UNRUH RADIATION

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Based on a collaboration with

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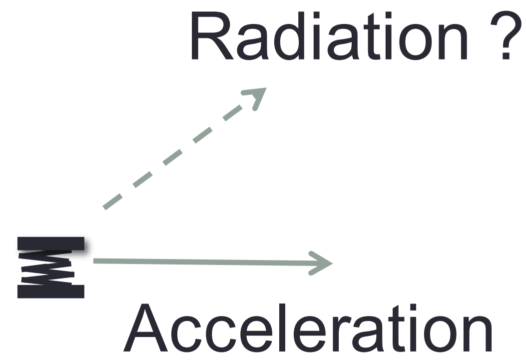
S. Iso, N. Oshita, R. Tatsukawa, K. Y., S. Zhang, **arXiv:1610.08158**

N. Oshita, K. Y., S. Zhang, PRD 93 085016 (2016)

N. Oshita, K. Y., S. Zhang, PRD 92 045027 (2015)

N. Oshita, K. Y., S. Zhang, PRD 89 124028 (2014)

S. Iso, K. Y., S. Zhang, PTEP 063B01 (2013)



2016/11/16 seminar at Kobe University

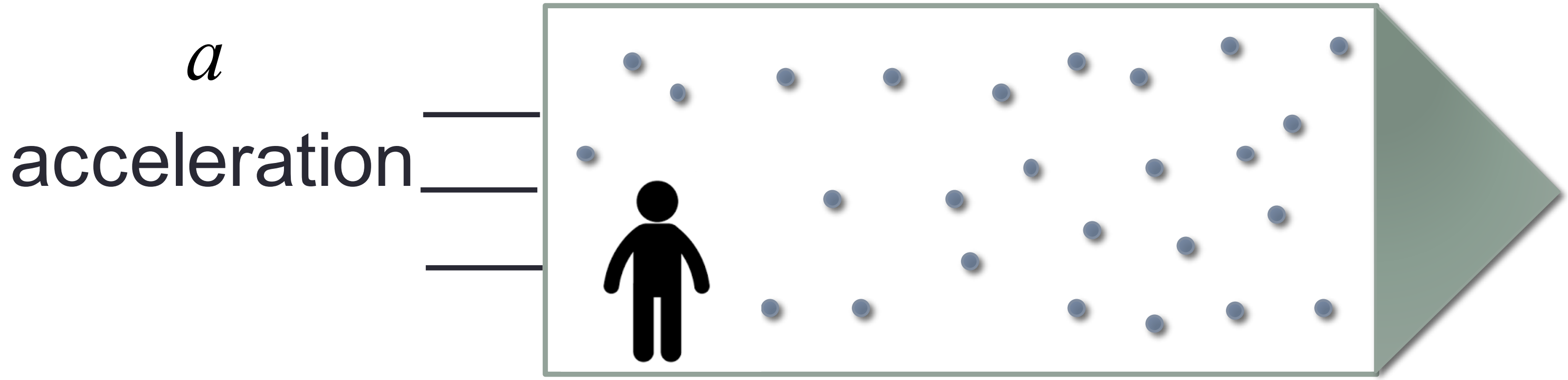
Outline

1. Introduction of Unruh effect
2. Quantum radiation from Unruh - de Witt detector
3. Interpretation of the origin of the quantum radiation
4. Quantum radiation from a particle model
5. Summary and conclusions

1.Introduction

Unruh effect

- A observer in an uniformly accelerated motion sees the Minkowski vacuum as a thermally excited state.



Unruh Temperature

$$T_U = \frac{a}{2\pi} = 4\times10^{-20}\,\mathrm{K}\left(\frac{a}{9.8\,\mathrm{m/s^2}}\right)$$

Signals of the Unruh effect will be tiny for the Unruh temperature is very low.

Physics related to the Unruh effect

Detection will have Impacts on the research of fundamental physics

- ✓ Unruh effect is the simplest system that the relativity and quantum mechanics play important roles, simultaneously.
- ✓ Unruh effect can be understood in analogy with the Hawking radiation by the equivalence principle.
- ✓ Hawking radiation and Unruh effect

  Quantum effects in a system with the horizon
- ✓ Experiment of detecting the Unruh effect might be possible.

