The phenomenology of the Minimal Theory of Massive Gravity

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[with prof. Mukohyama, PLB 2016, JCAP 2016, arXiv:1607.03368]

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Introduction

- Einstein theory: great, beautiful achievement
- Fantastic success
- Phenomenology: black holes, neutron stars, gravitational waves (detected)
- Sorry Einstein, but what if…

Revolution

- 2011 Nobel Prize: discovery of acceleration at large scales
- What drives it accounts for 68% of the total matter distribution
- What is it?

Gravity is changing?

- Maybe not
- Simply a cosmological constant
- But if it does, how?
- The theory must be a sensible one
- No ghosts, viable, and phenomenologically interesting

Modified gravity models

- f(R) models [Capozziello: IJMP 2002; ADF, Tsujikawa: LRR 2010]
- Extra dim.: DGP [Dvali, Gabadadze, Porrati: PLB 2000]
- Effective theories of ED (Galileons)
  [Nicolis, Rattazzi, Trincherini: PRD 2009]
- Horndeski (generalized Galileon) [Horndeski: IJTP 1974]
- Bigravity [Hassan, Rosen: JHEP 2012;
  ADF, Nakamura, Tanaka: PTEP 2014; Sakakihara, Soda: JCAP 2015]

dRGT Massive gravity

[de Rham, Gabadadze, Tolley: PRL 2011]

- What if the graviton has a mass?
- Boulware-Deser theorem: in general a ghost is present
- Can this ghost be removed?
- dRGT showed that it is possible
- Formal proofs [Hassen, Rosen: JHEP 2012; Kugo, Ohta: PTEP 2014]
- If so, what kind of theory is this?

Lagrangian of Massive gravity

- Introduce the Lagrangian

$$\mathscr{L} = \frac{M_P^2}{2}\sqrt{-g}\left[R - 2\Lambda + 2m_g^2 \mathscr{L}_{MG}\right], \qquad \mathscr{L}_{MG} = \mathscr{L}_2 + \alpha_3 \mathscr{L}_3 + \alpha_4 \mathscr{L}_4,$$

$$K^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu},$$

$$\mathscr{L}_2 = \frac{1}{2}\left([K^2] - [K]^2\right),$$

$$\mathscr{L}_3 = \frac{1}{6}\left([K]^3 - 3[K][K^2] + 2[K^3]\right),$$

$$\mathscr{L}_4 = \frac{1}{24}\left([K]^4 - 6[K]^2[K^2] + 3[K^2]^2 + 8[K][K^3] - 6[K^4]\right).$$

New ingredient: fiducial metric

- Non-dynamical object: fiducial metric
- In terms of 4 new scalars, it can be written as

$$f_{\mu\nu} = f_{ab}(\phi^c)\,\partial_\mu \phi^a \partial_\nu \phi^b$$

- The explicit form f_{ab} must be given
- What is this theory? How to fix the fiducial metric?

dRGT gravity: degrees of freedom

- Introducing 4 new scalar fields: Stuckelberg fields
- Then 4 sc dof, 4 vct dof, 2 Gws dof + 4 SF dof
- Unitary gauge (remove 4 SF dof): 4 sc , 4 vct, 2 Gws dof
- Constraints kill 2 sc dof and 2 vect dof: 2+2+2=6 dof
- dRGT kills one mode, the BD ghost. Finally only 5 dof.

No stable FLRW solutions

- FLRW background allowed
  [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011][Langlois, Naruko: CQG 12/13]
- But no stable FLRW exists: one of the 5 dof is ghost
  [ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]
- Inhomogeneity? Anisotropies? [D'Amico et al: PRD 2011]
  [E. Gumrukcuoglu, C. Lin, S. Mukohyama: JCAP 2011. ADF, EG, SM: JCAP 2012]
- Something else?

Introduction/motivation

- Massive gravity as an alternative to dark energy
- How to remove BD ghosts?
- dRGT action able to remove it.
- However, cosmology not viable
- One of the expected “good” modes becomes a ghost
- How to remove this unwanted mode?

