Stability of the Early Universe in Bigravity Theory

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1. Introduction

2. Massless limit = GR?

3. Stability of the Early Universe in Bigravity

4. Summary
Why massive?

What is graviton?
- It should be spin-2 field.
- Massless field or Massive field? How many gravitons?
  
  GR describes a massless spin-2 field.
  
  Is there a theory with a massive spin-2 field?
  
  If there is, which theory describes our Universe?

Experimental constraint on Yukawa-type potential

\[ m < 7.1 \times 10^{-23} \text{eV} \quad \text{(from the solar-system experiment)} \]

\[ \Phi \propto \frac{1}{r} \rightarrow \Phi \propto \frac{1}{r} e^{-mr} \]
Why massive?

GR can describe our Universe if we introduce unknown matters.

Dark components hint us that GR should be modified at large scale.

If we add a mass to graviton, gravitational behaviours may be modified at scales larger than the Compton wavelength, but may **not** be modified at small scales.

\[ \Phi \propto \frac{1}{r} \rightarrow \Phi \propto \frac{1}{r} e^{-mr} \]
How to give a mass to graviton?

To construct mass terms of tensor field, we need a reference metric (Here, $f_{\mu\nu}$ is non-dynamical metric).

\[
g_{\mu\nu}g^{\mu\nu} \quad g_{\mu\nu}f^{\mu\nu}
\]

not mass term \hspace{1cm} mass term

Mass term is given by an interaction between two tensors.

$\rightarrow \mathcal{U}(g, f)$

It breaks the gauge symmetry.

$\rightarrow$ Massive gravity generally has 6 DoFs

\[
6 = 5 \text{ (massive spin-2)} + 1 \text{ (additional scalar)}
\]

We have to eliminate the ghost mode! Ghost mode!
Fierz-Pauli theory

→ The linear ghost-free massive gravity (Fierz and Pauli, 1939)

\[
S = \frac{1}{2\kappa^2} \int d^4x \left[ \mathcal{L}_{EH}[h] - \frac{m^2}{4} (h_{\mu\nu} h^{\mu\nu} - h^2) \right]
\]

\[
(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad f_{\mu\nu} = \eta_{\mu\nu})
\]

Special choice of mass term eliminates the ghost mode

This theory describes a linear massive spin-2 field on Minkowski spacetime.

What is non-linear extension of FP theory?
dRGT theory

→ The nonlinear ghost-free massive gravity
  (de Rham, Gabadadze, and Tolley, 2011)

\[ S = \frac{1}{2\kappa^2 g} \int d^4x \sqrt{-g} R(g) - \frac{m^2}{\kappa^2 g} \int d^4x \sqrt{-g} \sum_{i=0}^{4} b_i \mathcal{U}_i(g, f) \]

\[ \mathcal{U}_n(g, f) = -\frac{1}{n!(4-n)!} \epsilon^{\cdots} \epsilon^{\cdots} (\gamma^\mu_\nu)^n \]

\[ \gamma^\mu_\alpha \gamma^\alpha_\nu = g^\mu_\alpha f_\alpha_\nu \]

Again, special choice of mass term eliminates the ghost mode.

Another choice of \( f_{\mu\nu} \) gives another theory.

How to determine \( f_{\mu\nu} \)?
One possibility is that $f_{\mu\nu}$ is also dynamical field. (Hassan, and Rosen, 2011)

\[
S = \frac{1}{2\kappa_g^2} \int d^4x \sqrt{-g} R(g) + \frac{1}{2\kappa_f^2} \int d^4x \sqrt{-f} \mathcal{R}(f)
- \frac{m^2}{\kappa^2} \int d^4x \sqrt{-g} \sum_{i=0}^{4} b_i \mathcal{U}_i(g, f) \quad \kappa^2 = \kappa_g^2 + \kappa_f^2
\]

\[
\mathcal{U}_n(g, f) = -\frac{1}{n!(4-n)!} \epsilon^{\cdots} \epsilon^{\cdots} (\gamma^\mu_\nu)^n
\]

\[
\gamma^\mu_\alpha \gamma^\alpha_\nu = g^{\mu\alpha} f_{\alpha\nu}
\]

$f_{\mu\nu}$ is determined by the equation of motion as well as $g_{\mu\nu}$.

