Stability of the Early Universe in Bigravity Theory

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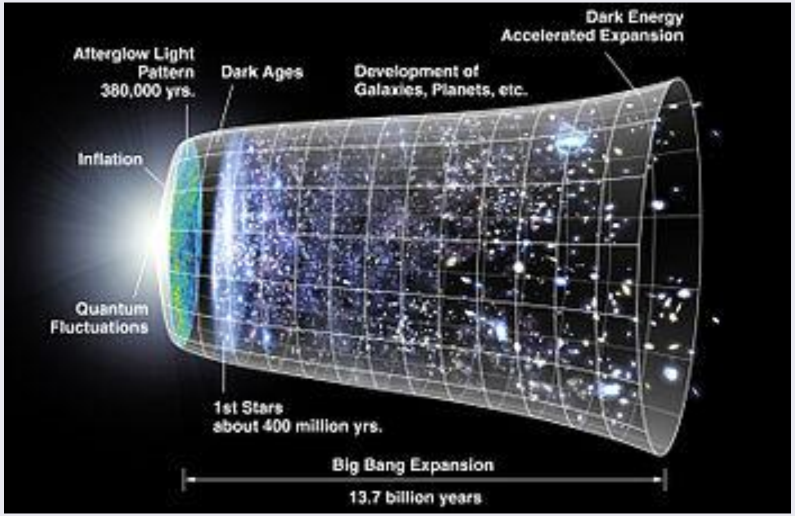


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

1. Introduction

2. Massless limit = GR ?
3. Stability of the Early Universe in Bigravity
4. Summary

Why massive?

What is graviton?

- It should be spin-2 field.
- Massless field or Massive field? How many gravitons?

GR describes a massless spin-2 field.

Is there a theory with a massive spin-2 field?

If there is, which theory describes our Universe?

Experimental constraint on Yukawa-type potential

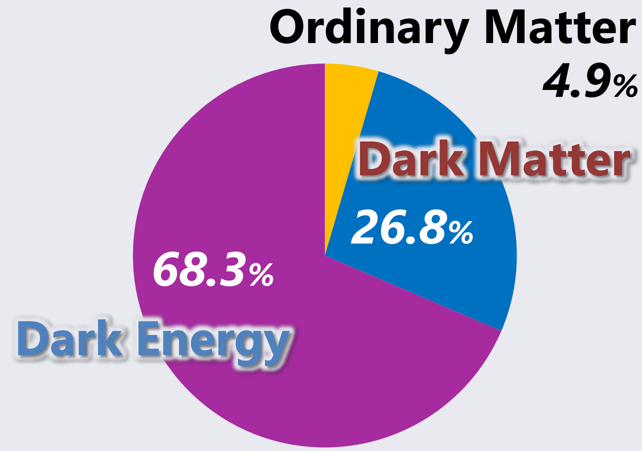
→ $m < 7.1 \times 10^{-23}\mathrm{eV}$ (from the solar-system experiment)

$$\Phi \propto \frac{1}{r} \rightarrow \Phi \propto \frac{1}{r} e^{-mr}$$

Why massive?

GR can describe our Universe
if we introduce unknown matters

Dark components hint us that
GR should be modified at large scale.



If we add a mass to graviton,
gravitational behaviours may be modified
at scales lager than the Compton wavelength,
but may not be modified at small scales.

$$\Phi \propto \frac{1}{r} \to \Phi \propto \frac{1}{r} e^{-mr}$$

How to give a mass to graviton?

To construct mass terms of tensor field,
we need a reference metric (Here, $f_{\mu\nu}$ is non-dynamical metric).

$g_{\mu\nu}g^{\mu\nu}$	$g_{\mu\nu}f^{\mu\nu}$
not mass term	mass term

Mass term is given by an interaction between two tensors.

→ $\mathscr{U}(g, f)$

It breaks the gauge symmetry.

→ Massive gravity generally has 6 DoFs

$$6 = 5 \text{ (massive spin-2)} + 1 \text{ (additional scalar)}$$

Ghost mode!

We have to eliminate the ghost mode!

Fierz-Pauli theory

→ The linear ghost-free massive gravity (Fierz and Pauli, 1939)

$$S = \frac{1}{2\kappa^2} \int d^4x \left[\mathcal{L}_{\mathrm{EH}}[h] - \frac{m^2}{4} (h_{\mu\nu} h^{\mu\nu} - h^2) \right]$$

$$(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,, \quad f_{\mu\nu} = \eta_{\mu\nu})$$

Special choice of mass term eliminates the ghost mode

This theory describes a linear massive spin-2 field on Minkowski spacetime.