[ADF, E. Gumrukcuoglu, S. Mukohyama: PRL 2012]

Theory of Minimal Massive gravity (part I)

[ADF, S. Mukohyama: arXiv:1506.01594; PLB752 2016]

- Reconsider dRGT in vielbein formalism $g_{\mu\nu} = e^A{}_\mu e^B{}_\nu \eta_{AB}$
- Introduce ADM vielbein

$$\bar{e}^0{}_0 = N, \bar{e}^I{}_0 = \bar{e}^I{}_k N^k, \bar{e}^0{}_i = 0, \bar{e}^I{}_j, \quad \bar{e}^A{}_\mu = \begin{pmatrix} N & 0 \\ N^k e^I{}_k & e^I{}_j \end{pmatrix}$$

- Do not boost/rotate: break Lorentz inv. [BLI: Comelli et al JHEP '13]
- Consider dRGT Lagrangian but substitute ADM vielbein

Precursor action

- ADM fiducial vielbein $e^A{}_\mu = \begin{pmatrix} N & 0 \\ N^k e^I{}_k & e^I{}_j \end{pmatrix}, \quad E^A{}_\mu = \begin{pmatrix} M & 0 \\ M^k E^I{}_k & E^I{}_j \end{pmatrix}$
- Build 3D metric $g_{ij} = e^I{}_i e^J{}_j \delta_{IJ}$
- We define $\mathscr{L}_{pre} = \mathscr{L}_{GR} + \frac{1}{2} M_P^2 m^2 \sum_{n=0}^{4} c_n \mathscr{L}_n$
- $\mathscr{L}_0 = \frac{1}{24} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu E^C{}_\varrho E^D{}_\sigma \qquad \mathscr{L}_1 = \frac{1}{6} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu E^C{}_\varrho e^D{}_\sigma$

$$\mathscr{L}_2 = \frac{1}{4} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu E^B{}_\nu e^C{}_\varrho e^D{}_\sigma \qquad \mathscr{L}_3 = \frac{1}{6} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} E^A{}_\mu e^B{}_\nu e^C{}_\varrho e^D{}_\sigma$$

$$\mathscr{L}_4 = \frac{1}{24} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{ABCD} e^A{}_\mu e^B{}_\nu e^C{}_\varrho e^D{}_\sigma$$

Degrees of freedom for precursor action

- We consider $e^I{}_j$ as the dynamical variables
- N, N^i Lagrange multipliers
- Unitary gauge. Fiducial veilbein: given functions of time
- In phase space there are 2*9 = 18 variables
- Hamiltonian formalism

Precursor Hamiltonian

- Primary Hamiltonian

$$H^{(p)}_{pre} = \int d^3x \left[-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} \right]$$

where

$$R_0 = R_0^{GR} - m^2 H_{0,} \quad R_i = 2\gamma_{ik} D_j \pi^{kj}, \quad \pi^{jk} = \delta^{IJ} \Pi^j{}_I e_J{}^k$$

$$H_0 = \sqrt{\tilde{\gamma}} \left(c_1 + c_2 Y_I{}^I \right) + \sqrt{\gamma} \left(c_3 X_I{}^I + c_4 \right), \quad Y_I{}^J = E_I{}^k e^J{}_k$$

$$H_1 = \sqrt{\tilde{\gamma}} \left[c_1 Y_I{}^I + \frac{1}{2} c_2 \left(Y_I{}^I Y_J{}^J - Y_I{}^J Y_J{}^I \right) \right] + c_3 \sqrt{\gamma}, \quad X_I{}^J = e_I{}^k E^J{}_k,$$

$$P^{[MN]} = e^M{}_j \Pi^j{}_I \delta^{IN} - e^N{}_j \Pi^j{}_I \delta^{IM}$$

Secondary precursor Hamiltonian

- Time derivative of the primary constraints
- $\dot{P}^{[MN]} \approx 0 \;\Rightarrow\; Y^{[MN]} \approx 0$
- Only two other secondary contraints: $\tilde{C}^{\sigma}, \;\; \sigma = 1,2$

$$\dot{R}_0 = \{R_0, H^{(p)}\} + \frac{\partial R_0}{\partial t} \approx 0, \;\; \dot{R}_i = \{R_i, H^{(p)}\} \approx 0,$$

$$\{R_0, R_0\} \approx 0, \;\; \{R_i, R_j\} \approx 0, \;\; \{R_0, R_i\} \neq 0$$

so

$$H^{(s)}_{pre} = \int d^3x [-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} + \tilde{\lambda}^{\sigma} \tilde{C}_{\sigma}]$$

