Bigravity from DGP 2-brane model

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based on

JCAP 1406 (2014) 004 YY and Tanaka

arXiv 1510.07551 YY and Tanaka

bigravity and Boulware-Deser ghost

bigravity : gravitational theory which contains two gravitons interacting each other

$$S = \frac{M_{pl}^2}{2} \int d^4x\sqrt{-g}\,[R + \underline{V(g,\,\tilde{g})}] + \cancel{\frac{\chi M_{pl}^2}{2} \int d^4x\sqrt{-\tilde{g}}\tilde{R}}$$

fix $\tilde{g}$

The interaction term breaks general covariance for g

→ $\underbrace{\text{GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0,}}_{\text{massive graviton}}$ $\boxed{\text{helicity-0}}$)

This mode's kinetic term has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is removed by constraints.

ghost-free bigravity

To avoid BD ghost, the interaction *V* should be tuned as

$$V = m^2 \sum_{n=0}^{4} c_n \epsilon^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} \mathcal{K}^{\nu_1}_{\mu_1} \ldots \mathcal{K}^{\nu_n}_{\mu_n} \,, \; \mathcal{K}^{\nu}_{\mu} = \sqrt{g^{\nu\rho} \tilde{g}_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)
Hassan and Rosen (2012)

- We can construct a realistic cosmological model at low energies.
- The gravitational wave has a characteristic feature.

… two gravitons cause "graviton oscillation" like neutrino oscillation

Questions in ghost-free bigravity

- What is the hidden metric?
- The form of the interaction is derived technically and artificially.

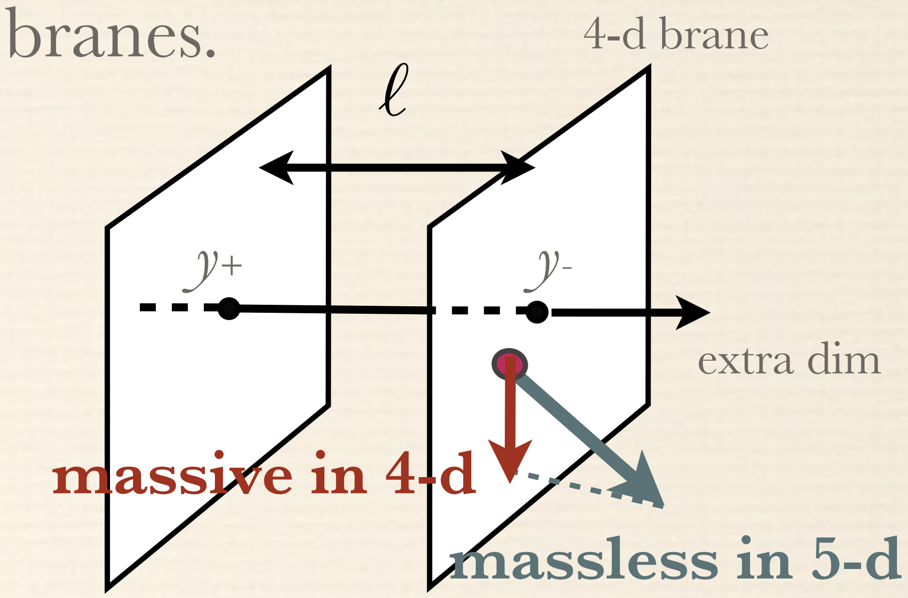
  … What is the mechanism that tune the interaction to the ghost-free one?

embed ghost-free bigravity

to higher dimensional gravity.

Why higher dimensional theory?

Consider 5-dim braneworld model sandwiched by two branes.



$$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g}\,R \;+\; (\text{boundary term})$$

- There is no BD ghost problem.
- two metrics induced on two branes ⇔ two metrics in bigravity
- 5-dim massless graviton

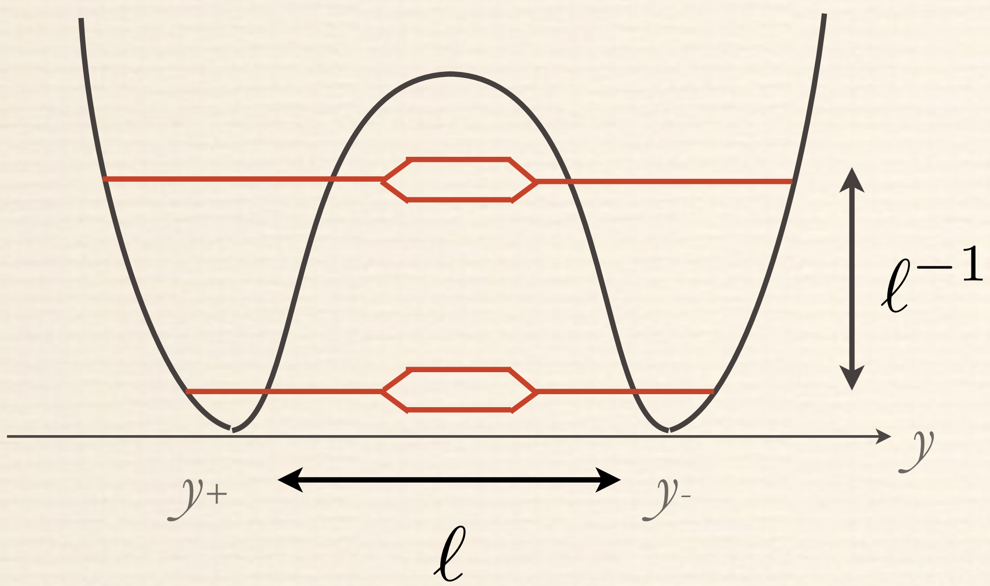
  = 1 massless and infinite # of massive gravitons on the branes

The 4-d effective theory contains **one massless graviton**, **infinite # of massive gravitons** and **one scalar** (radion=brane separation).

Model

In order to obtain bigravity, only one massive mode is to be kept at low energies.

effective potential of gravity



high potential barrier (ℓ << depth)

→ nearly degenerate two small mass

Dvali-Gabadadze-Poratti model

potential depth

$$S = \frac{M_5^3}{2}\left[\int d^5x\sqrt{-{}^5g}\,{}^5R + 2r_c^{(+)}\int d^4x\sqrt{-g_+}\,(R_+ - 2\sigma_+) + 2r_c^{(-)}\int d^4x\sqrt{-g_-}\,(R_- - 2\sigma_-)\right]$$