Possibility of detecting the Unruh effect?

Electron acceleration with an intense laser

Chen Tajima (99)

Schutzhold, et al (06

Electric field ~ $eE = 10^{13}\,\mathrm{eV/cm}$

For an electron, $T_U = \dfrac{\hbar a}{2\pi c k_B} = 7\times 10^{5}\,\mathrm{K}\left(\dfrac{eE}{10^{13}\,\mathrm{eV/cm}}\right)$

They argued that an electron emits radiation in an accelerated motion in the thermally excited state due to the Unruh effect.

(Unruh radiation)?

Iso, et al (11)

Oshita, et al (15,16)

Naively expected quantum radiations from thermal excitation induced by the Unruh effect almost cancel out due to the interference effect.

It is also shown that the cancellation is not complete, and some quantum radiation may remain, its origin is not understood.

➡ Today's topic

Trajectory of a observer in a uniformly accelerated motion

$$x^\mu = z^\mu(\tau)$$

$$v^\mu = \dot{z}^\mu(\tau)$$

$$a^\mu = \ddot{z}^\mu(\tau)$$

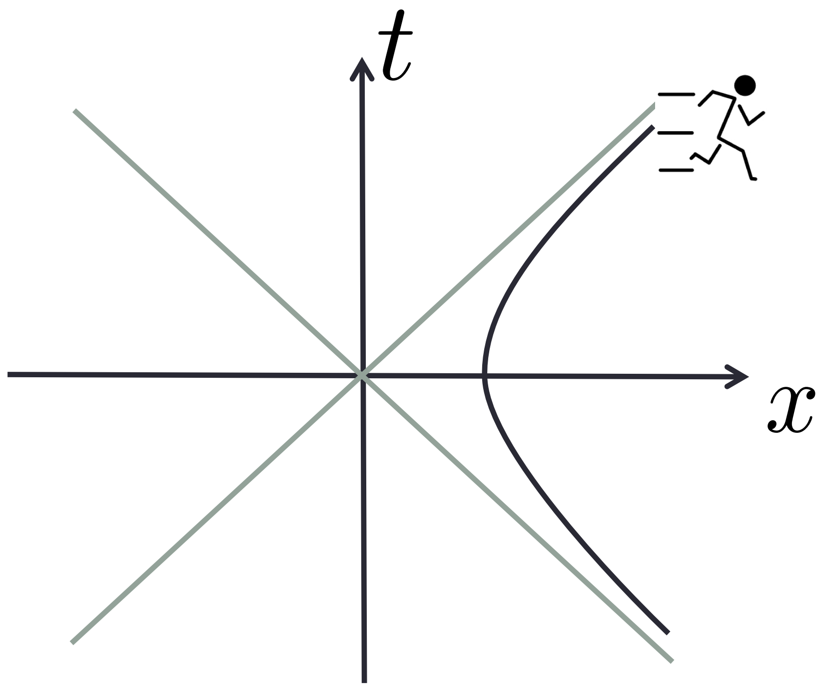
$$v^\mu v_\mu = 1$$

$$a^\mu a_\mu = -a^2$$

$$\begin{cases} \dot{t} = \sqrt{1 + \dot{x}^2} \\ \ddot{t}^2 - \ddot{x}^2 = -a^2 \end{cases}$$