Dof for precursor theory

- No more (tertiary) constraints
- All constraints are of second class
- Therefore 12 s.c. constraints: $R_0, \; R_i, \; P^{[MN]}, \; Y^{[MN]}, \; \widetilde{C}^\sigma$
- Therefore (9 * 2 – 12) / 2 = 3 modes remaining
- In fact, Gws + 1 scalar mode, on normal branch
- Strong coupling expected

Removing the scalar dof

- Consider that $H_{pre}^{(s)} \approx \bar{H}_1 \equiv \int d^3x\, m^2 M H_1$
- Consider now

$$C_0 \equiv \{R_0, \bar{H}_1\} + \frac{\partial R_0}{\partial t},$$
$$C_i \equiv \{R_i, \bar{H}_1\}$$

- $\tilde{C}_\sigma \approx$ two linear combinations of C_i

Theory of minimal massive gravity (part II)

- The theory is defined by imposing 4 constraints (instead of 2)

$$C_0 \approx 0, \quad C_i \approx 0$$

- Hamiltonian

$$H = \int d^3x \left[-N R_0 - N^i R_i + m^2 M H_1 + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} + \lambda C_0 + \lambda^i C_i \right]$$

- 14 second class constraints
- (9 * 2 – 14) / 2 = 2 dof

FLRW vacuum

- Let us study the FLRW background for given $M(t), E^I{}_j = \tilde{a}(t)\delta^I_j$
- Since

$$\dot{R}_0 = \{R_0, H\} + \frac{\partial R_0}{\partial t} = C_0 + \int dy\,\lambda\{R_0, C_0(y)\} + \ldots$$

$\lambda(t) = 0$ on the background

- Friedmann equation

$$3H^2 = \frac{m^2}{2}(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3), \quad H \equiv \frac{\dot{a}}{Na}, \quad X \equiv \frac{\tilde{a}}{a}$$

- Two branches exist

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0.$$

As for the background...

- The theory has the the same background of dRGT
- In particular: existence of self-accelerating sol.

$$X = X_{\pm} = \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1}$$

- Existence of normal branch but only with 2 GWs

As for the perturbations…

- No scalar perturbations
- No vector perturbations
- Tensor perturbations do exist

$$S=\frac{M_P^2}{8}\sum_{\sigma}\int d^4x\,N(t)\,a(t)^3\left[\frac{\dot{h}_\sigma^2}{N^2}-\frac{(\partial h_\sigma)^2}{a^2}-\mu^2 h_\sigma^2\right],$$

$$\mu^2=\frac{1}{2}m^2(c_3+c_2X)\left(X-\frac{M}{N}\right)>0$$

Matter fields

- How to couple matter fields?
- Lorentz breaking terms present only in gravity sector
- Matter Lagrangian coupled only to physical metric
- Standard matter Lagrangians are invariant under a general Lorentz transformation of vielbeins

$$g_{\mu\nu} = e^{A}{}_{\mu} e^{B}{}_{\nu} \eta_{AB},$$

Lagrangian of MTMG

- Having defined the theory, can we find the Lagrangian?
- As usual, use inverse Legendre transformation
- Find $\dot{e}^I{}_j = \{e^I{}_j, H_T\}$
- Invert to find momentum as function of $\dot{e}^I{}_j$
- Lagrangian: $\mathscr{L} = \int d^3x \, \Pi_I{}^j \dot{e}^I{}_j - H_T$

Metric formalism

- Consider two given external fields and 3D metric

$$\tilde{\gamma}_{ij}=\delta_{IJ}E^I{}_i E^J{}_j, \quad \tilde{\zeta}^i{}_j=\frac{1}{M}E_L{}^i\dot{E}^L{}_j, \quad \gamma_{ij}=\delta_{IJ}e^I{}_i e^J{}_j,$$