What is non-linear extension of FP theory?

dRGT theory

→ The nonlinear ghost-free massive gravity

(de Rham, Gabadadze, and Tolley, 2011)

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) - \frac{m^2}{\kappa_g^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f)$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdots}\epsilon_{\cdots\cdots}(\gamma^\mu{}_\nu)^n$$

$$\gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu}$$

Again, special choice of mass term eliminates the ghost mode.

Another choice of $f_{\mu\nu}$ gives another theory.

How to determine $f_{\mu\nu}$?

Non-linear bigravity theory (Hassan,Rosen, '11)

One possibility is that $f_{\mu\nu}$ is also dynamical field.

(Hassan, and Rosen, 2011)

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f) \qquad \kappa^2 = \kappa_g^2 + \kappa_f^2$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdot}\epsilon_{\cdots\cdot}(\gamma^{\mu}{}_{\nu})^n$$

$$\gamma^{\mu}{}_{\alpha}\gamma^{\alpha}{}_{\nu} = g^{\mu\alpha}f_{\alpha\nu}$$

$f_{\mu\nu}$ is determined by the equation of motion as well as $g_{\mu\nu}$.

Bigravity contains a massive field as well as a massless field

Non-linear bigravity theory (Hassan,Rosen, '11)

It can explain the origin of dark matter or dark energy if

$$m \sim 10^{-33}\text{eV} \Rightarrow \text{DE} \quad \text{or} \quad m \gtrsim 10^{-27}\text{eV} \Rightarrow \text{DM}$$

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f)$$

Gives accelerating expansion

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4}\boxed{b_i\mathscr{U}_i(g,f)} + S^{[\mathrm{m}]}(g,f,\psi) \qquad \kappa^2 = \kappa_g^2 + \kappa_f^2$$

$$\gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu} \qquad \mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots}\epsilon_{\cdots}(\gamma^\mu{}_\nu)^n$$

$$S^{[\mathrm{m}]} = S_g^{[\mathrm{m}]}(g,\psi_g) + \boxed{S_f^{[\mathrm{m}]}(f,\psi_f)}$$

Physical matter Dark matter (KA and K. Maeda, '14)

~~1. Introduction~~

2. Massless limit = GR ?

3. Stability of the early Universe in Bigravity

4. Summary

Massless limit = GR ?

The mass term should be negligible beyond the mass scale.
→ GR should be recovered.

However, the linear massive gravity is not restored to GR even in massless limit.

On flat spacetime → vDVZ discontinuity

On FLRW spacetime → Higuchi ghost or gradient instability

Massless limit = GR ?

The mass term should be negligible beyond the mass scale.
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On flat spacetime → vDVZ discontinuity

→ It can be resolved by Vainshtein mechanism

On FLRW spacetime → Higuchi ghost or gradient instability

vDVZ discontinuity

Linear massive spin-2 field has a discontinuity

(van Dam and Veltman, 1970, Zakharov, 1970)

$$S = \frac{1}{2\kappa_g^2}\int d^4x \left[-\frac{1}{2}h^{\mu\nu}\mathcal{E}_{\mu\nu}^{\alpha\beta}h_{\alpha\beta} - \frac{1}{4}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \kappa_g^2 h_{\mu\nu}T^{\mu\nu}\right]$$

Introducing Struckelberg fields

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu, \qquad A_\mu \to A_\mu + \partial_\mu \phi$$

Canonical scaling and massless limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}_{\mu\nu}^{\alpha\beta}h_{\alpha\beta} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (h^{\mu\nu}\partial_\mu\partial_\nu\phi - h\partial_\mu\partial^\mu\phi) + \kappa h^{\mu\nu}T_{\mu\nu}$$

Kinetic mixing

vDVZ discontinuity

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (h^{\mu\nu}\partial_\mu\partial_\nu\phi - h\partial_\mu\partial^\mu\phi) + \kappa h^{\mu\nu}T_{\mu\nu}$$

Kinetic mixing

$$\Downarrow \quad \tilde{h}_{\mu\nu} = h_{\mu\nu} - \phi\eta_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{2}\tilde{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\tilde{h}_{\alpha\beta} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \kappa\tilde{h}^{\mu\nu}T_{\mu\nu} + \kappa\phi T$$

Scalar mode cannot be decoupled even in massless limit!

Fierz-Pauli theory cannot be restored to Newtonian gravity due to the existence of scalar graviton mode.