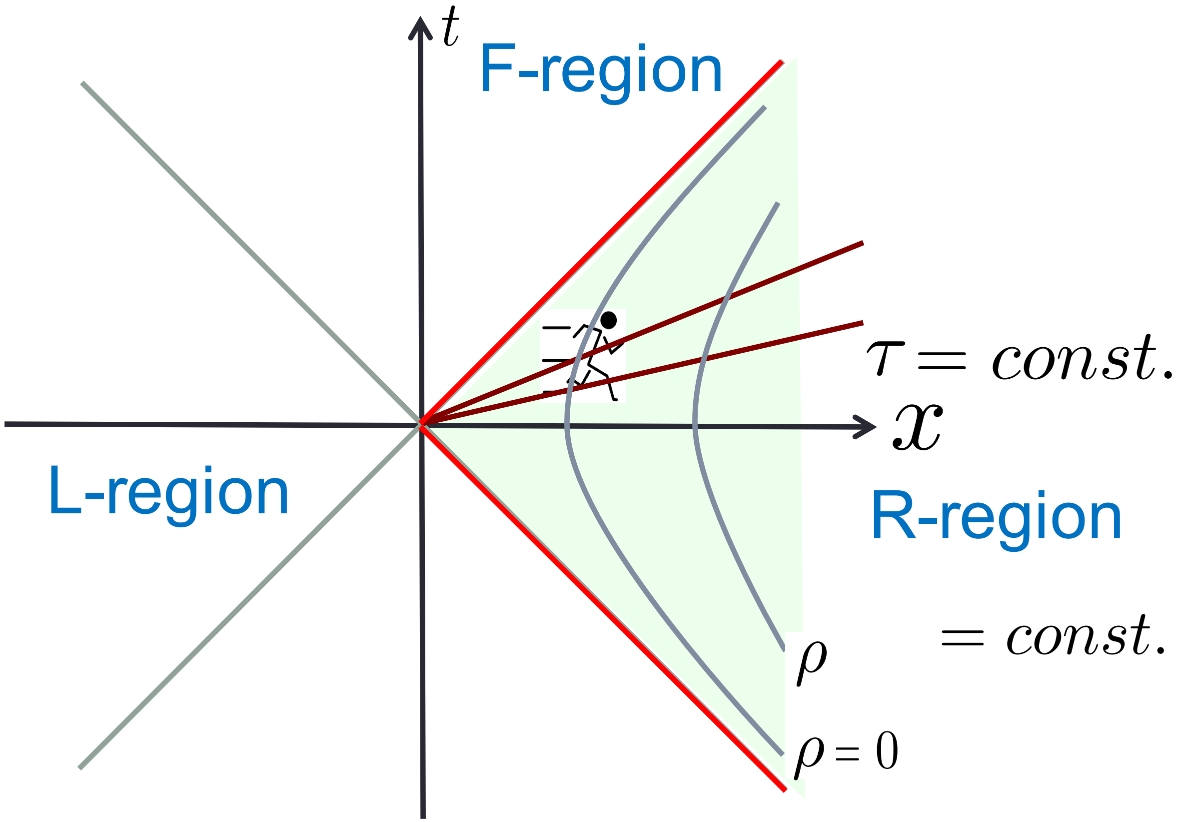
$$t = \frac{1}{a} \sinh a\tau$$

$$x = \frac{1}{a} \cosh a\tau$$



Rindler spacetime is the coordinate of a uniformly accelerating observer

Minkowski coordinate, t, x ⟷ Rindler coordinate τ, ρ



$$t = \frac{1}{a}\sinh a\tau + \rho \sinh a\tau$$

$$x = \frac{1}{a}\cosh a\tau + \rho \cosh a\tau$$

$$ds^2 = dt^2 - dx^2$$

$$ds^2 = (1 + a\rho)^2 d\tau^2 - d\rho^2$$