Bigravity contains a massive field as well as a massless field.
Non-linear bigravity theory (Hassan, Rosen, ’11)

It can explain the origin of dark matter or dark energy if

\[ m \sim 10^{-33} \text{eV} \Rightarrow \text{DE} \quad \text{or} \quad m \gtrsim 10^{-27} \text{eV} \Rightarrow \text{DM} \]

\[
S = \frac{1}{2\kappa_g^2} \int d^4x \sqrt{-g} R(g) + \frac{1}{2\kappa_f^2} \int d^4x \sqrt{-f} \mathcal{R}(f)
- \frac{m^2}{\kappa^2} \int d^4x \sqrt{-g} \sum_{i=0}^{4} b_i \mathcal{U}_i(g, f) + S^{[m]}(g, f, \psi)
\]

Gives accelerating expansion

\[
\gamma^\mu_{\alpha} \gamma^{\alpha}_{\nu} = g^\mu_{\alpha} f_{\alpha\nu} \quad \mathcal{U}_n(g, f) = -\frac{1}{n!(4-n)!} \epsilon^{.... \epsilon .... (\gamma^\mu_\nu)^n}
\]

\[
S^{[m]} = S_{g}^{[m]}(g, \psi_g) + S_{f}^{[m]}(f, \psi_f)
\]

Physical matter \quad Dark matter (KA and K. Maeda, ’14)
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4. Summary
Massless limit = GR?

The mass term should be negligible beyond the mass scale. → GR should be recovered.

However, the linear massive gravity is not restored to GR even in massless limit.

On flat spacetime → vDVZ discontinuity

On FLRW spacetime → Higuchi ghost or gradient instability
Massless limit = GR?

The mass term should be negligible beyond the mass scale. → GR should be recovered.

However, the linear massive gravity is not restored to GR even in massless limit.

On flat spacetime → vDVZ discontinuity
→ It can be resolved by Vainshtein mechanism

On FLRW spacetime → Higuchi ghost or gradient instability
vDVZ discontinuity

Linear massive spin-2 field has a discontinuity

\[ S = \frac{1}{2\kappa^2} \int d^4x \left[ -\frac{1}{2} h^{\mu\nu} \epsilon_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \kappa^2 h_{\mu\nu} T_{\mu\nu} \right] \]

Introducing Struckelberg fields

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu, \quad A_\mu \rightarrow A_\mu + \partial_\mu \phi \]

Canonical scaling and massless limit

\[ \mathcal{L} = -\frac{1}{2} h^{\mu\nu} \epsilon_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (h^{\mu\nu} \partial_\mu \partial_\nu \phi - h \partial_\mu \partial^{\mu} \phi) + \kappa h^{\mu\nu} T_{\mu\nu} \]

Kinetic mixing

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vDVZ discontinuity

\[ \mathcal{L} = -\frac{1}{2} h^{\mu\nu} \varepsilon^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (h^{\mu\nu} \partial_\mu \partial_\nu \phi - h \partial_\mu \partial^\mu \phi) + \kappa h^{\mu\nu} T_{\mu\nu} \]

Kinetic mixing

\[ \tilde{h}_{\mu\nu} = h_{\mu\nu} - \phi \eta_{\mu\nu} \]

\[ \mathcal{L} = -\frac{1}{2} \tilde{h}^{\mu\nu} \varepsilon^{\alpha\beta}_{\mu\nu} \tilde{h}_{\alpha\beta} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2} (\partial_\mu \phi)(\partial^\mu \phi) + \kappa \tilde{h}^{\mu\nu} T_{\mu\nu} + \kappa \phi T \]

Scalar mode cannot be decoupled even in massless limit!

Fierz-Pauli theory cannot be restored to Newtonian gravity due to the existence of scalar graviton mode.