→ The discontinuity can be resolved by non-linear interactions

Vainshtein mechanism (Vainshtein, 1972)

$$\mathcal{L} = -\frac{3}{2}(\partial\phi)^2 - \frac{c_{\rm NL}}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots + \frac{c_n}{\Lambda^{3(n-1)}}h^{\mu\nu}X^{(n)}_{\mu\nu} + \cdots + \kappa\phi T$$

$$\Lambda^3 = (M_{\rm pl}m^2)^{1/3}, \quad X^{(n)}_{\mu\nu} \sim (\partial\partial\phi)^n$$

Splitting the source into a background T_0 and a perturbation δT and the scalar field into $\phi = \pi_0 + \pi$

$$\mathcal{L}_{\rm scalar} \simeq -\frac{1}{2}Z^{\mu\nu}\partial_\mu\pi\partial_\nu\pi + \kappa\pi\delta T$$

with $Z \sim 1 + \dfrac{\partial\partial\pi_0}{\Lambda^3} + \cdots + \dfrac{M_{\rm pl}R}{\Lambda^3} + \cdots$

The effective coupling constant is given by $\kappa_{\rm eff} = \dfrac{\kappa}{\sqrt{Z}}$

The interaction is suppressed in the nonlinear regime ($r \ll r_V$)

High-energy regime of bigravity

The mass term should be negligible beyond the mass scale.
→ GR should be recovered.

However, the linear massive gravity is not restored to GR even in massless limit.

On flat spacetime → vDVZ discontinuity

→ It can be resolved by Vainshtein mechanism

On FLRW spacetime → Higuchi ghost or gradient instability

→ Instability can be stabilized by non-linear interactions

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Massive spin-2 field on curved spacetime

Assumption:
we consider a linear massive spin-2 field on a GR solution.
→ There is only massless spin-2 field in the background.

*This is realized by perturbation around homothetic solution in bigravity

The action is given by linearized EH action with FP mass term

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}\left[\mathcal{L}_{\mathrm{EH}}[h;\Lambda_g] - \frac{m^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^2)\right]$$

To recover gauge symmetry, we introduce Stueckelberg fields

$$h_{\mu\nu} \to h_{\mu\nu} + 2\bar{\nabla}_{(\mu}A_{\nu)} + 2\bar{\nabla}_{\mu}\bar{\nabla}_{\nu}\phi$$

Massive spin-2 field on curved spacetime

The decoupling limit $(\Lambda^3 = (M_{\rm pl} m^2)^{1/3})$

$$\mathcal{L} = -\left(\frac{3}{4}\bar{g}^{\mu\nu} - \frac{M_{\rm pl}}{\Lambda^3}\bar{R}^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi + \cdots$$

→ Standard kinetic term if $\bar{R} \ll m^2$

(FP theory on Minkowski is recovered → vDVZ discontinuity)

How about $\bar{R} \gg m^2$?

= Massless limit on curved background

Massive spin-2 field on curved spacetime

The decoupling limit $(\Lambda^3 = (M_{\rm pl} m^2)^{1/3})$

$$\mathcal{L} = -\left(\cancel{\frac{3}{4}\bar{g}^{\mu\nu}} - \frac{M_{\rm pl}}{\Lambda^3}\bar{R}^{\mu\nu} \right) \partial_\mu \phi \partial_\nu \phi + \cdots$$

The fifth force can be screened due to curvature coupling.

However, the curvature coupling produces the instability

e.g. $d\bar{s}^2 = a^2(-d\eta^2 + \delta_{ij} dx^i dx^j)$

$$\bar{R}^{\mu\nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi = \frac{3H^2}{2a^2}(1+3w)\left((\partial_\eta \phi)^2 - \frac{w-1}{1+3w}(\partial_i \phi)^2 \right)$$

Ghost in $w < -1/3$

Gradient instability in $-1/3 < w < 1$

Instability of cosmological sol. in bigravity

For simplicity, we assume background solution is homothetic

$$S_2 = \frac{1}{\kappa_+^2}\int d^4x\sqrt{-\bar{g}}\mathcal{L}_{\rm EH}\left[h^{[+]};\Lambda_g\right] + \frac{1}{\kappa_-^2}\int d^4x\sqrt{-\bar{g}}\left[\mathcal{L}_{\rm EH}\left[h^{[-]};\Lambda_g\right] + \mathcal{L}_{\rm FP}\left[h^{[-]};m_{\rm eff}^2\right]\right],$$

The perturbations can be decomposed into a massless mode $h^{[+]}$ and a massive mode $h^{[-]}$.

