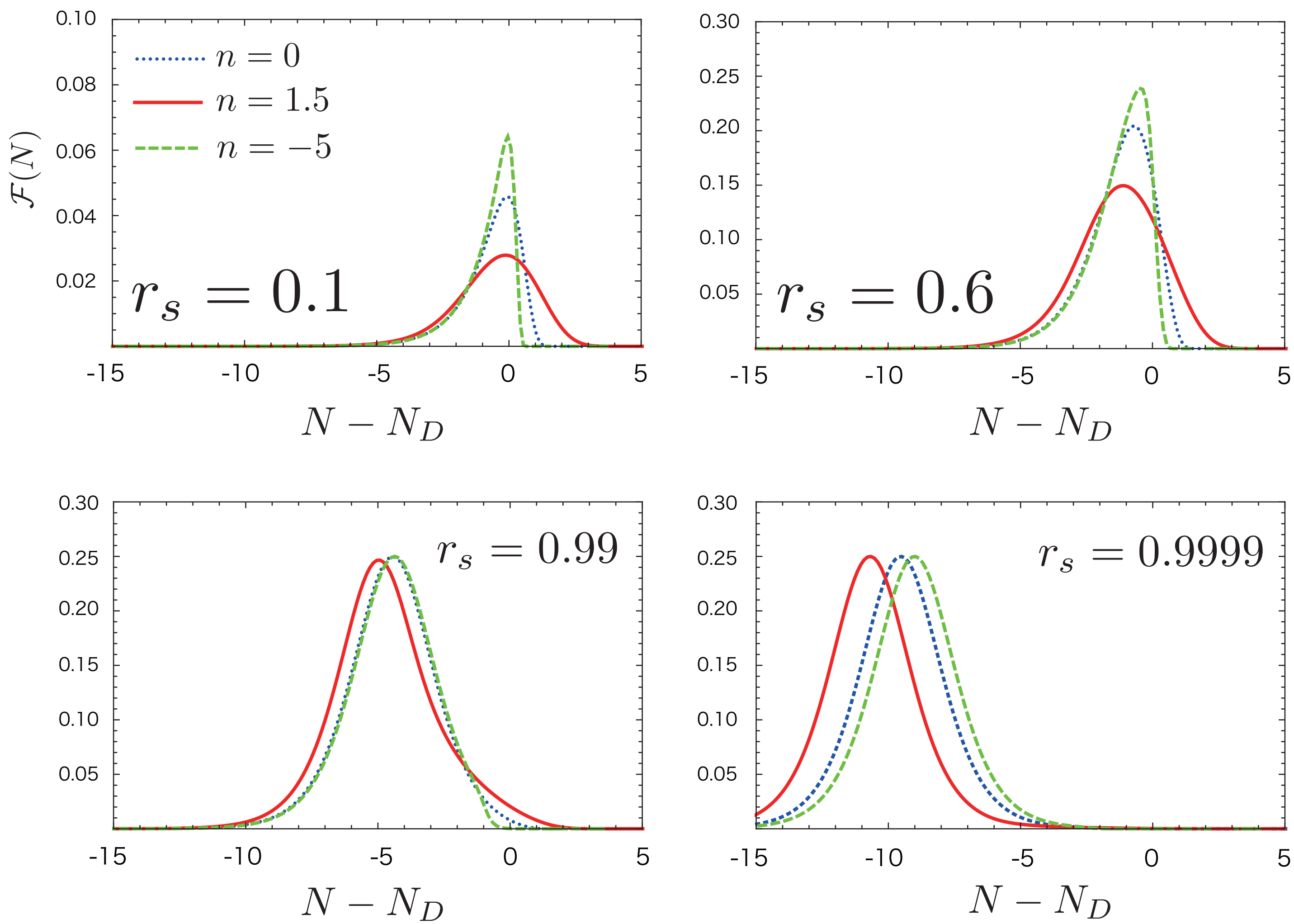
blue box; numerically evaluated $r_{\rm dec}$

red triangle; Gupta et al. (2004) fitting formula $r(p) = 1 - (1 + \frac{0.924}{1.24}p)^{-1.24}$ with $p = \left(\Omega_\sigma \sqrt{\frac{H}{\Gamma}}\right)_{H=m}$

green circle; entropy production rate
(we newly introduce.
briefly explain this later.)

$$r_s(q) = 1 - \left(1 + \frac{3}{4}q\right)^{-1} \quad \text{with } q := \left(\frac{S_f}{S_i}\right)^{4/3} - 1$$

Evolution of transfer func. I



Evolution of transfer func. II