- Define the tensors

$$\kappa^m{}_l\kappa^l{}_n=\tilde{\gamma}^{ms}\gamma_{sn}, \quad k^m{}_j\kappa^j{}_n=\delta^m{}_n,$$

$$\Theta^{ij}=\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}\{c_1(\gamma^{il}\kappa^j{}_l+\gamma^{jl}\kappa^i{}_l)+c_2[\kappa(\gamma^{il}\kappa^j{}_l+\gamma^{jl}\kappa^i{}_l)-2\tilde{\gamma}^{ij}]\}+2c_3\gamma^{ij}$$

Precursor Lagrangian

- GR action $S_{GR}=\frac{M_P^2}{2}\int d^4x\,N\sqrt{\gamma}[{}^{(3)}R+K^{ij}K_{ij}-K^2]$
- Precursor action

$$S_{pre}=S_{GR}+\frac{M_P^2}{2}\sum_{i=1}^{4}\int d^4x\,S_i,$$

$$S_1=-m^2c_1\tilde{a}^3(N+M\kappa),$$

$$S_2=-\frac{1}{2}m^2c_2\tilde{a}^3(2N\kappa+M\kappa^2-M\kappa^i{}_j\kappa^j{}_i),$$

$$S_3=-m^2c_3\sqrt{\gamma}(M+Nk),$$

$$S_4=-m^2c_4\sqrt{\gamma}.$$

Lagrangian of MTMG in metric formalism

- The action is found to be

$$S=S_{pre}+\frac{M_P^2}{2}\int d^4x\,N\sqrt{\gamma}\left(\frac{m^2N\lambda}{4N}\right)^2\left(\Theta_{ij}\Theta^{ij}-\Theta^2/2\right)$$
$$-\frac{M_P^2}{2}\int d^4x\sqrt{\gamma}\left[\lambda C_0-\left(D_n\lambda^i\right)C^n{}_i\right]+S_{mat}$$

- where

$$C_0=\frac{1}{2}m^2MK_{ij}\Theta^{ij}-m^2M\left(\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}\left[c_1\xi+c_2\left(\kappa\xi-\kappa^m{}_n\xi^n{}_m\right)\right]+c_3k^m{}_n\xi^n{}_m\right)$$
$$C^n{}_i=m^2M\left(\frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}}\left[c_1\kappa^n{}_i+c_2\left(\kappa\kappa^n{}_i-\kappa^n{}_l\kappa^l{}_i\right)\right]+c_3\delta^n{}_i\right)$$

What is the content of MTMG?

- The Lagrangian written in 1+ 3 ADM formalism
- Non-trivial constraints, which are not only potential-like
- The Lagrangian has constraints
- The theory is simple(r)
- Why? Only Gws exists (non-linearly, on any background)

FLRW in the presence of matter field

- The background is the same as of dRGT (with matter)
- Still self-accelerating and normal branch exist

$$\left(c_1 X^2 + 2c_2 X + c_3\right)\left(X H_f - H\right) = 0, \quad H = \frac{\dot{a}}{N a}, \quad H_f = \frac{\dot{\tilde{a}}}{M \tilde{a}}, \quad X = \frac{\tilde{a}}{a}$$

- On the background $\lambda = 0$
- Discussion of the two branches

Self-accelerating branch

- X is constant: effective cosmological constant
- Introduce a perfect fluid, as test matter field
- In cosmological perturbation theory, at linear order, the theory exactly reduces to GR for scalar and vector, but Gws are massive
- No more ghost in dRGT

$$\mu^2 = \frac{1}{2} m^2 X [c_2 X + c_3 r X (c_1 X + c_2)]$$

- Phenomenology reduces to GR except for GWs

Normal branch

- The background is non-trivial

$$3M_P^2H^2=\frac{1}{2}c_4m^2M_P^2+\rho_X+\rho_m,\quad \rho_X=\frac{1}{2}m^2M_P^2\left(3c_3X+3c_2X^2+c_1X^3\right)$$