The massive mode is given by FP theory on a GR solution!

→ Massive mode has an instability as in FP theory.

→ Cosmology in bigravity is also unstable in $m_{\rm eff} \ll H$

(c.f., for general solution, Comelli et al. '12, '14, De Felice et al. '14)

Instability of massive spin-2 field

✓ Higuchi ghost (Higuchi, 1972, Grisa and Sorbo, 2010)

de Sitter background (or the accelerating universe) with $m/H \to 0$.

⇨ Scalar graviton has ghost instability

✓ Gradient instability (Grisa and Sorbo, 2010)

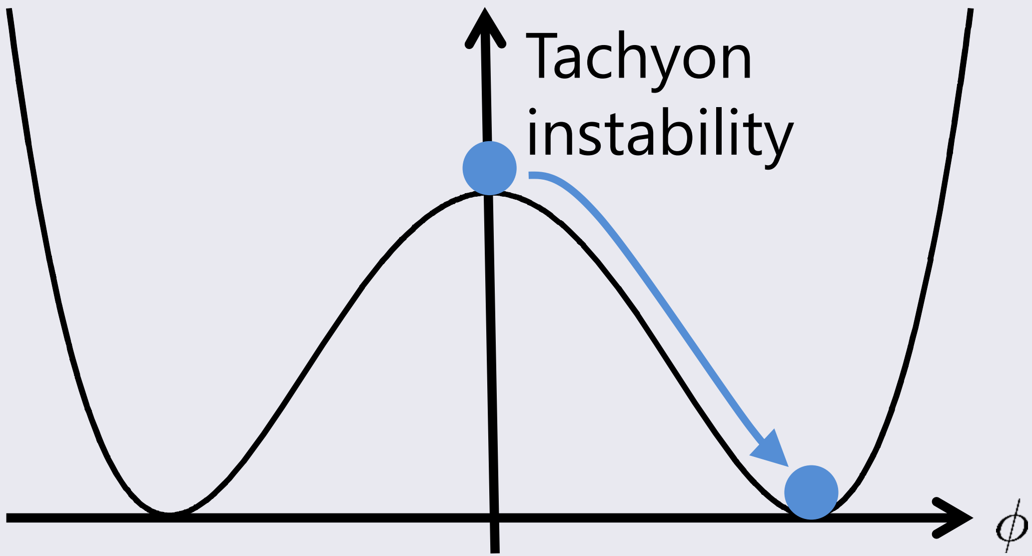
the decelerating universe ($-1/3 < w < 1$) with $m/H \to 0$.

⇨ Scalar graviton has gradient instability

Why? Massive field should be massless in $m/H \to 0$.

Condensation of scalar field?

Linear instability → field should be non-linear
Is there a stable point?



Simple example

$$V(\phi) = (\phi^2 - v^2)^2$$

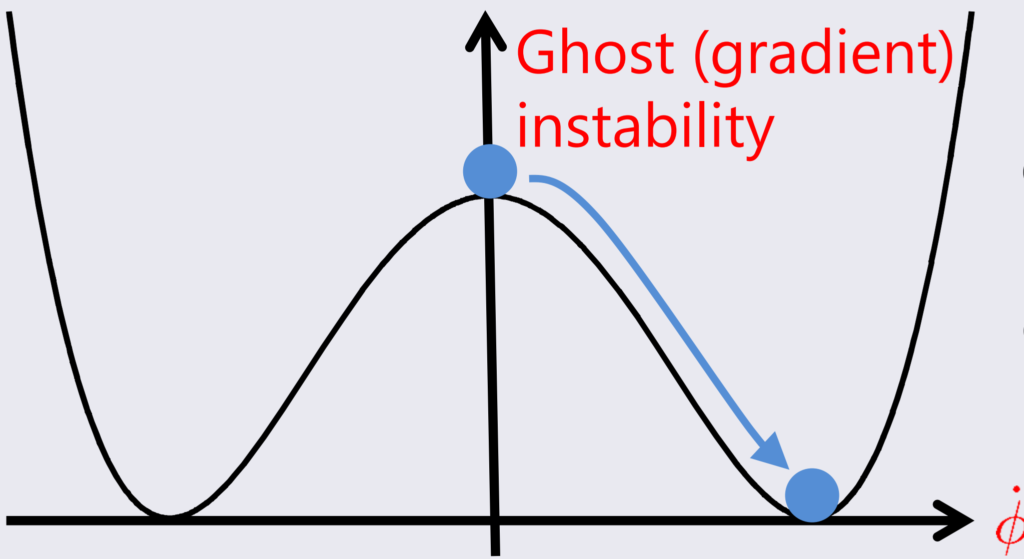
Scalar field is stable if $\langle \phi \rangle = v$

Although the solution $\phi = 0$ is unstable, the system is stable.