→ The discontinuity can be resolved by non-linear interactions
Vainshtein mechanism (Vainshtein, 1972)

\[
\mathcal{L} = -\frac{3}{2} (\partial \phi)^2 - \frac{c_{NL}}{\Lambda^3} (\partial \phi)^2 \Box \phi + \cdots + \frac{c_n}{\Lambda^{3(n-1)}} h^{\mu \nu} X^{(n)}_{\mu \nu} + \cdots + \kappa \phi T
\]

\[
\Lambda^3 = (M_{pl} m^2)^{1/3}, \quad X^{(n)}_{\mu \nu} \sim (\partial \partial \phi)^n
\]

Splitting the source into a background \( T_0 \) and a perturbation \( \delta T \) and the scalar field into \( \phi = \pi_0 + \pi \)

\[
\mathcal{L}_{\text{scalar}} \sim -\frac{1}{2} Z^{\mu \nu} \partial_\mu \pi \partial_\nu \pi + \kappa \pi \delta T
\]

with \( Z \sim 1 + \frac{\partial \partial \pi_0}{\Lambda^3} + \cdots + \frac{M_{pl} R}{\Lambda^3} + \cdots \)

The effective coupling constant is given by \( \kappa_{\text{eff}} = \frac{\kappa}{\sqrt{Z}} \)

The interaction is suppressed in the nonlinear regime \( (r \ll r_V) \)
High-energy regime of bigravity

The mass term should be negligible beyond the mass scale. → GR should be recovered.

However, the linear massive gravity is not restored to GR even in massless limit.

On flat spacetime → vDVZ discontinuity

→ It can be resolved by Vainshtein mechanism

On FLRW spacetime → Higuchi ghost or gradient instability

→ Instability can be stabilized by non-linear interactions

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Massive spin-2 field on curved spacetime

Assumption:
we consider a linear massive spin-2 field on a GR solution. → There is only massless spin-2 field in the background.

*This is realized by perturbation around homothetic solution in bigravity

The action is given by linearized EH action with FP mass term

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \mathcal{L}_{EH}[h; \Lambda_g] - \frac{m^2}{4} (h_{\mu\nu} h^{\mu\nu} - h^2) \right] \]

To recover gauge symmetry, we introduce Stueckelberg fields

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + 2 \tilde{\nabla}_{(\mu} A_{\nu)} + 2 \tilde{\nabla}_\mu \tilde{\nabla}_\nu \phi \]
Massive spin-2 field on curved spacetime

The decoupling limit \( \Lambda^3 = (M_{\text{Pl}} m^2)^{1/3} \)

\[
\mathcal{L} = - \left( \frac{3}{4} g^{\mu \nu} - \frac{M_{\text{Pl}}}{\Lambda^3} \bar{R}^{\mu \nu} \right) \partial_\mu \phi \partial_\nu \phi + \ldots
\]

→ Standard kinetic term if \( \bar{R} \ll m^2 \)

(FP theory on Minkowski is recovered → vDVZ discontinuity)

How about \( \bar{R} \gg m^2 \)?

= Massless limit on curved background
Massive spin-2 field on curved spacetime

The decoupling limit \( (\Lambda^3 = (M_{\text{pl}} m^2)^{1/3}) \)

\[
\mathcal{L} = - \left( \frac{3}{4} g_{\mu \nu} - \frac{M_{\text{pl}}}{\Lambda^3} \bar{R}^{\mu \nu} \right) \partial_\mu \phi \partial_\nu \phi + \ldots
\]

The fifth force can be screened due to curvature coupling.