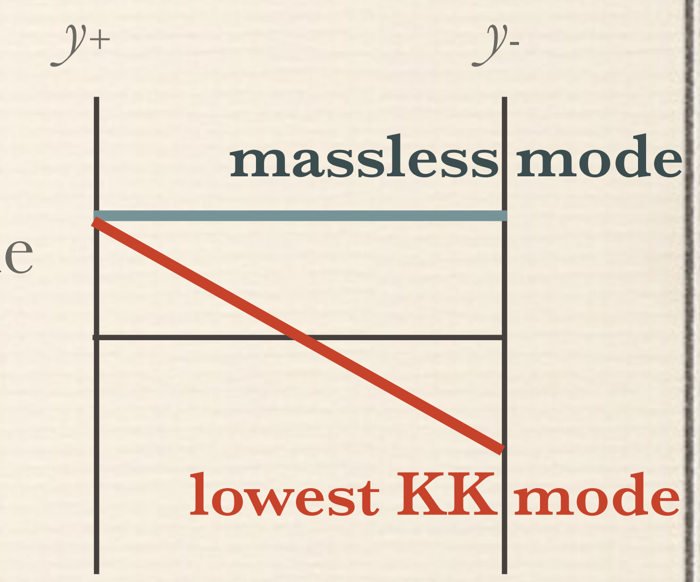
induced gravity terms = potential wells

graviton's mass spectrum

bulk equation: $G_{ab} = 0 \quad \sim \quad \left(\partial_y^2 + \Box^{(4)}\right) \delta g_{ab} = 0\,,\ \Box^{(4)} = m^2$

junction condition: $K_{\mu\nu}^{(\pm)} = r_c^{(\pm)} \left(G_{\mu\nu}^{\pm(4)} - \frac{1}{3} G^{\pm(4)} g_{\mu\nu} \right) \sim \partial_y \delta g_{\mu\nu} = r_c m_1^2 \delta g_{\mu\nu}$

For $\ell \ll r_c$, the mode functions for the mass eigenstates become



When $\ell \ll r_c$, the junction condition becomes

$$\frac{\delta g_{\mu\nu}}{\ell} \sim r_c m_1^2 \delta g_{\mu\nu} \quad \Rightarrow \quad \boxed{m_1^2 \sim \frac{1}{r_c \ell} \ll \frac{1}{\ell^2} \sim m_2^2}$$

hierarchy

Stabilization mechanism (Goldberger & Wise)

There is an extra scalar d.o.f. corresponding to the brane separation.

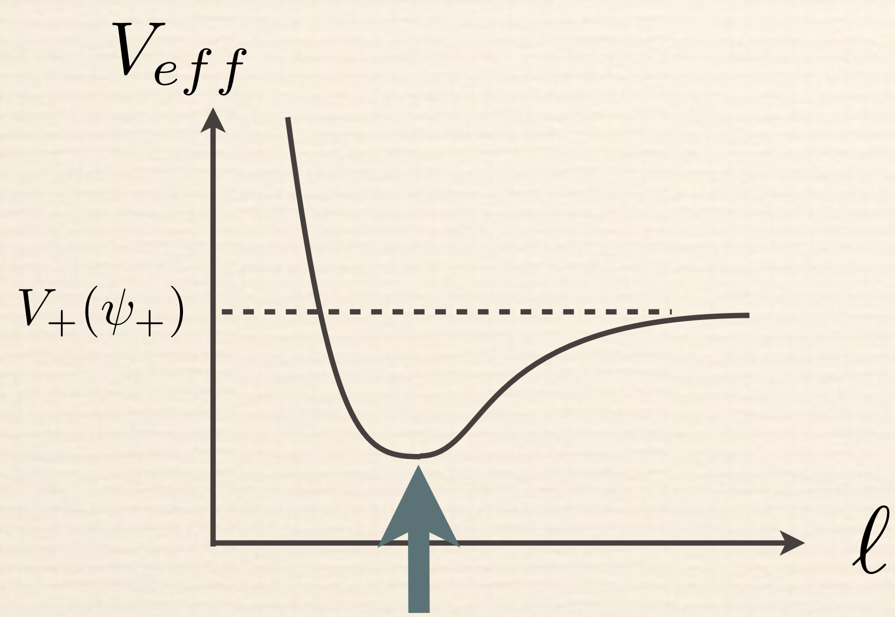
… We should remove it to reproduce a pure bigravity.

We introduce a stabilization scalar field to fix the brane separation.

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} \underline{V_{(\sigma)}(\psi)\delta(y-y_\sigma)}\right)$$

$\psi(y_\pm)$: fixed

$$\partial_y\psi \to \infty \quad \text{as} \quad \ell \to 0$$

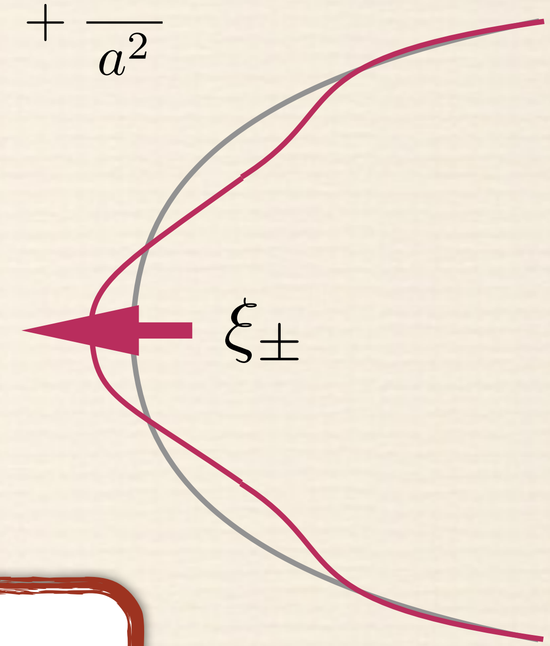


The distance between two branes are stabilized.

Normal and Self-accelerating branches

For simplicity, we consider a perturbation around a de Sitter brane solution with 4-d comoving curvature H.

$$ds^2 = dy^2 + a^2(y)\gamma_{\mu\nu}dx^\mu dx^\nu \qquad K^2 := \left(\frac{\partial_y a}{a}\right)^2 = \frac{1}{6M_5^3}\left(\frac{1}{2}\psi'^2 - V_B\right) + \frac{H^2}{a^2}$$



brane bending mode $\xi_\pm$

$$\left(1 \mp 2r_c^{(\pm)}K_\pm\right)\xi_\pm \propto \pm\frac{1}{(\Box + 4H^2)}T^{(\pm)}$$

To choose the healthy branch (normal branch),

$1 \mp 2r_c^{(\pm)}K_\pm > 0$ must be satisfied.

junction condition $K \sim r_c H^2 \longrightarrow H \lesssim \frac{1}{r_c^{(\pm)}}$: the 4-d curvature cannot be large.