How about the case of ghost or gradient instability?

Higuchi ghost condensation?

Ghost (and gradient) instability can be stabilized by non-linear kinetic terms.



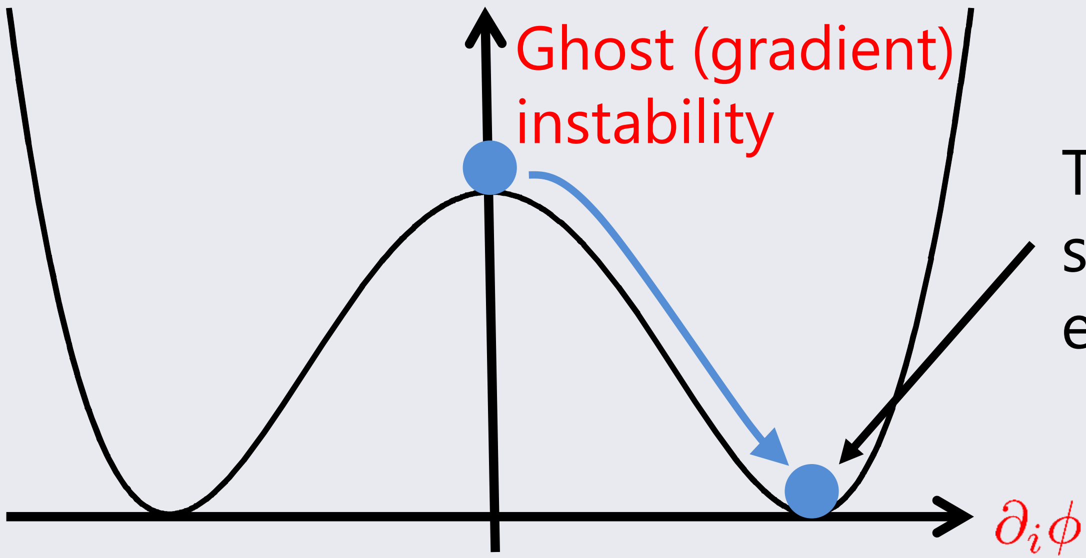
Non-zero $\langle\dot{\phi}\rangle$ can stabilize in the ghost condensation
(Arkani-Hamed, et al., 2004)

Ghost condensation in bigravity?

There can be homogeneous solution in ghost condensation.
However, general homogeneous solution is unstable.
→ Inhomogeneity of scalar graviton? We cannot obtain FLRW?

Ghost condensation + Vainshtein

Although the scalar mode has an inhomogeneity,
the spacetime can be homogenous by screening mechanism.



The spacetime can be still homogeneous even if $\langle \partial_i \phi \rangle \neq 0$

Is there a stable (approximative) FLRW solution with inhomogeneous scalar graviton?

1. ~~Introduction~~
2. ~~Massless limit = GR ?~~

3. Stability of the Early Universe in Bigravity

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Ghost condensation + Vainshtein

We must take into account non-linear effects.
However, full non-linear analysis is quite difficult.

Strategy

The interaction between tensor mode and scalar mode
is suppressed by the screening

→ We must retain non-linearities of scalar graviton,
but non-linearities of other fields could be ignored.

Scalar graviton arises from Stueckelberg fields.

We only consider non-linear effects of Stueckelberg fields.

Set up

What is Stueckelberg field in bigravity?

→ Stueckelberg fields is introduced to recover gauge symmetry.

$$ds_g^2 = g_{\mu\nu} dx_g^\mu dx_g^\nu \,, \quad ds_f^2 = f_{\mu\nu} dx_g^\mu dx_g^\nu = f_{ab} dx_f^a dx_f^b$$

$x_f^a = x_f^a(x_g^\mu)$ ← Physical dof, since we have only one diffeo.

We assume spacetime deviations are small $(g, f \simeq \mathrm{FLRW})$

but coordinate deviations are not small. $(x_f \not\simeq x_g)$

→ Two spacetime are almost homogeneous and isotropic,
but two foliations do not coincide!

We restrict analysis to spherically symmetric configuration.