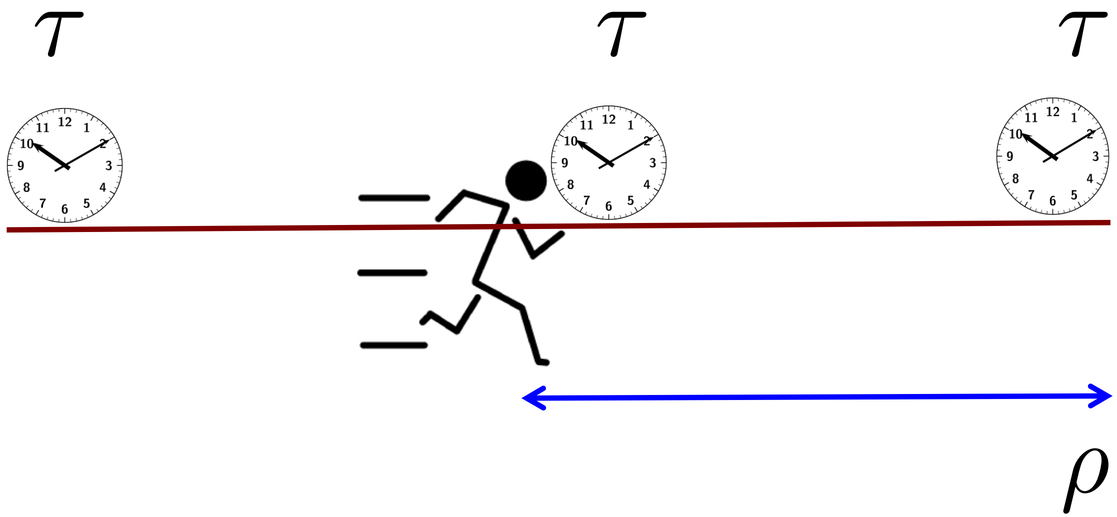
$$\xi = \frac{1}{a}\ln(1 + a\rho)$$

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2)$$

Rindler spacetime covers only the R-region,
a quarter of the Minkowski spacetime.
No information from the L-region, F-region,
→ Rindler horizon

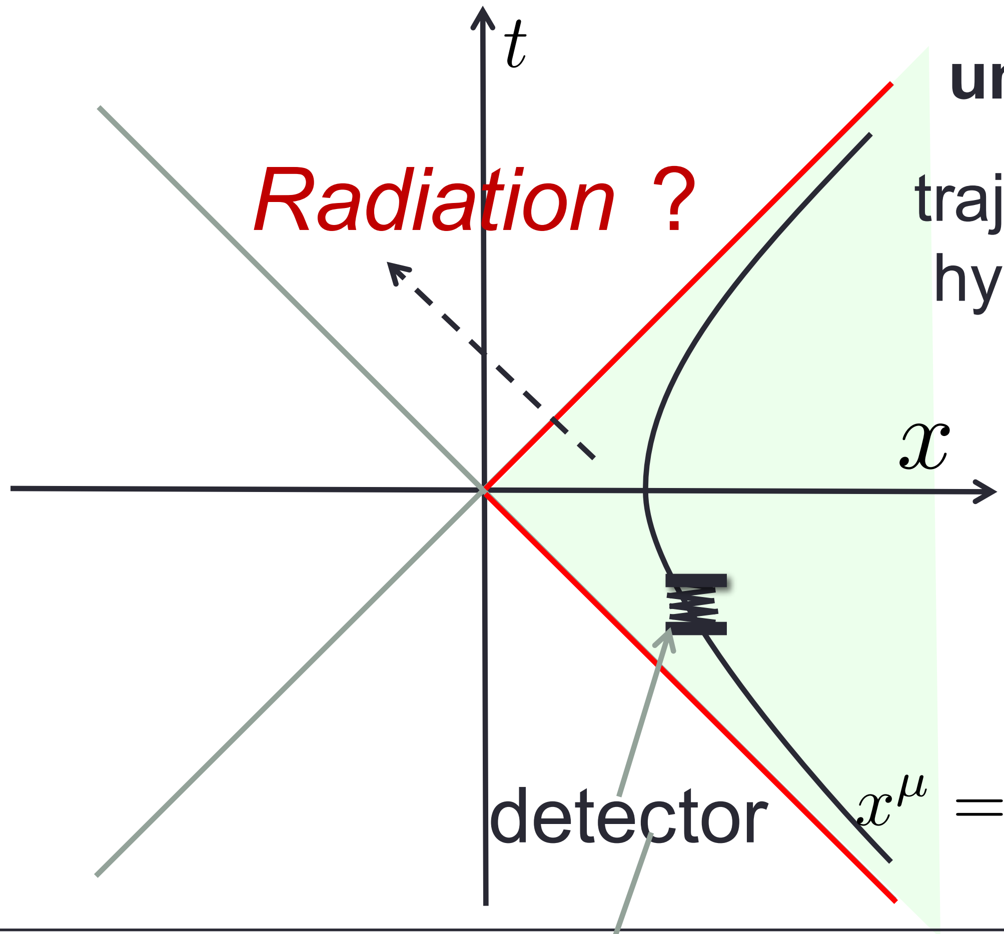
Rindler spacetime : coordinate of a observer in a uniformly accelerating motion.