However, the curvature coupling produces the instability

e.g. \( ds^2 = a^2 (-d\eta^2 + \delta_{ij} dx^i dx^j) \)

\[
\bar{R}^{\mu \nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi = \frac{3H^2}{2a^2} (1 + 3w) \left( (\partial_\eta \phi)^2 - \frac{w - 1}{1 + 3w} (\partial_i \phi)^2 \right)
\]

Ghost in \( w < -1/3 \)

Gradient instability in \( -1/3 < w < 1 \)
For simplicity, we assume background solution is homothetic

\[ S_2 = \frac{1}{\kappa^2_+} \int d^4 x \sqrt{-\bar{g}} \mathcal{L}_{\text{EH}}[h^+; \Lambda_g] \]
\[ + \frac{1}{\kappa^2_-} \int d^4 x \sqrt{-\bar{g}} \left[ \mathcal{L}_{\text{EH}}[h^-; \Lambda_g] + \mathcal{L}_{\text{FP}}[h^-; m^2_{\text{eff}}] \right], \]

The perturbations can be decomposed into a massless mode \( h^+ \) and a massive mode \( h^- \).

The massive mode is given by FP theory on a GR solution!

\[ \rightarrow \] Massive mode has an instability as in FP theory.

\[ \rightarrow \] Cosmology in bigravity is also unstable in \( m_{\text{eff}} \ll H \)

(c.f., for general solution, Comelli et al. ‘12, ‘14, De Felice et al. ‘14)
Instability of massive spin-2 field

✓ Higuchi ghost (Higuchi, 1972, Grisa and Sorbo, 2010)
  de Sitter background (or the accelerating universe) with $m/H \to 0$.
  ▸ Scalar graviton has ghost instability

✓ Gradient instability (Grisa and Sorbo, 2010)
  the decelerating universe ($-1/3 < w < 1$) with $m/H \to 0$.
  ▸ Scalar graviton has gradient instability

Why? Massive field should be massless in $m/H \to 0$. 
Condensation of scalar field?

Linear instability $\rightarrow$ field should be non-linear
Is there a stable point?

Tachyon instability

Simple example

$$V(\phi) = (\phi^2 - v^2)^2$$

Scalar field is stable if $\langle \phi \rangle = v$

Although the solution $\phi = 0$ is unstable, the system is stable.

How about the case of ghost or gradient instability?
Higuchi ghost condensation?

Ghost (and gradient) instability can be stabilized by non-linear kinetic terms.

Non-zero $\langle \dot{\phi} \rangle$ can stabilize in the ghost condensation (Arkani-Hamed, et al., 2004)

Ghost (gradient) instability can be stabilized by non-linear kinetic terms.

There can be homogeneous solution in ghost condensation. However, general homogeneous solution is unstable. → Inhomogeneity of scalar graviton? We cannot obtain FLRW?
Although the scalar mode has an inhomogeneity, the spacetime can be homogenous by screening mechanism.

Is there a stable (approximative) FLRW solution with inhomogeneous scalar graviton?
3. Stability of the Early Universe in Bigravity
We must take into account non-linear effects. However, full non-linear analysis is quite difficult.

**Strategy**

The interaction between tensor mode and scalar mode is suppressed by the screening

→ We must retain non-linearities of scalar graviton, but non-linearities of other fields could be ignored.

Scalar graviton arises from Stueckelberg fields.

We only consider non-linear effects of Stueckelberg fields.
Set up

What is Stueckelberg field in bigravity?

→ Stueckelberg fields is introduced to recover gauge symmetry.

\[ ds_g^2 = g_{\mu\nu} dx_g^\mu dx_g^\nu, \quad ds_f^2 = f_{\mu\nu} dx_g^\mu dx_g^\nu = f_{ab} dx_f^a dx_f^b \]

\[ x_f^a = x_f^a(x_g^\mu) \leftarrow \text{Physical dof, since we have only one diffeo.} \]

We assume spacetime deviations are small \((g, f \simeq \text{FLRW})\) but coordinate deviations are not small. \((x_f \not\simeq x_g)\)

→ Two spacetime are almost homogeneous and isotropic, but two foliations do not coincide!