DGP 2-brane model with stabilization mechanism which reproduces bigravity

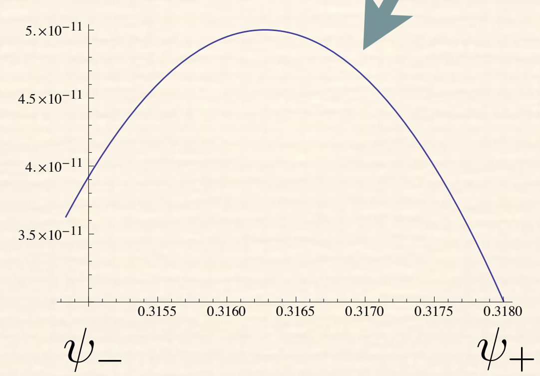
parameters

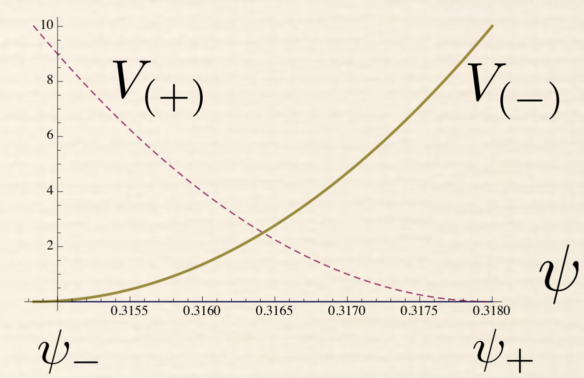
$$M_5 = 1.00$$

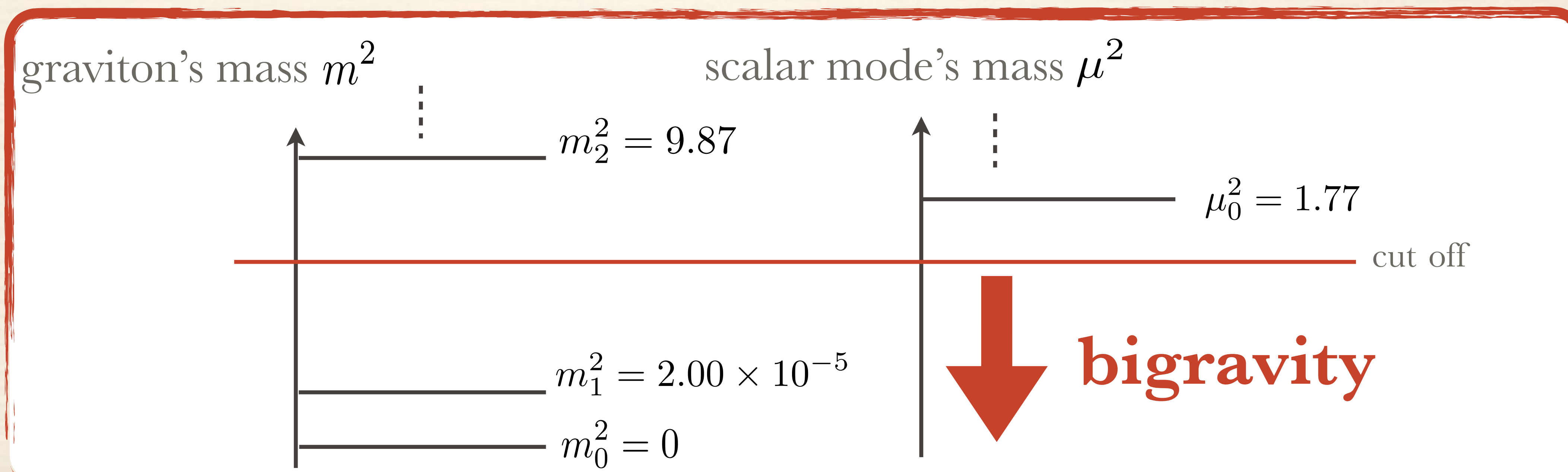
$$r_c^{(\pm)} = 1.00 \times 10^5\,, \quad \ell = 1.00$$

potential of scalar field

$$S_s = \int d^5x\sqrt{-g}\left(-\frac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - V_B(\psi) - \sum_{\sigma=\pm} V_{(\sigma)}(\psi)\delta(y-y_\sigma)\right)$$







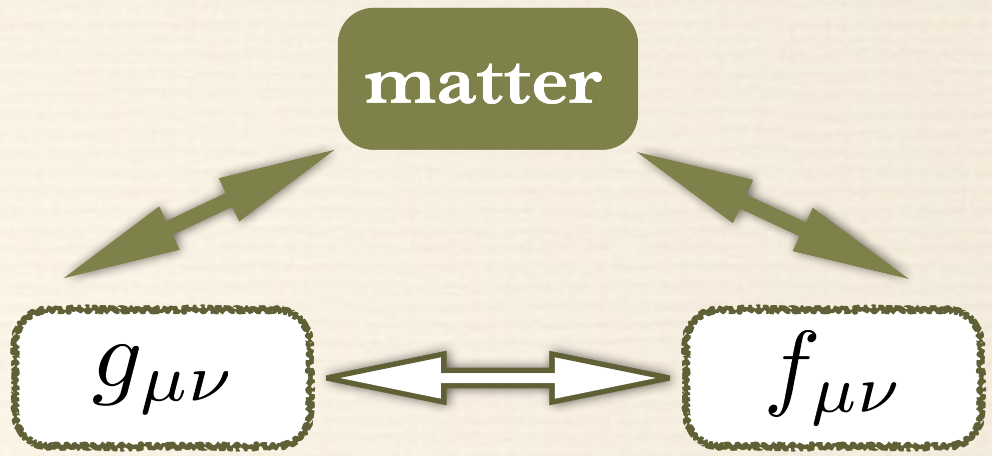
Let us see how bigravity arises as an effective theory from DGP 2-brane model.

- For simplicity, we neglect radion stabilization.
- Therefore we consider a system which contains two gravitons and one scalar.

…radion as a doubly coupled matter?

radion as a doubly coupled matter

In bigravity, doubly coupled matter generally breaks the ghost-free interaction structure.



In braneworld model, **radion couples both metrics ghost-freely**.

We expect to obtain a ghost-free doubly coupled matter model from radion.

Strategy to obtain bigravity action

We solve the bulk equations for given boundary metrics $g^{(\pm)}_{\mu\nu}$

$$\frac{1}{N}\partial_y K_{\mu\nu} = -2K^{\rho}_{\mu}K_{\rho\nu} + KK_{\mu\nu} + \frac{4}{\ell_\Lambda^2}g_{\mu\nu} - R_{\mu\nu} + \frac{1}{N}\nabla_\mu\nabla_\nu N$$

$$K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu} = -\frac{12}{\ell_\Lambda^2} + R \qquad K_{\mu\nu} = -\frac{1}{2N}\partial_y g_{\mu\nu}$$

gauge fix: $\partial_y N = 0\,,\ N^\mu = 0$

The momentum constraints is automatically imposed by the junction conditions:

$$K^{(\pm)}_{\mu\nu} - K^{(\pm)}g_{\mu\nu} = r_c^{(\pm)}G_{\mu\nu} \quad \to \quad \nabla_\mu K^{\mu}_{\nu} - \nabla_\nu K = 0$$

and obtain the effective action from

$$S = \frac{M_{pl}^2}{2r_c^{(+)}}\oint d^5x\sqrt{-g}(R + K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu} - \frac{12}{\ell_\Lambda^2}) + (\text{induced gravity term})$$

by substituting back the bulk metric solution $g_{\mu\nu}(y)$

and integrating out the bulk degree of freedom.