A observer in a uniformly accelerating motion has a stick of the length ρ and the clock τ



Rindler space-time coordinate

Massless scalar field + Unruh-deWitt detector



$x^\mu = z^\mu(\tau)$

uniformly accelerated motion

trajectory of the detector $Q(\tau)$
hyperbolic curve

$$z^0 = \frac{1}{a}\sinh a\tau$$

$$z^1 = \frac{1}{a}\cosh a\tau$$

Lin & Hu (06)
Iso, KY, Zhang (13)

$$S[Q,\phi] = \frac{m}{2}\int d\tau\left((\dot{Q}(\tau))^2 - \Omega_0^2 Q^2(\tau)\right) + \frac{1}{2}\int d^4x\partial^\mu\phi(x)\partial_\mu\phi(x) + S_{\text{int}}[Q,\phi]$$

$$S_{\text{int}}[Q,\phi] = \lambda\int d^4x d\tau Q(\tau)\phi(x)\delta^4(x - z(\tau))$$

Linear coupling between Q and ϕ

2. Quantum radiation from Unruh de Witt detector model

$$S[Q,\phi]=\frac{m}{2}\int d\tau\left((\dot{Q}(\tau))^2-\Omega_0^2Q^2(\tau)\right)+\frac{1}{2}\int d^4x\partial^\mu\phi(x)\partial_\mu\phi(x)+S_{\rm int}[Q,\phi]$$

$$S_{\rm int}[Q,\phi]=\lambda\int d^4xd\tau Q(\tau)\phi(x)\delta^4(x-z(\tau))$$

☑ $$\boxed{\partial^\mu\partial_\mu\phi(x)=\lambda\int d\tau Q(\tau)\delta_D^{(4)}(x-z(\tau))}$$

①

$$\phi_{inh}(x)=\lambda\int d\tau Q(\tau)G_R(x-(z(\tau))$$

$$\phi(x)=\phi_{inh}(x)+\phi_h(x)$$

④ Two point function

→energy momentum tensor

③

☑ $$\boxed{(\partial_\tau^2+\Omega_0^2)Q(\tau)=\frac{\lambda}{m}\phi(z(\tau))}$$

②

$$\left(\frac{d^2}{d\tau^2}+\Omega^2\right)Q(\tau)-\frac{\lambda^2}{m}\int d\tau' Q(\tau')G_R(\bar{z}(\tau),\bar{z}(\tau'))=\frac{\lambda}{m}\phi_h(z(\tau))$$

Dissipation due to the coupling

Fluctuation random force from the vacuum fluctuations

②Solution of the Unruh-de Witt detector

$$\left(\frac{d^2}{d\tau^2}+\Omega_0^2\right)Q(\tau)-\frac{\lambda^2}{m}\int d\tau' Q(\tau')G_R(\bar{z}(\tau),\bar{z}(\tau'))=\frac{\lambda}{m}\phi_h(z(\tau))$$

$$G_R(x,y)=\frac{1}{4\pi}\delta\left(\sigma\right)\theta(x^0-y^0) \qquad \sigma=\frac{1}{2}(x_\mu-y_\mu)(x^\mu-y^\mu)$$

$$G_R^{reg.}(x,y)=\frac{1}{4\pi}\sqrt{\frac{8}{\pi}}\Lambda^2 e^{-2\Lambda^4\sigma^2}\theta(x^0-y^0)$$

regularized retarded Green function

$$\int d\tau' Q(\tau')G_R(\bar{z}(\tau),\bar{z}(\tau'))=\frac{1}{4\pi}\left[\Lambda 2^{7/4}\Gamma(5/4)/\sqrt{\pi}Q(\tau)-\dot{Q}(\tau)+\mathcal{O}(1/\Lambda)\right]$$

$$\left(\frac{d^2}{d\tau^2}+2\gamma\frac{d}{d\tau}+\Omega^2\right)Q(\tau)=\frac{\lambda}{m}\phi_h(z(\tau))$$

$$\Omega^2=\Omega_0^2-\frac{\lambda^2\Lambda 2^{7/4}\Gamma(5/4)}{4\pi m\sqrt{\pi}} \qquad \gamma=\frac{\lambda^2}{8\pi m}$$