Stability of the early Universe in bigravity

The background spacetimes:

$$d\bar{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,,$$
$$d\bar{s}_f^2 = K^2 a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,.$$

We consider spherically symmetric configurations:

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2\right],$$
$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2\right],$$
$$\eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r),$$

Small perturbation around homogenous and isotropic “spacetimes” → $\Phi_{g/f}, \Psi_{g/f} \ll 1$

However, it does not mean $\boldsymbol{\eta_f \approx \eta, r_f \approx r}$

Stabili

The back

We are interested in scalar graviton
→ Spherically symmetric configurations

For bigravity, there are 6 independent variables

$$6 = 2\ (g_{\mu\nu}) + 2\ (f_{\mu\nu}) + 2\ (\text{Stueckelberg fields})$$

We consider spherically symmetric configurations:

$$ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g}d\eta^2 + e^{2\Psi_g}dr^2 + r^2 d\Omega^2\right],$$

$$ds_f^2 = K^2 a^2(\eta_f)\left[-e^{2\Phi_f}d\eta_f^2 + e^{2\Psi_f}dr_f^2 + r_f^2 d\Omega^2\right],$$

$$\eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r),$$

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Stability of the early Universe in bigravity

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$$d\bar{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2 d\Omega^2)\,,$$

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We consider spherically symmetric configurations:

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$$\eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r),$$

Small perturbation around homogenous and isotropic “spacetimes” → $\Phi_{g/f}, \Psi_{g/f} \ll 1$

However, it does not mean $\boldsymbol{\eta_f \approx \eta, r_f \approx r}$

Strategy

- ✓ Assume $\Phi_{g/f}, \Psi_{g/f} \ll 1$, but do not assume $\nu, \mu \ll 1$
- ✓ Consider only sub-horizon scale. $\eta_f = (1+\nu)\eta\,,\; r_f = (1+\mu)r$
- ✓ Decompose all variables into adiabatic modes and oscillation modes.

$$X = X^{\rm ad} + X^{\rm osc}$$

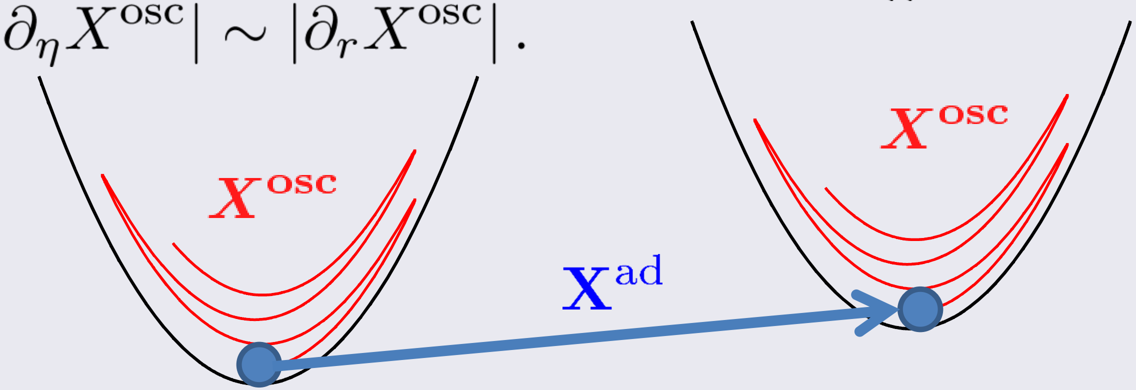
with

$$\left|\partial_\eta X^{\rm ad}\right| \sim \left|aHX^{\rm ad}\right|\,,$$
$$\left|\partial_\eta X^{\rm osc}\right| \sim \left|\partial_r X^{\rm osc}\right|\,.$$

$X^{\rm ad} = \langle X \rangle$

= expectation value

$X^{\rm osc} \ll 1$



Stability in pure graviton case

We concentrate on the early stage of the Universe ($m_{\rm eff} \ll H$)
We solve the equations up to ϵ^2. $\epsilon \sim aLH \ll 1$

If there is no matter perturbation

$$\rightarrow \quad \Phi_{g/f} \sim (arm_{\rm eff})^2 \approx 0, \quad \Psi_{g/f} \sim (arm_{\rm eff})^2 \approx 0$$

Pure scalar graviton solution: oscillation mode

$$\eta_f \approx \eta - \frac{1}{2}Har^2(2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0)r + \delta r$$

adiabatic mode

where $\mu_0 = 0 \text{ or } \mathcal{O}(1)$

$$\delta\eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 arH}{1+\mu_0}\frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 arH \partial_\eta \pi}{a^2(1+\mu_0)}$$