We restrict analysis to spherically symmetric configuration.
Stability of the early Universe in bigravity

The background spacetimes:

\[ d\tilde{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2), \]
\[ d\tilde{s}_f^2 = K^2a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2). \]

We consider spherically symmetric configurations:

\[ ds_g^2 = a^2(\eta) \left[ -e^{2\Phi_g}d\eta^2 + e^{2\Psi_g}dr^2 + r^2d\Omega^2 \right], \]
\[ ds_f^2 = K^2a^2(\eta_f) \left[ -e^{2\Phi_f}d\eta_f^2 + e^{2\Psi_f}dr_f^2 + r_f^2d\Omega^2 \right], \]
\[ \eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r), \]

Small perturbation around homogenous and isotropic “spacetimes” \( \rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, it does not mean \( \eta_f \approx \eta, r_f \approx r \)
Stability of the early Universe in bigravity

We are interested in scalar graviton → Spherically symmetric configurations

For bigravity, there are 6 independent variables

\[ 6 = 2 \left( g_{\mu\nu} \right) + 2 \left( f_{\mu\nu} \right) + 2 \left( \text{Stueckelberg fields} \right) \]

The background spacetimes:

We consider spherically symmetric configurations:

\[ ds_g^2 = a^2(\eta) \left[ -e^{2\Phi_g} d\eta^2 + e^{2\Psi_g} dr^2 + r^2 d\Omega^2 \right], \]

\[ ds_f^2 = K^2 a^2(\eta_f) \left[ -e^{2\Phi_f} d\eta_f^2 + e^{2\Psi_f} dr_f^2 + r_f^2 d\Omega^2 \right], \]

\[ \eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r), \]

Small perturbation around homogenous and isotropic "spacetimes" → \( \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, it does not mean \( \eta_f \approx \eta, r_f \approx r \)
Stability of the early Universe in bigravity

The background spacetimes:
\[ d\tilde{s}_g^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2), \]
\[ d\tilde{s}_f^2 = K^2a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2). \]

We consider spherically symmetric configurations:
\[ ds_g^2 = a^2(\eta)\left[-e^{2\Phi_g}d\eta^2 + e^{2\Psi_g}dr^2 + r^2d\Omega^2\right], \]
\[ ds_f^2 = K^2a^2(\eta_f)\left[-e^{2\Phi_f}d\eta_f^2 + e^{2\Psi_f}dr_f^2 + r_f^2d\Omega^2\right], \]
\[ \eta_f = \eta_f(\eta, r), \quad r_f = r_f(\eta, r), \]

Small perturbation around homogenous and isotropic “spacetimes” \( \rightarrow \Phi_{g/f}, \Psi_{g/f} \ll 1 \)

However, it does not mean \( \eta_f \approx \eta, r_f \approx r \)
Strategy

✓ Assume $\Phi_{g/f}, \Psi_{g/f} \ll 1$, but do not assume $\nu, \mu \ll 1$
✓ Consider only sub-horizon scale.
✓ Decompose all variables into adiabatic modes and oscillation modes.

$$X = X^{\text{ad}} + X^{\text{osc}}$$

with

$$|\partial_\eta X^{\text{ad}}| \sim |\partial_r X^{\text{ad}}|, \quad |\partial_\eta X^{\text{osc}}| \sim |\partial_r X^{\text{osc}}|.$$
Stability in pure graviton case

We concentrate on the early stage of the Universe \( m_{\text{eff}} \ll H \). We solve the equations up to \( \epsilon^2 \).

If there is no matter perturbation

\[
\Phi_{g/f} \sim (a r m_{\text{eff}})^2 \approx 0, \quad \Psi_{g/f} \sim (a r m_{\text{eff}})^2 \approx 0
\]

Pure scalar graviton solution:

\[
\eta_f \approx \eta - \frac{1}{2} H a r^2 (2 \mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0) r + \delta r
\]

where \( \mu_0 = 0 \) or \( O(1) \)

\[
\delta \eta = - \frac{\partial \eta \pi}{a^2} + \frac{\mu_0 a r H}{1 + \mu_0} \frac{\partial r \pi}{a^2}, \quad \delta r = \frac{\partial r \pi + \mu_0 a r H \partial \eta \pi}{a^2 (1 + \mu_0)}
\]
Stability in pure graviton case