②Solution of the Unruh-de Witt detector

$$\left(\frac{d^2}{d\tau^2}+\Omega_0^2\right)Q(\tau)-\frac{\lambda^2}{m}\int d\tau' Q(\tau')G_R(\bar{z}(\tau),\bar{z}(\tau'))=\frac{\lambda}{m}\phi_h(z(\tau))$$

$$\downarrow$$

$$\left(\frac{d^2}{d\tau^2}+2\gamma\frac{d}{d\tau}+\Omega^2\right)Q(\tau)=\frac{\lambda}{m}\phi_h(z(\tau)) \qquad \gamma=\frac{\lambda^2}{8\pi m}$$

$$Q(\tau)=\frac{1}{2\pi}\int d\omega e^{-i\omega\tau}\tilde{Q}(\omega)$$

$$\phi_h(z(\tau))=\frac{1}{2\pi}\int d\omega e^{-i\omega\tau}\varphi(\omega)$$

solution $\downarrow$

$$\tilde{Q}(\omega)=\lambda h(\omega)\varphi(\omega) \qquad h(\omega)=\frac{1}{-m\omega^2+m\Omega^2-i\frac{\omega\lambda^2}{4\pi}}$$

Energy equi-partition relation

$$\langle E\rangle=\frac{m}{2}\left(\langle\dot{Q}^2(\tau)\rangle+\Omega^2\langle Q^2(\tau)\rangle\right) \longrightarrow \langle E\rangle\simeq\frac{a}{2\pi}=T_U$$

Energy equi-partition relation

$$Q(\tau) = \frac{1}{2\pi}\int d\omega e^{-i\omega\tau}\tilde{Q}(\omega)$$

$$\tilde{Q}(\omega) = \lambda h(\omega)\varphi(\omega)$$

$$h(\omega) = \frac{1}{-m\omega^2 + m\Omega^2 - i\frac{\omega\lambda^2}{4\pi}}$$

Property of interaction

$$\langle Q(\tau)Q(\tau')\rangle = \frac{1}{(2\pi)^2}\int\int d\omega d\omega' e^{-i\omega\tau}e^{-i\omega'\tau'}\lambda^2 h(\omega)h(\omega')\langle\varphi(\omega)\varphi(\omega')\rangle$$

$$\langle\varphi(\omega)\varphi(\omega')\rangle = \int d\tau\int d\tau'\langle\phi(z(\tau))\phi(z(\tau'))\rangle$$

$$= -\frac{\omega}{1-e^{-2\pi\omega/a}}\delta(\omega+\omega)$$

Determined by the Minkowski vacuum two point function

$$\frac{m}{2}\Omega^2\langle Q(\tau)^2\rangle = \frac{a}{4\pi}$$

$$\frac{m}{2}\langle\dot{Q}(\tau)^2\rangle = \frac{a}{4\pi}$$

$$\langle E\rangle = \frac{m}{2}\Big(\langle\dot{Q}^2(\tau)\rangle + \Omega^2\langle Q^2(\tau)\rangle\Big)$$

$$= \frac{a}{2\pi} = T_U$$

Thermal property

③Two point function of the field

$$h(\omega)=\frac{1}{-m\omega^2+m\Omega^2-i\frac{\omega\lambda^2}{4\pi}}$$

$$\phi(x)=\phi_{\rm h}(x)+\phi_{\rm inh}(x)$$

$$\phi_{\rm inh}(x)=\lambda^2\int\int d\tau\frac{d\omega}{2\pi}e^{-i\omega\tau}h(\omega)G_R(x-z(\tau))\varphi(\omega)$$