Gradient expansion

To obtain bigravity, the parameter is to be tuned as $\dfrac{\ell}{r_c^{(\pm)}} \ll 1 \;\to\; m^2 \simeq \dfrac{1}{r_c^{(\pm)}\ell}$

$r_c^{(\pm)} K \lesssim 1$ should be satisfied for the ghost-free branch.

gradient expansion

We calculate the effective action at the leading order the expansion in $K\ell \ll 1$

$$\Delta g_{\mu\nu} := g^{(+)}_{\mu\nu} - g^{(-)}_{\mu\nu} \sim \mathcal{O}\left(K\ell\right)$$

$$r_c^{(\pm)} \sim \mathcal{O}\left(1/K\right)\,, \quad m^2 \sim k^2 \sim \mathcal{O}\left(K/\ell\right)$$

To compute the effective action

Expand the metric around the middle point of the branes *(y=0)*

$$g^{(\pm)}_{\mu\nu} = \bar{g}_{\mu\nu} + \underbrace{\overline{\partial_y g_{\mu\nu}}}_{= -2\bar{K}_{\mu\nu}} y^{\pm} + \frac{1}{2}\underbrace{\overline{\partial_y{}^2 g_{\mu\nu}}}_{= -2N\overline{\partial_y K_{\mu\nu}}}(y^{\pm})^2 + \cdots$$

$$\frac{1}{N}\partial_y K_{\mu\nu} = -2K^{\rho}_{\mu}K_{\rho\nu} + KK_{\mu\nu} + \frac{4}{\ell_\Lambda^2}g_{\mu\nu} - R_{\mu\nu} + \frac{1}{N}\nabla_\mu\nabla_\nu N$$

$\Rightarrow$ $\bar{K}_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ can be written in terms of $g^{(\pm)}_{\mu\nu}$ and $\Phi := \frac{1}{2}N\ell$

Hamiltonian constraint $K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu} = -\frac{12}{\ell_\Lambda^2} + R$ determines Φ in terms of $g^{(\pm)}_{\mu\nu}$

The bulk action is expanded as

$$S_b \propto \int d^4x N dy \left[\sqrt{-\bar{g}}\left(\bar{R} - \frac{12}{\ell_\Lambda^2}\right) + y\overline{\frac{\delta\left(\sqrt{-g}\left(R - \frac{12}{\ell_\Lambda^2}\right)\right)}{\delta g_{\mu\nu}}}\,\overline{\partial_y g_{\mu\nu}} + \frac{y^2}{2}\overline{\partial_y\left(\frac{\delta\left(\sqrt{-g}\left(R - \frac{12}{\ell_\Lambda^2}\right)\right)}{\delta g_{\mu\nu}}\partial_y g_{\mu\nu}\right)} + \cdots\right]$$

Result

At the **leading order of gradient expansion**,

$$S = \frac{M_{pl}^2}{2}\frac{2}{r_c^{(+)}}\int d^4x\sqrt{-g}\left[\frac{\Delta g^2 - \Delta g_{\mu\nu}\Delta g^{\mu\nu}}{16\Phi} + \frac{\Phi}{3}\left(\nabla^\mu\nabla^\nu - g^{\mu\nu}\Box - R^{\mu\nu}\right)\left((\nabla_\mu\Phi)(\nabla_\nu\Phi) - \frac{\Phi^2}{\ell_\Lambda^2}g_{\mu\nu}\right)\right]$$

$$+\,(\text{induced gravity terms})$$

ℓ_Λ : 5-d cosmological constant

∇ is the covariant differentiation with respect to $g_{\mu\nu}$,

which is **indistinguishable** from $g^{(+)}_{\mu\nu}$, $g^{(-)}_{\mu\nu}$, and $\frac{1}{2}\left(g^{(+)}_{\mu\nu} + g^{(-)}_{\mu\nu}\right)$.

$\Phi := \frac{1}{2}N\ell$ is determined by the Hamiltonian constraint:

$$C := \bar{R} - \frac{12}{\ell_\Lambda^2} - \frac{\Delta g^2 - \Delta g_{\mu\nu}\Delta g^{\mu\nu}}{16\Phi^2} = 0 \qquad \text{where} \quad \bar{g}_{\mu\nu} = \frac{g^{(+)}_{\mu\nu} + g^{(-)}_{\mu\nu}}{2} + \Phi\nabla_\mu\nabla_\nu\Phi + \frac{\Phi^2}{\ell_\Lambda^2}g_{\mu\nu}$$

Result

treat Φ as an independent variable by adding $\lambda\left(\bar{R}-\frac{12}{\ell_\Lambda^2}-\frac{\Delta g^2-\Delta g_{\mu\nu}\Delta g^{\mu\nu}}{\Phi^2}\right)$

and eliminate Lagrange multiplier λ using EOM of Φ

absorbed by conformal trsf.

$$S=\frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-g_+}R_{(+)}+\chi\int d^4x\sqrt{-g_-}R_{(-)}\right.$$
$$+\frac{2}{r_c^{(+)}}\int d^4x\sqrt{-g}\left\{\frac{\Delta g^2-\Delta g_{\mu\nu}\Delta g^{\mu\nu}}{32\Phi}-\frac{1}{2\ell_\Lambda^2}\Phi^2\left(\Box+\frac{4}{\ell_\Lambda^2}\right)\Phi+\frac{\Phi}{2}\left(R-\frac{12}{\ell_\Lambda^2}\right)\right.$$
$$\left.\left.-\frac{1}{6}\underbrace{\left(\nabla_\mu\Phi\right)\left(\nabla_\nu\Phi\right)\left(\nabla^\mu\nabla^\nu-g^{\mu\nu}\Box-R^{\mu\nu}\right)\Phi}_{\text{cubic Galileon}}\right\}\right]$$

We obtain a well-known ghost-free system with two interacting gravitons and a scalar.

At the leading order of the gradient expansion,

we cannot examine
- form of nonlinear mass interactions
- the coupling of radion as a doubly coupled matter

Summary

- We want to derive the ghost-free bigravity from some more fundamental theory.

  … **DGP 2-brane model** mass spectrum can reproduce bigravity.

- We calculate the effective action under **gradient expansion**, in which the brane separation is so small that the metric does not change significantly along y-direction, by solving the bulk equations and integrating out the bulk degrees of freedom.
  We obtain **a well-known ghost-free bigravity and one scalar system**.

- The extension to the higher order of gradient expansion is difficult because it will produce complicated and higher-derivative interactions, which may correspond to the appearance of the other massive KK modes.

Future work

In order to investigate the higher order term in $g_{\mu\nu}^{(+)} - g_{\mu\nu}^{(-)}$,

we should avoid $K \lesssim 1/r_c^{(\pm)}$ (the ghost-free branch condition).

$\cdots$

- Choose the pathological branch and fix the radion by hand.

- Relax the ghost-free branch condition:

  The branch crossing occurs at the point that the scalar mode strongly couples to the source: $\left(1 \mp 2r_c^{(\pm)} K_\pm\right) \xi_\pm \propto \pm \frac{1}{(\Box + 4H^2)} T^{(\pm)}$

$\rightarrow$ Introduce Gauss-Bonnet term and weaken the coupling between the radio and metric effectively.

Correspondence between ghost-free bigravity and DGP 2-brane model with stabilization mechanism

When the two branes are almost flat,

DGP 2-brane model is identical to ghost-free bigravity.

ghost-free bigravity		**DGP 2-brane model**
two metrics	⟷	two metrics induced on the two branes
graviton's mass	⟷	the mass of the lowest massive mode

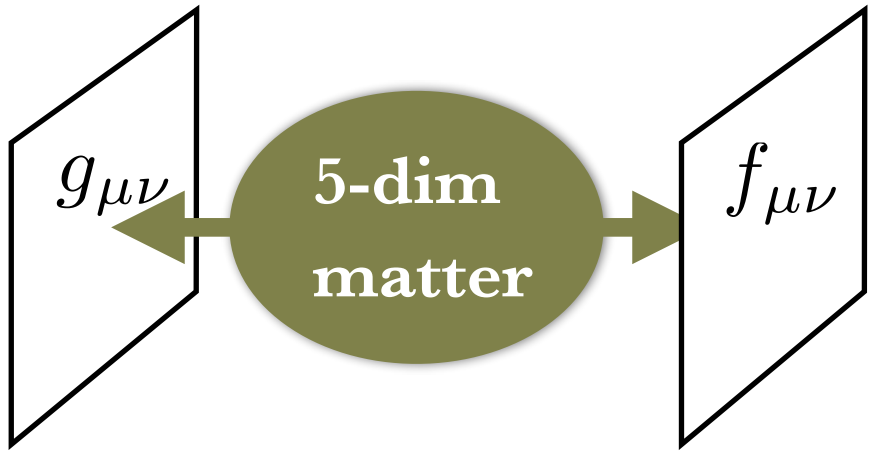
YY and Tanaka (2014)

However, can we really embed bigravity to braneworld setup?