- Perturbation scalar equation

$$\mathscr{L}=M_P^2Na^3Q\left[\frac{1}{N^2}\dot{\delta}_m^2+4\pi G_{eff}\rho_m\delta_m^2\right]$$

- Geff is model dependent

Normal branch

- Also this branch is regularized: no more Higuchi ghost
- The phenomenology is different from GR for both scalar and tensor degrees of freedom
- It depends on the choice of the fiducial metric
- Let us put MTMG to the test

Normal branch to the test

- Let us consider simplest choice for external fields
- X = constant, (implies r = 1)
- Background becomes Lambda-CDM, exactly

$$3H^2 = 8\pi G_N(\rho_\Lambda + \rho_m), \quad (H^2)' = -8\pi G_N(\rho_m + P_m),$$
$$\rho_\Lambda = \frac{1}{2} m^2 M_P^2 (c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3), \quad N = -\ln(1+z)$$

- Therefore it will have same background constraints

Scalar perturbations

- No ghost condition is always verified: it is a square
- Modified high-k dynamics

$$\delta_m'' + \left(2 - \frac{3}{2}\Omega_m\right)\delta_m' + \left(\frac{9\theta Y\Omega_m}{2(\theta Y - 2)^2} + \frac{3}{\theta Y - 2}\right)\Omega_m\delta_m = 0,$$

$$\Omega_m' = 3\Omega_m(\Omega_m - 1), \qquad \Omega_M = 8\pi G_N \rho_m/(3H^2)$$

$$Y' = 3Y\Omega_m, \qquad Y = (H_0/H)^2$$

$$\theta = \frac{\mu^2}{H_0^2} = \frac{1}{2}\frac{m^2}{H_0^2}X_0(c_1 X_0^2 + 2c_2 X_0 + c_3)$$

Dynamics of matter perturbation

- Dynamical eom for matter perturbation

$$\delta_m'' + \left(2 - \frac{3}{2}\Omega_m\right)\delta_m' + \left(\frac{9\theta Y \Omega_m}{2(\theta Y - 2)^2} + \frac{3}{\theta Y - 2}\right)\Omega_m \delta_m = 0,$$

- It is k-independent as in Lambda-CDM
- Early time Lambda-CDM, as Y approaches 0
- Smooth GR limit, $\theta = 0$

Redshift space distortion measurements

- Dynamics of perturbation is modified at late times
- Observable to be constrained:

$$y = f(z)\sigma_8(z), \quad f = \frac{\delta_m{}'}{\delta_m}, \quad \sigma_8 \propto \delta_m,$$

- If constant of proportionality is left as a free parameter
- In Lambda-CDM, RSD: $\sigma_8(0) \simeq 0.7,$
- In Lambda-CDM, Planck: $\sigma_8(0) \simeq 0.8159 \pm 0.0086,$

RSD and MTMG

- Constraint on MTMG using RSD
- Same observable $y = f(z)\sigma_8(z), \quad f = \frac{\delta_m'}{\delta_m}, \quad \sigma_8 = \sigma_8(N_i)\frac{\delta_m}{\delta_m(N_i)},$
- N_i initial time during deep matter era ($N_i = -6$)
- $\sigma_8(N_i)$ same for GR and MTMG (taken at high redshift, and known using GR evolution + Planck data)
- ICs: $\delta_m'(N_i) = \delta_m(N_i), \; Y(N=0) = 1, \; \Omega_m(N=0) = 0.3089$