$$\boxed{\begin{array}{l}\langle\phi(x)\phi(y)\rangle=\langle\phi_{\rm h}(x)\phi_{\rm h}(y)\rangle+\underline{\langle\phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle+\langle\phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle}+\langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle\\ \text{Vacuum fluctuation + Interference term + Inhomogeneous term}\end{array}}$$

$$\underline{\langle\phi_{inh}(x)\phi(y)\rangle+\langle\phi_h(x)\phi_{inh}(y)\rangle}$$

$$=\lambda^2\int d\tau\frac{d\omega}{2\pi}e^{-i\omega\tau}h(\omega)[G_R(x-z(\tau))\langle\varphi(\omega)\phi_h(y)\rangle+G_R(y-z(\tau))\underline{\langle\phi_h(x)\varphi(\omega)\rangle}]$$

Minkowki vacuum two point function

$$\langle\phi_h(x)\varphi(\omega)\rangle=\int d\tau e^{i\omega\tau}\langle\phi_h(x)\phi_h(z(\tau))\rangle$$

Key quantity

$$=-\frac{1}{4\pi^2}\int d\tau\frac{e^{i\omega\tau}}{(t-z^0(\tau)-i\epsilon)^2-(x^1-z^1(\tau))^2-x_\perp^2}$$

$$=-\frac{1}{4\pi^2}P(x,\omega)$$

$$P(x,\ \omega) \equiv \int d\tau \frac{e^{i\omega\tau}}{(x^0 - z^0(\tau) - i\epsilon)^2 - (x^1 - z^1(\tau))^2 - x_\perp^2}$$

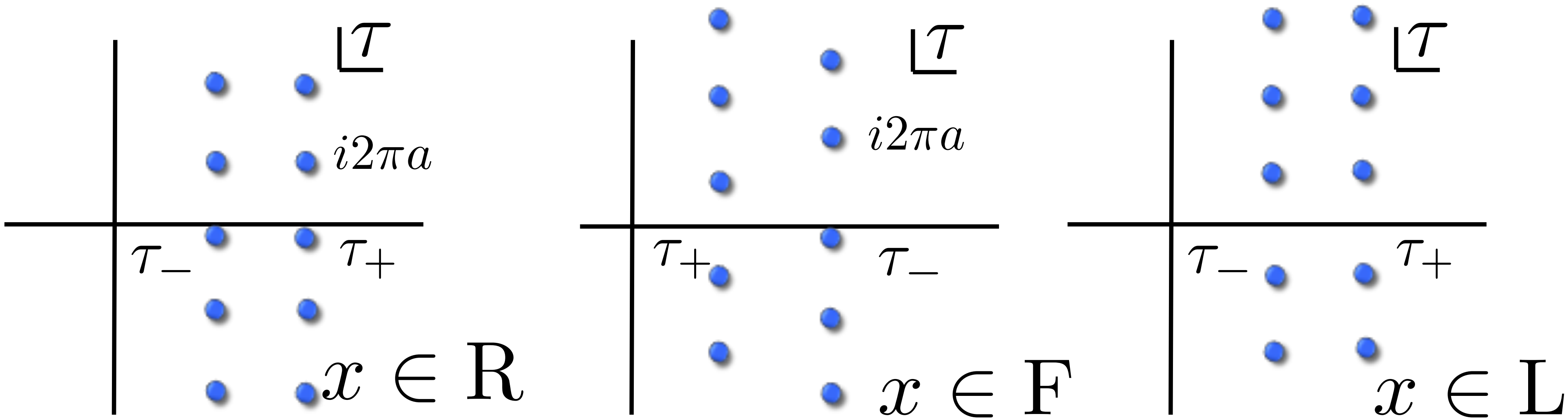
Poles of $(x^0 - z^0(\tau) - i\epsilon)^2 - (x^1 - z^1(\tau))^2 - x_\perp^2 = 0$

$n = 0, \pm 1, \pm 2, \cdots$

$x^\mu \in \mathrm{R}$ region $\quad \tau = \tau_\pm - i\epsilon + i2\pi n a$

$x^\mu \in \mathrm{F}$ region $\quad \tau = \tau_- - i\epsilon + i2\pi n a, \quad \tau = \tau_+ + i\pi + i2\pi n a$