Pure scalar graviton solution: \((\mu_0 = 0 \text{ or } \mathcal{O}(1))\)

\[
\eta_f \approx \eta - \frac{1}{2} H ar^2 (2\mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0) r + \delta r
\]

\[
\delta \eta = -\frac{\partial_{\eta} \pi}{a^2} + \frac{\mu_0 a r H \partial_r \pi}{1 + \mu_0} \frac{1}{a^2}, \quad \delta r = \frac{\partial_r \pi + \mu_0 a r H \partial_{\eta} \pi}{a^2 (1 + \mu_0)}
\]

Quadratic action: \(\pi\) is the scalar graviton mode

\[
S_2 = \frac{m_{\text{eff}}^2}{\kappa_{-}^2} \int d\Omega \int d\eta dr (ar H)^2 \mathcal{K}_S \left[ (\partial_{\eta} \pi)^2 - c_S^2 (\partial_r \pi)^2 \right]
\]

\[\checkmark \quad \mu_0 = 0 \Rightarrow \text{Ghost or gradient instability appears for } w < 1\]

\[\checkmark \quad \mu_0 \sim 1 \Rightarrow \text{Stability depends on the background dynamics as well as the coupling constants}\]

\[b_2^2 - b_1 b_3 > 0, \quad b_2 < 0 \Rightarrow \mathcal{K}_S \geq 0, \quad c_S^2 > 0 \quad \text{for } w < 1 \quad (m_{\text{eff}}^2 > 0)\]
Stability in pure graviton case

As a result, we find a stable cosmological solution as

\[ ds_g^2 \simeq a^2(\eta) \left[ -d\eta^2 + dr^2 + r^2 d\Omega^2 \right], \]

\[ ds_f^2 \simeq K^2 a^2(\eta_f) \left[ -d\eta_f^2 + dr_f^2 + r_f^2 d\Omega^2 \right], \]

\[ \eta_f \approx \eta - \frac{1}{2} Har^2(2\mu_0 + \mu_0^2) + \delta \eta, \quad r_f \approx (1 + \mu_0)r + \delta r \]

Although two spacetimes are homogeneous and isotropic, two foliations are related by the non-linear coordinate transformation.

Cosmological evolution is same as the homothetic background.

When \( w > 1 \rightarrow \mu_0 = 0 \) is stable (linear Stueckelberg field)

When \( w < 1 \rightarrow \mu_0 \sim 1 \) is stable (non-linear Stueckelberg field)
Including matter perturbations

When there are matter perturbations

$$\Phi_g \sim \Phi_{GR} + (ar m_{\text{eff}})^2,$$
$$\Psi_g \sim \Psi_{GR} + (ar m_{\text{eff}})^2$$

$$\Phi_{GR}, \Psi_{GR} \sim (ar H)^2 \times \tilde{\delta}_g$$ for $$\mu \sim 1$$

The fifth force is screened in

$$\tilde{\delta}_g := \frac{\int 4\pi r^2 \delta_g dr}{\int 4\pi r^2 dr} \gg \frac{m_{\text{eff}}^2}{H^2} \rightarrow 0$$ in the early Universe

$$\Leftrightarrow r \ll r_V := \left(\frac{G\delta M}{m_{\text{eff}}^2}\right)^{1/3} \quad G\delta M := G \int 4\pi r^2 \delta \rho_g dr$$

$$\rightarrow$$ Vainshtein mechanism on a cosmological background
The result is a generalization of the Vainshtein mechanism.

Conventional Vainshtein mechanism (on Minkowski):
→ Non-linear terms are necessary to screen the fifth force in the case with matter perturbation.