Fit to the data

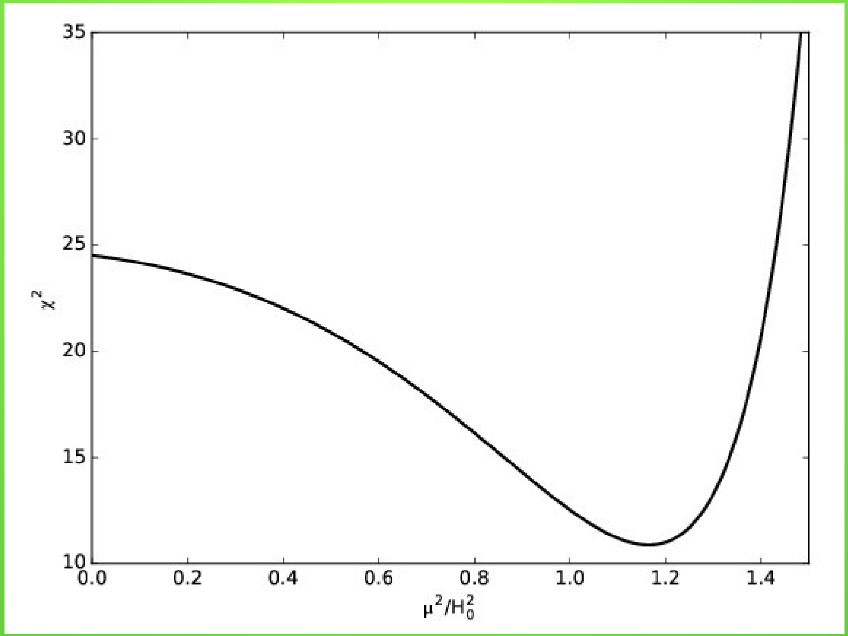
- Define chi-square functions

$$\chi^2_{MTMG} = \sum_n \frac{\left(y_n - y_n^{MTMG}\right)^2}{\sigma_n^2}, \quad \chi^2_{GR} = \sum_n \frac{\left(y_n - y_n^{GR}\right)^2}{\sigma_n^2},$$

- Fix all ICs and $\sigma_8(N_i)$ using Lambda-CDM best fit
- Only one left-over free parameter, θ
- Therefore $\chi^2_{MTMG} = \chi^2_{MTMG}(\theta), \quad \chi^2_{GR} = \chi^2_{MTMG}(\theta = 0)$

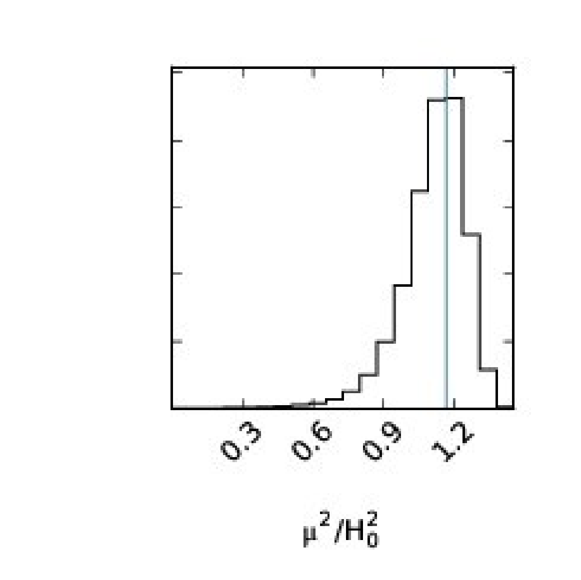
Chi square function

- Plot of chi-square
- Minimum exists for non-zero mass



MCMC sampling

- Using MCMC to sample the chi square
- At 1-sigma, $\theta = \frac{\mu^2}{H_0^2} = 1.165^{+0.070}_{-0.176}$



MTMG vs Lambda-CDM

- Minimum of chi square for MTMG

$$\bar{\chi}^2_{MTMG} \equiv \chi^2_{MTMG}(\theta = 1.165) = 10.876, \quad \chi^2_{GR} = \chi^2_{MTMG}(\theta = 0) = 24.51,$$

- AKAIKE criterion: relative likelihood

$$\exp[(\bar{\chi}^2_{MTMG} + 2 - \chi^2_{GR})/2] = 3 \times 10^{-3}$$

- Data pin down the mass of the gravitational waves

Conclusions

- Stable dRGT-like cosmology, if $\mu = 9.8\ 10^{-33}$ eV , with $\mu < 10^{-22}$ eV

[Abbott et al, 16]

- Simpler theory (only tensor modes propagate)
- IR Lorentz-violations in gravity sector
- Phenomenology: self-acc branch: same of GR, except Gws
- Normal branch has interesting phenomenology which can relax tension between early and late time cosmological data