Stability in pure graviton case

Pure scalar graviton solution: $(\mu_0 = 0 \text{ or } \mathcal{O}(1))$

$$\eta_f \approx \eta - \frac{1}{2}Har^2(2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0)r + \delta r$$

$$\delta\eta = -\frac{\partial_\eta \pi}{a^2} + \frac{\mu_0 arH}{1+\mu_0}\frac{\partial_r \pi}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 arH\partial_\eta \pi}{a^2(1+\mu_0)}$$

Quadratic action: π is the scalar graviton mode

$$S_2 = \frac{m_{\rm eff}^2}{\kappa_-^2}\int d\Omega \int d\eta dr (arH)^2 \mathcal{K}_S\left[(\partial_\eta \pi)^2 - c_S^2 (\partial_r \pi)^2\right]\,,$$

- ✓ $\mu_0 = 0$ ⇒ Ghost or gradient instability appears for $w < 1$
- ✓ $\mu_0 \sim 1$ ⇒ Stability depends on the background dynamics as well as the coupling constants

$b_2^2 - b_1 b_3 > 0, b_2 < 0 \Rightarrow \mathcal{K}_S \geq 0, c_S^2 > 0$ for $w < 1$ $(m_{\rm eff}^2 > 0)$

Stability in pure graviton case

As a result, we find a stable cosmological solution as

$$ds_g^2 \simeq a^2(\eta)\left[-d\eta^2 + dr^2 + r^2 d\Omega^2\right],$$

$$ds_f^2 \simeq K^2 a^2(\eta_f)\left[-d\eta_f^2 + dr_f^2 + r_f^2 d\Omega^2\right],$$

$$\eta_f \approx \eta - \frac{1}{2}Har^2(2\mu_0 + \mu_0^2) + \delta\eta\,, \quad r_f \approx (1+\mu_0)r + \delta r$$

Although two spacetimes are homogeneous and isotropic, two foliations are related by the non-linear coordinate transformation.

Cosmological evolution is same as the homothetic background.

When $w > 1 \rightarrow \mu_0 = 0$ is stable (linear Stueckelberg field)

When $w < 1 \rightarrow \mu_0 \sim 1$ is stable (non-linear Stueckelberg field)

Including matter perturbations

When there are matter perturbations

$$\to \quad \Phi_g \sim \Phi_{\rm GR} + (arm_{\rm eff})^2,$$

$$\Psi_g \sim \Psi_{\rm GR} + (arm_{\rm eff})^2$$

$$\Phi_{\rm GR}, \Psi_{\rm GR} \sim (arH)^2 \times \tilde{\delta}_g$$ for $\mu \sim 1$

The fifth force is screened in

$$\tilde{\delta}_g := \frac{\int 4\pi r^2 \delta_g dr}{\int 4\pi r^2 dr} \gg \frac{m_{\rm eff}^2}{H^2} \to 0$$ in the early Universe

$$\Leftrightarrow r \ll r_{\rm V} := \left(\frac{G\delta M}{m_{\rm eff}^2}\right)^{1/3} \qquad G\delta M := G \int 4\pi r^2 \delta\rho_g dr$$

→ Vainshtein mechanism on a cosmological background

Cosmological Vainshtein mechanism

The result is a generalization of the Vainshtein mechanism

Conventional Vainshtein mechanism (on Minkowski)

→ Non-linear terms are necessary to screen the fifth force

in the case with matter perturbation

Cosmological Vainshtein mechanism (on FLRW)

→ Non-linear terms are necessary to stabilize the fluctuation

even in the case without matter perturbation

Cosmological Vainshtein mechanism

= Ghost condensate + Vainshtein mechanism

$$\mathcal{L}_{\rm eff} = -\frac{3}{4}(\partial\phi)^2 + \frac{c_{\rm NL}}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots$$
$$+ \frac{\bar{R}^{\mu\nu}}{2m_{\rm eff}^2}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{c}_{\rm NL}}{\Lambda^3}\frac{\bar{R}^{\mu\nu\rho\sigma}}{m_{\rm eff}^2}\partial_\mu\phi\partial_\rho\phi\,\partial_\nu\partial_\sigma\phi + \cdots + \kappa\phi\delta T$$

When $R_0 \gg m_{\rm eff}^2\,,\quad R_0 \sim R_{\mu\nu} \qquad \kappa_{\rm eff} = \frac{m}{\sqrt{R_0}}\kappa \ll \kappa$

Fifth force can be screened even at linear order.