➡ Consider **doubly coupled matter** to test this idea.

doubly coupled matter

brane model

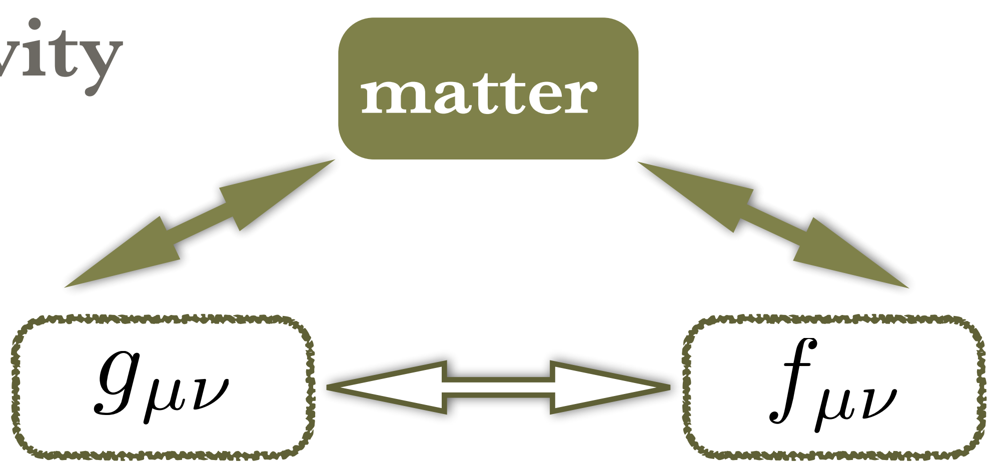


Introducing 5-d matter, we can naturally obtain a matter field which couples to both metrics.

$\rightarrow$ **BD ghost seems absent.**

contradiction

bigravity



coupling through the matter generally detunes the ghost-free structure of the interaction.

$\rightarrow$ **BD ghost appears?**

…There seems to be a difficulty in our attempt.

Seeking for models with doubly coupled matter which have no BD ghost

Introduce a k-essence scalar field

$$\mathcal{L}_m = \sqrt{-g}\, P(X,\phi) + \sqrt{-f}\, \tilde{P}(\tilde{X},\phi)$$

$$X = -\frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\,, \quad \tilde{X} = -\frac{1}{2} f^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$

Consider perturbation around FLRW and Bianchi type-1 spacetime

and evaluate the determinant and the eigenvalues of the kinetic matrix A.

When $\det A \neq 0$, an extra d.o.f. exists.

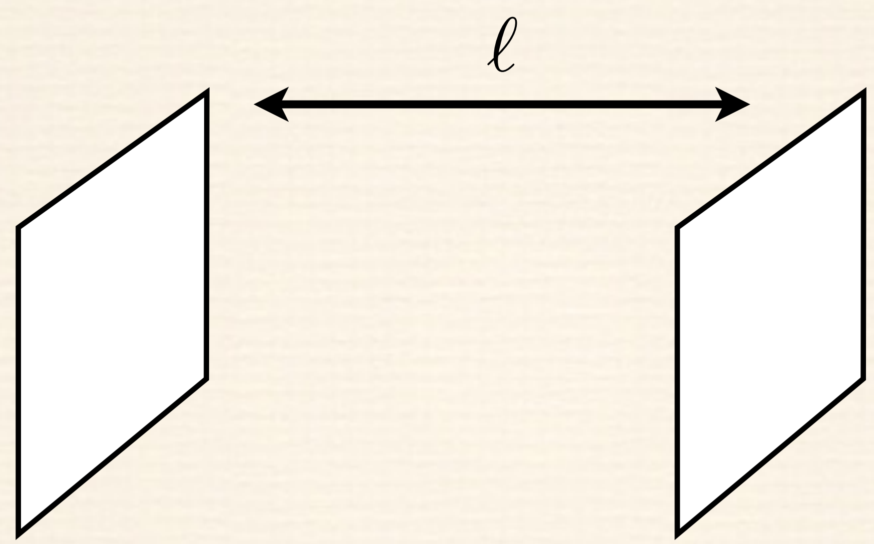
their signs clarify whether the d.o.f. is a ghost mode or not.

$\Rightarrow$ $\boxed{\textbf{BD ghost appears unless} \quad \tilde{P} = \tilde{P}(\phi) \ \text{ or } \ P = P(\phi)}$

YY, De Felice and Tanaka (2014)

Radion as a doubly coupled matter

Radion: a degree of freedom which corresponds to the brane separation



We will check how radion couples to the two metrics in 4-dim effective theory.

…We can obtain a ghost free model in bigravity with doubly coupled matter
or find how the correspondence breaks between ghost-free bigravity
and braneworld model.

bigravity and Boulware-Deser ghost

bigravity : gravity which contains two interacting gravitons

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R^{(g)} + \underline{2m^2V(g,\,f)}\right] + \frac{M_f^2}{2}\cancel{\int d^4x\sqrt{-f}\,R^{(f)}}$$

fix *f*

The interaction term breaks general covariance for *g*

$\Rightarrow$ $\underbrace{\text{GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0,}}_{\text{massive graviton}}\ \boxed{\text{helicity-0}}\text{)}$

This mode's kinetic term has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is removed by constraints.

ghost-free bigravity

Choosing the form of the interaction as

$$V = \sum_{n=0}^{4} c_n \epsilon^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} K^{\nu_1}_{\mu_1} \ldots K^{\nu_n}_{\mu_n} \qquad K^{\nu}_{\mu} = \sqrt{g^{\nu\rho} f_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)

ADM decomposition

$$N^{-2} = -g^{00}\,, \quad N_i = g_{0i}\,, \quad \gamma_{ij} = g_{ij}\,,$$
$$L^{-2} = -f^{00}\,, \quad L_i = f_{0i}\,, \quad {}^3f_{ij} = f_{ij}\,.$$

define new shift-like vector n^i
and rewrite N^i with n^i

Then Hamiltonian becomes linear in $N,\ L,\ L^i.$

$$H = N\mathcal{C} + LC^L + L^iC^L_i.$$

conjugate momentum

$\mathcal{C},\ C^L,\ C^L_i$ are functions of $\{\gamma_{ij},\ \pi^{ij},\ {}^3f_{ij},\ p^{ij}\}$

→ One of the Hamiltonian constraints kills BD ghost.