Cosmological Vainshtein mechanism (on FLRW):
→ Non-linear terms are necessary to stabilize the fluctuation even in the case without matter perturbation.
Cosmological Vainshtein mechanism

= Ghost condensate + Vainshtein mechanism

\[ \mathcal{L}_{\text{eff}} = -\frac{3}{4} (\partial \phi)^2 + \frac{c_{\text{NL}}}{\Lambda^3} (\partial \phi)^2 \Box \phi + \cdots \]

\[ + \frac{\bar{R}^{\mu\nu}}{2m_{\text{eff}}^2} \partial_\mu \phi \partial_\nu \phi + \frac{\tilde{c}_{\text{NL}}}{\Lambda^3} \frac{\bar{R}^{\mu\nu\rho\sigma}}{m_{\text{eff}}^2} \partial_\mu \phi \partial_\rho \phi \partial_\nu \partial_\sigma \phi + \cdots + \kappa \phi \delta T \]

When \( R_0 \gg m_{\text{eff}}^2 \), \( R_0 \sim R_{\mu\nu} \)

\[ \kappa_{\text{eff}} = \frac{m}{\sqrt{R_0}} \kappa \ll \kappa \]

Fifth force can be screened even at linear order.

However, third term produces an instability

e.g., \( \bar{R}^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = + \Lambda_g (\partial \phi)^2 \rightarrow \text{Higuchi ghost} \)
Cosmological Vainshtein mechanism

\[ \mathcal{L}_{\text{eff}} = -\frac{3}{4}(\partial \phi)^2 + \frac{c_{\text{NL}}}{\Lambda^3} (\partial \phi)^2 \Box \phi + \cdots \]

\[ + \frac{\bar{R}^{\mu \nu}}{2m_{\text{eff}}^2} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{\tilde{c}_{\text{NL}}}{\Lambda^3} \frac{\bar{R}^{\mu \nu \rho \sigma}}{m_{\text{eff}}^2} \partial_{\mu} \phi \partial_{\rho} \phi \partial_{\nu} \partial_{\sigma} \phi + \cdots + \kappa \phi \delta T \]

Non-zero expectation value \( \langle \pi_0^{'} \rangle \) can stabilize the fluctuation.

(=spatial derivative)

c.f. Non-zero \( \langle \pi_0 \rangle \) can stabilize in the ghost condensation (Arkani-Hamed, et al., 2004)

\[ \phi = \pi_0 + \pi \quad \text{oscillation mode} \]

\[ \text{adiahabatic mode} \]

Although the scalar mode has an inhomogeneity, the spacetime is homogenous due to the screening mechanism.
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Summary and Discussion

Bigravity is attractive related to dark matter and dark energy. We show that Higuchi ghost and the gradient instability can be resolved by the nonlinear self-interactions of the scalar graviton in bigravity theory.

This result suggests following cosmic history:

- GR phase
- Bigravity phase

\[ m_{\text{eff}}^{-1} \quad H^{-1} \]

The early stage \((m_{\text{eff}} \ll H) \rightarrow \text{GR with nonlinear Stueckelberg}\)

The late stage \((m_{\text{eff}} \gg H) \rightarrow \text{FP with linear Stueckelberg}\)
Summary and Discussion

The early stage \((m_{\text{eff}} \ll H) \rightarrow \text{GR with nonlinear Stueckelberg}\)

The late stage \((m_{\text{eff}} \gg H) \rightarrow \text{FP with linear Stueckelberg}\)

Is it realized that GR transits to FP as the universe expands?

We also find that the transition is not realized with Hubble time scale unless \(w > 1/3\).

\(\rightarrow\) The transition should be **instantaneous** if it is possible.

We do not conclude the cosmology is completely viable yet.

However, the parameter space \((b_2^2 - b_1 b_3 > 0, b_2 < 0)\) is a necessary condition to obtain the viable cosmology.
Summary and Discussion

The cosmological Vainshtein mechanism is stable.

Stability of Vainshtein mechanism on flat spacetime?

\[ \mathcal{L}_{\text{eff}} = -\frac{m^2}{\sqrt{\beta_3}} \frac{M_{\text{pl}}^2 G M}{r^3} \left[ 2(\partial_r \phi)^2 - \frac{(D_i \phi)^2}{r^2} \right] + \ldots \]

Gradient instability

*Vector graviton is not pathological.

\((\partial_t \phi)^2\) does not appear at leading order.

\(\rightarrow\) strong coupling

Unstable? or Perturbed approach breaks down?

Boundedness in nonlinear system?