However, third term produces an instability

e.g., $\bar{R}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = +\Lambda_g(\partial\phi)^2$ → Higuchi ghost

Cosmological Vainshtein mechanism

$$\mathcal{L}_{\rm eff} = -\frac{3}{4}(\partial\phi)^2 + \frac{c_{\rm NL}}{\Lambda^3}(\partial\phi)^2\Box\phi + \cdots$$
$$+ \frac{\bar{R}^{\mu\nu}}{2m_{\rm eff}^2}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{c}_{\rm NL}}{\Lambda^3}\frac{\bar{R}^{\mu\nu\rho\sigma}}{m_{\rm eff}^2}\partial_\mu\phi\partial_\rho\phi\,\partial_\nu\partial_\sigma\phi + \cdots + \kappa\phi\delta T$$

Non-zero expectation value $\langle\pi_0'\rangle$ can stabilize the fluctuation.
(=spatial derivative)

c.f. Non-zero $\langle\dot{\pi}_0\rangle$ can stabilize in the ghost condensation (Arkani-Hamed, et al., 2004)

$\phi = \pi_0 + \pi$ ← oscillation mode

adiabatic mode

Although the scalar mode has an inhomogeneity,
the spacetime is homogenous due to the screening mechanism.

1. ~~Introduction~~
2. ~~Massless limit = GR ?~~
3. ~~Stability of the Early Universe in Bigravity~~

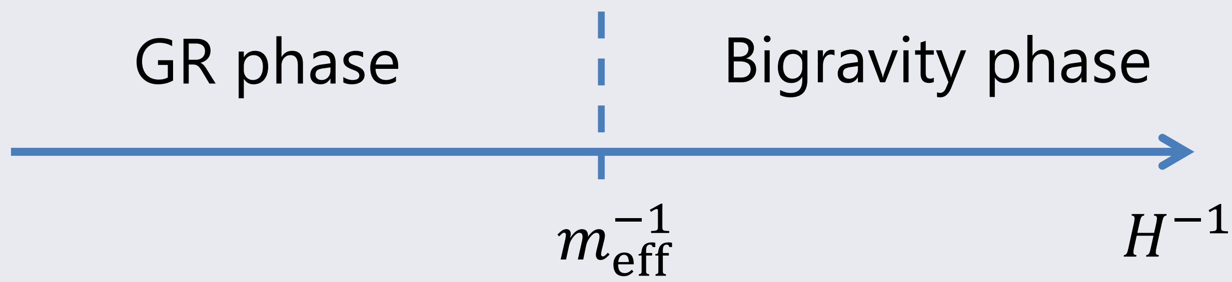
4.Summary

Summary and Discussion

Bigravity is attractive related to dark matter and dark energy.

We show that Higuchi ghost and the gradient instability can be resolved by the nonlinear self-interactions of the scalar graviton in bigravity theory.

This result suggests following cosmic history;



The early stage ($m_{\rm eff} \ll H$) → GR with nonlinear Stueckelberg

The late stage ($m_{\rm eff} \gg H$) → FP with linear Stueckelberg

Summary and Discussion

The early stage ($m_{\mathrm{eff}} \ll H$) → GR with nonlinear Stueckelberg
The late stage ($m_{\mathrm{eff}} \gg H$) → FP with linear Stueckelberg

Is it realized that GR transits to FP as the universe expands?

We also find that the transition is not realized with Hubble time scale unless $w > 1/3$.
→ The transition should be instantaneous if it is possible.

We do not conclude the cosmology is completely viable yet.

However, the parameter space ($b_2^2 - b_1 b_3 > 0, b_2 < 0$) is a necessary condition to obtain the viable cosmology.

Summary and Discussion

The cosmological Vainshtein mechanism is stable.

Stability of Vainshtein mechanism on flat spacetime?

$$\mathcal{L}_{\mathrm{eff}} = -\frac{m_{\mathrm{eff}}^2 M_{\mathrm{pl}}^2}{\sqrt{\beta_3}} \frac{GM}{r^3} \left[2(\partial_r \phi)^2 - \frac{(D_i \phi)^2}{r^2} \right] + \cdots$$

Gradient instability

*Vector graviton is not pathological.

$(\partial_t \phi)^2$ does not appear at leading order.

→ **strong coupling**

Unstable? or Perturbed approach breaks down?

Boundedness in nonlinear system?