Hassan and Rosen (2012)

mass spectrum (scalar mode)

stabilization mechanism $\rightarrow$ **no massless mode**

If stabilization is weak: $\left|\dfrac{\partial_y \mathcal{H}}{\mathcal{H}^2}\right| \sim \dfrac{(\partial_y \psi)^2}{M_5^3 \mathcal{H}^2} \lessapprox 1$

the lowest mass becomes
$$\mu^2 \approx \frac{2\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^2} + \sum_\sigma \frac{2r_c^{(\sigma)}}{a_\sigma^2} \frac{1}{1-\sigma 2 r_c^{(\sigma)} \mathcal{H}_\sigma}}{\displaystyle\int_{y_+}^{y_-} \frac{dy}{a^4(-\mathcal{H}')}}$$

$\mathcal{H}$: 5-d curvature scale

- stronger stabilization ($\boxed{\text{large } |\mathcal{H}'|}$) $\longrightarrow$ **large μ^2**
- $\boxed{1 \mp 2 r_c^{(\pm)} \mathcal{H}_\pm < 0}$ make μ^2 negative

$\longrightarrow$ corresponds to the **self accelerating branch**

: K.Izumi et al. (2007)

ghost in DGP model

H : 4-dim comoving curvature scale

the regularity on +brane imposes

$$2\left(\sum_i \frac{u_i^2(y_+)}{m_i^2 - 2H^2}\right) + \frac{1}{H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\frac{2\kappa^2}{3H_+^2(2r_c\mathcal{H}_+ - 1)}\left(\sum_i \frac{v_i^2(y_+)}{\mu_i^2 + 4H^2}\right) + \mathcal{H}_+\right) = 0$$

diverges as $m^2 \to 2H^2$: Higuchi bound

diverges as $\mu^2 \to -4H^2$

: critical mass that scalar ghost appears

$2r_c\mathcal{H}_+ - 1 > 0$: self-accelerating branch

$\mu_i^2 + 4H^2 \to \mp\epsilon$ means $m_i^2 - 2H^2 \to \pm\epsilon$

ghost never disappears K.Izumi et. al. (2007)

$2r_c\mathcal{H}_+ - 1 < 0$: normal branch

The same identity prohibits m_i^2 & μ_i^2 from crossing their critical masses

no ghost

Normal and Self-accelerating branches

For simplicity, we consider the perturbation around a de Sitter brane solution, whose curvature is given as H.

$$ds^2 = dy^2 + a^2(y)\gamma_{\mu\nu}dx^\mu dx^\nu$$

$$K^2 := \left(\frac{\partial_y a}{a}\right)^2 = \frac{1}{6M_5^3}\left(\frac{1}{2}\psi'^2 - V_B\right) + \frac{H^2}{a^2}$$

$$\pm K_\pm = r_c^{(\pm)}\frac{H^2}{a^2} - \frac{1}{6M_5^3}V_{(\pm)}(\psi_\pm)$$

$$\rightarrow \quad K_\pm^2 \pm \frac{1}{r_c^{(\pm)}}K_\pm + \frac{1}{6M_5^3}\bar{V}_{(\pm)} = 0$$

$$\bar{V}_{(\pm)} = -\frac{1}{2}\psi_\pm'^2 + V_B(\psi_\pm) + \frac{1}{r_c^{(\pm)}}V_{(\pm)}(\psi_\pm)$$

two branches for each brane:

$$1 - 2r_c^{(+)}K_+ = \pm\sqrt{1 - \frac{2}{3M_5^3}r_c^{(+)}\bar{V}_{(+)}} \qquad 1 + 2r_c^{(-)}K_- = \pm\sqrt{1 - \frac{2}{3M_5^3}r_c^{(-)}\bar{V}_{(-)}}$$

collapse of the structure in DGP model

junction condition

$$K_{\mu\nu}^{(\pm)} = r_c^{(\pm)} \left(G_{\mu\nu}^{\pm(4)} - \frac{1}{3} G^{\pm(4)} g_{\mu\nu} \right)$$

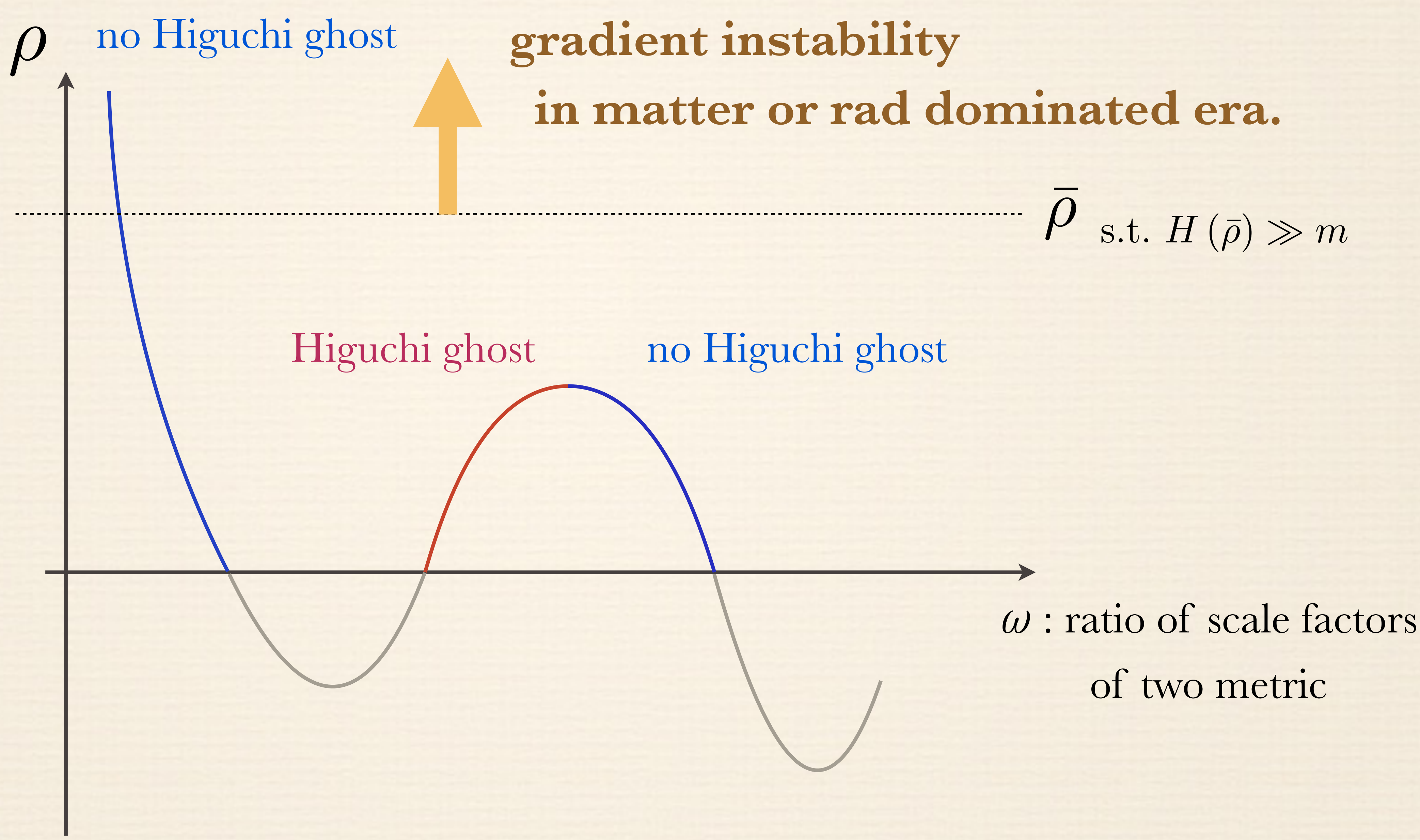
When we consider to increase the energy scale on the branes, the curvature scale also increase.

On the other hand,

$|\mathcal{H}| \lesssim \dfrac{1}{r_c^{\pm}}$ must be satisfied to avoid scalar-mode instability

→ slightly curved branes cause instability and break the stabilization!

Cosmological solution in ghost-free bigravity



Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1+(\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

this sign determines the ghost appearance

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

For flat vacuum solution, $H \to 0$ as $\omega \to \omega_0$ where $\rho_m(\omega_0) \to 0$,

$$f'(\omega_0) = -3\left(1+\frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

negative when $\Gamma > 0$ i.e. $m_{eff}^2 > 0$

→ **no Higuchi ghost**

Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

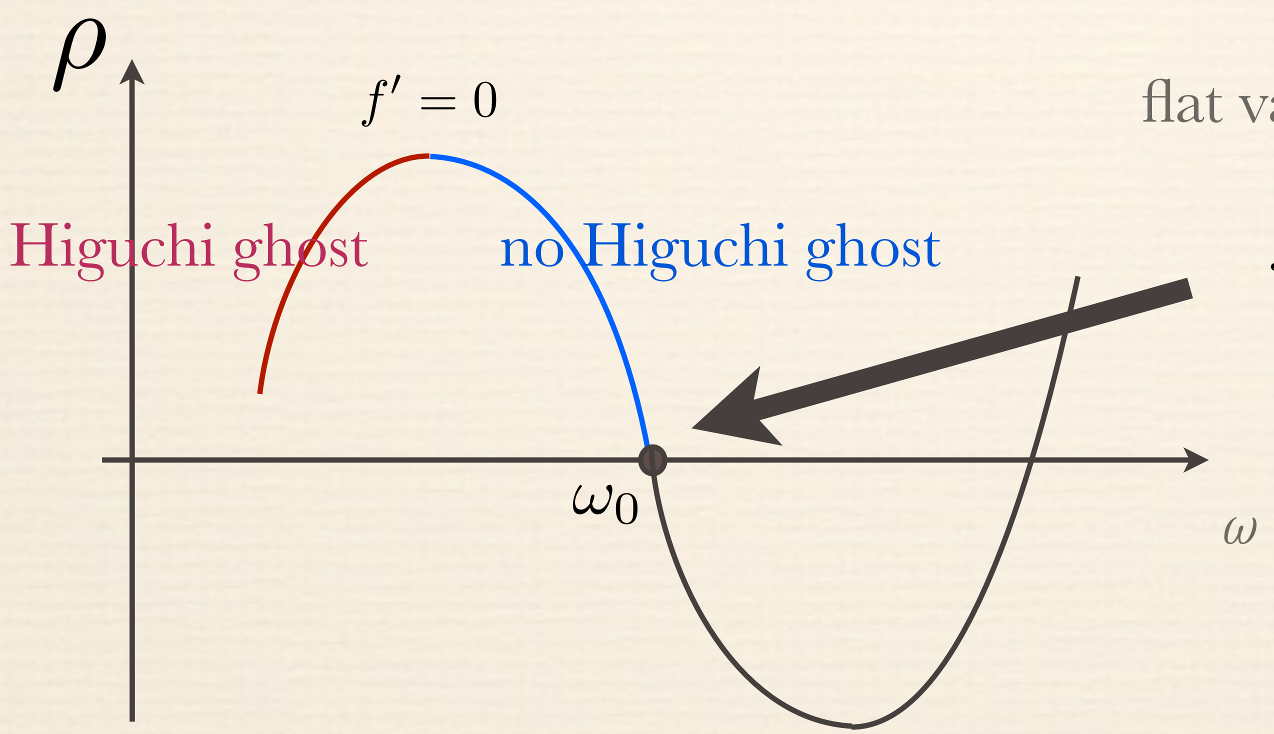
ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1+(\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

this sign determines the ghost appearance



flat vacuum $H = 0, \rho = 0$

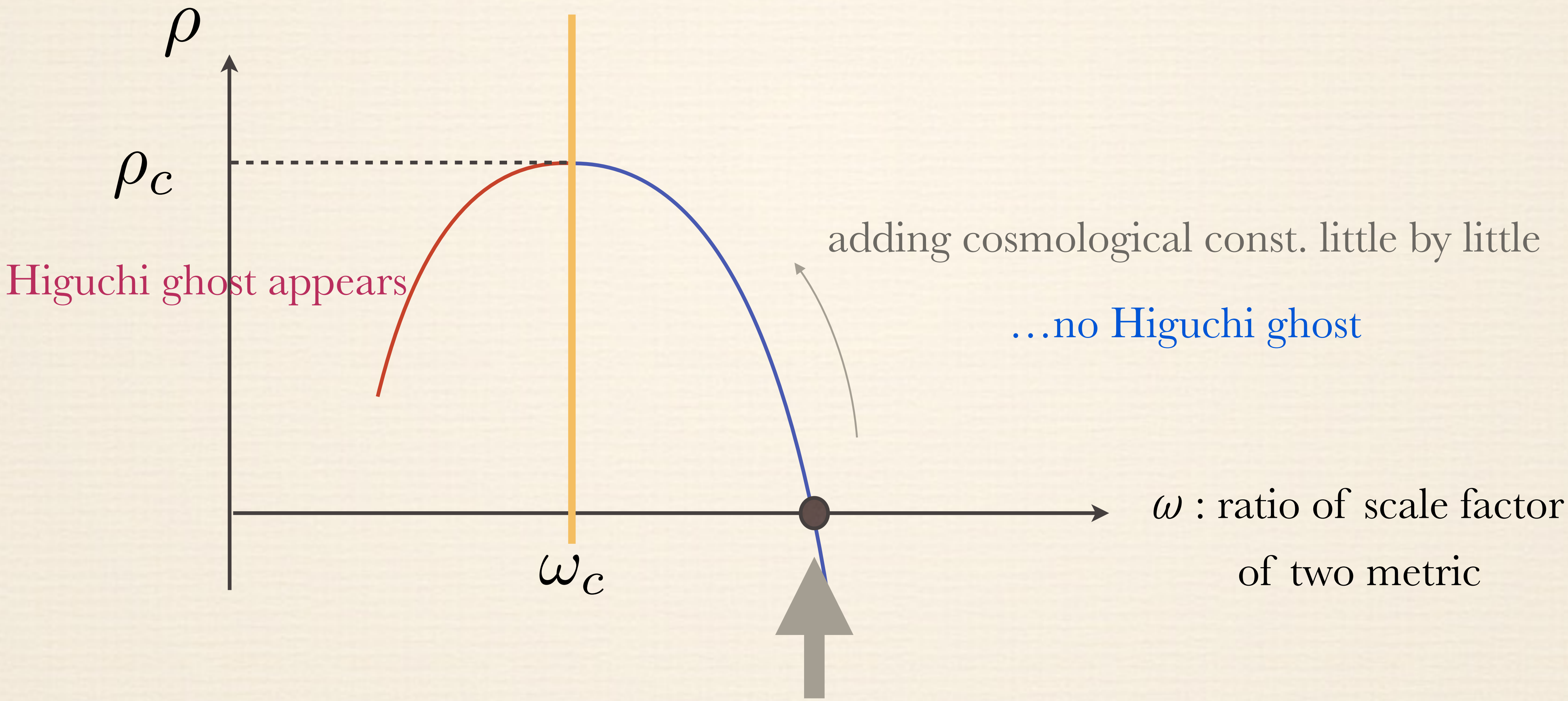
$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

...negative

when $\Gamma > 0 \Leftrightarrow m_{eff}^2 > 0$

Higuchi ghost in dRGT bigravity

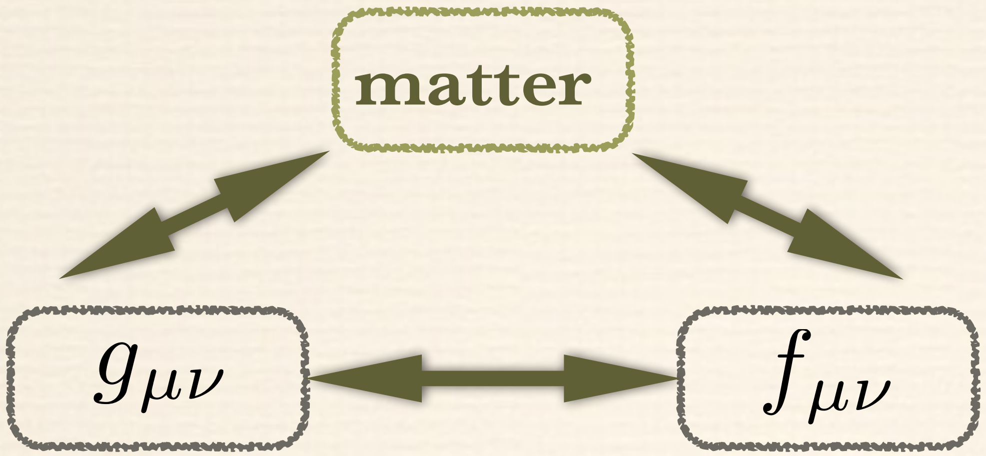
de Sitter solution does not exist above this critical density,
and Higuchi ghost appears after crossing the critical ω.



choose the branch connected to the vacuum flat spacetime
with positive graviton mass

doubly coupled matter

However,



coupling through the matter generally detunes the ghost-free structure of the interaction.

$\rightarrow$ **BD ghost?**

Consider a free scalar field which couples to both metric:

$$\mathcal{L}_m = \sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) + \sqrt{-f}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

conjugate momentum $\quad \pi_\phi \sim \left(\frac{1}{N} + \frac{1}{L}\right)\partial_t\phi$

Hamiltonian $\quad \mathcal{H} \ni \frac{NL}{N+L}\pi_\phi^2 \quad$ …nonlinear in the lapse fcns $\rightarrow$ **BD ghost!**

Seeking for models with doubly coupled matter which have no BD ghost

- another ghost-free model motivated by the quasi-dilaton massive gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_g^2 R^{(g)}}{2} + 2m^2 M_{\rm eff}^2 \sum_n c_n e_n\left(\sqrt{g^{\mu\nu}(f_{\mu\nu} + \alpha\partial_\mu\phi\,\partial_\nu\phi)}\right)\right] + \int d^4x\sqrt{-f}\left[\frac{M_f^2 R^{(f)}}{2} - \frac{1}{2} f^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$$

YY, De Felice and Tanaka (2014)

- matter which couples to an effective metric

$$g_{\mu\nu}^{\rm eff} = a^2 g_{\mu\nu} + 2ab g_{\mu\alpha}\sqrt{g^{\alpha\beta}f_{\beta\nu}} + b^2 f_{\mu\nu}$$

This model has BD ghost, but it appears beyond the strong coupling scale.

de Rham, Heisenberg and Rebeiro (2014)

The model of doubly coupled matter is considerably restricted.

… inconsistent with the intuition in braneworld models.

Result

$$S = \frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-\gamma}\, m_*^2 \left\{\Delta h^2 - \Delta h^{\mu\nu}\Delta h_{\mu\nu} - \frac{3}{4}\Phi\left(1-\alpha^2 H^2\left(\Box + 4H^2\right)\right)\Phi\right\}\right.$$
$$\left. + \int d^4x\sqrt{-g_{(+)}}\left(R_{(+)} - \frac{6H^2}{a_+^2}\right) + \chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)} - \frac{6H^2}{a_-^2}\right)\right]$$

$$\Phi := \Delta h + \frac{4\alpha}{3}\bar{R}^{(1)}$$

$$\alpha := \frac{-y_0^+\mathcal{H}^{-1}(0)}{2}$$

treat Φ as an independent variable

by adding $\lambda\left(\Phi - \Delta h - \frac{4\alpha}{3}\bar{R}^{(1)}\right)$

$$S = \frac{M_{pl}^2}{2}\left[\int d^4x\sqrt{-\gamma}\, m_*^2 \left\{\Delta h^2 - \Delta h^{\mu\nu}\Delta h_{\mu\nu} - \frac{3}{4}\alpha^2 H^2\Phi\left(\Box + 4H^2\right)\Phi + \frac{3}{4}\Phi\left(\Phi - 2\Delta h\right) - \alpha\Phi\left(R_{(+)}^{(1)} + R_{(-)}^{(1)}\right)\right\}\right.$$
$$\left. + \int d^4x\sqrt{-g_{(+)}}\left(R_{(+)} - \frac{6H^2}{a_+^2}\right) + \chi\int d^4x\sqrt{-g_{(-)}}\left(R_{(-)} - \frac{6H^2}{a_-^2}\right)\right]$$

conformal trsf

…two gravitons interacting through Fierz-Pauli mass term

and one scalar whose kinetic term couples to γ …no BD ghost

Equations of motion

$$h^{(i)\,TT}_{\mu\nu} = \frac{-2M_{pl}^{-2}}{a_+^2 + a_-^2\chi}\left[\frac{1}{\Box - 2H^2 - m_i^2}\left\{T^{(i)}_{\mu\nu} - \frac{1}{4}T^{(i)}\gamma_{\mu\nu} + \frac{1}{3\left(m_i^2 - 2H^2\right)}\left(\nabla_\mu\nabla_\nu - \frac{\Box}{4}\gamma_{\mu\nu}\right)T^{(i)}\right\}\right.$$
$$\left. - \frac{1}{3\left(m_i^2 - 2H^2\right)}\left(\nabla_\mu\nabla_\nu - \frac{\Box}{4}\gamma_{\mu\nu}\right)\frac{1}{\Box + 4H^2}T^{(i)}\right]$$

$$T^{(0)}_{\mu\nu} := T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}$$
$$T^{(m)}_{\mu\nu} := \frac{T^{(+)}_{\mu\nu}}{a_+^2} - \frac{T^{(-)}_{\mu\nu}}{a_-^2\chi}$$

Poles at $\Box - 2H^2 = 0,\ m^2$ and $\Box + 4H^2 = 0$

…one massless and one massive gravitons and one scalar (radion)

We find the sign of the coefficient of the pole $\Box + 4H^2 = 0$ flips at

$$2a_\pm^2\chi_\pm r_c^{(+)}\mathcal{H}_\pm - 1 = 0$$

…equivalent to the condition for the ghost-free branch

We succeeded to obtain a ghost-free bigravity+scalar system.