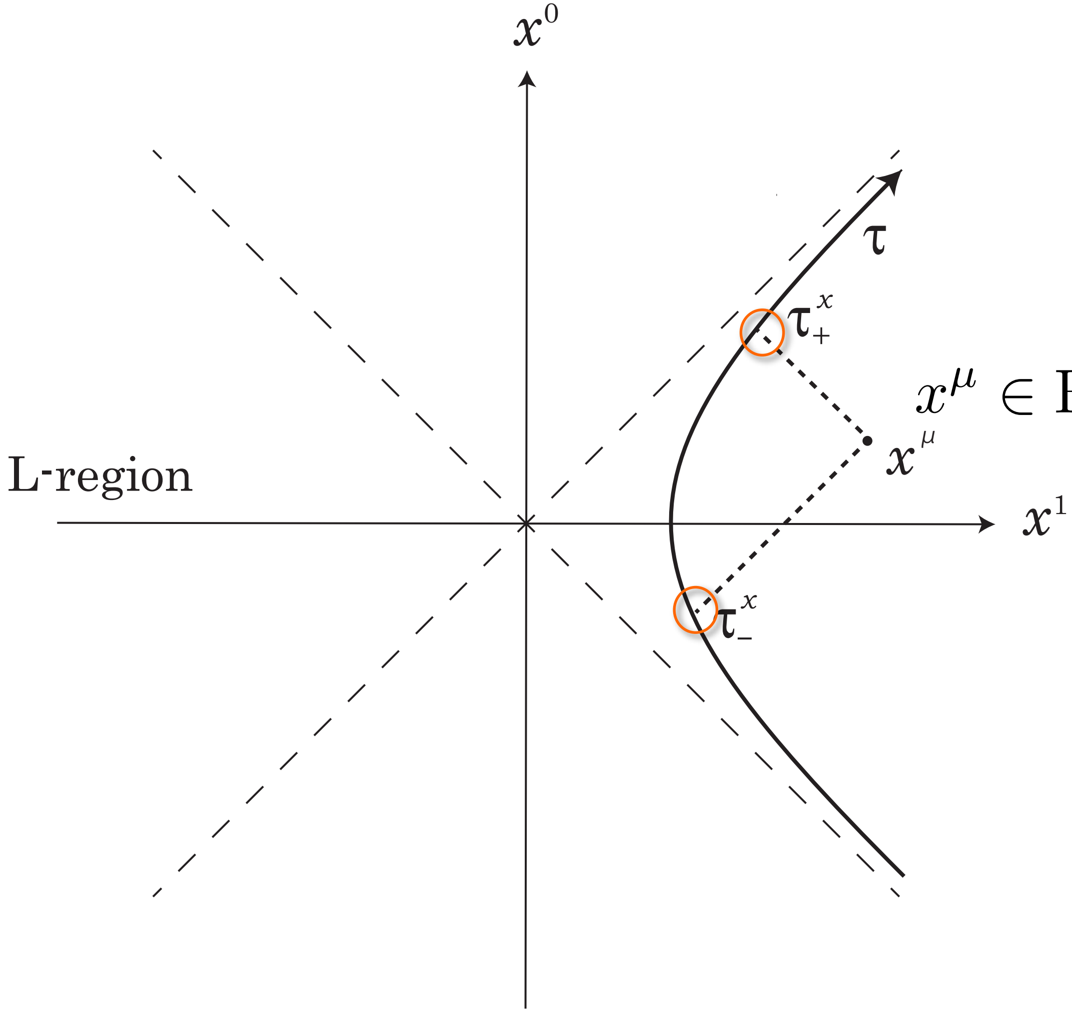
$x^\mu \in \mathrm{L}$ region $\quad \tau = \tau_\pm + i\pi + i2\pi n a$



Physical meaning $x^{\mu} \in \mathrm{R}$ region

$$\tau = \tau_{\pm} - i\epsilon + i2\pi n a$$

$\tau_{\pm}$ is Rindler coordinate time of the point that the light cone of x intersect with the trajectory of the detector.

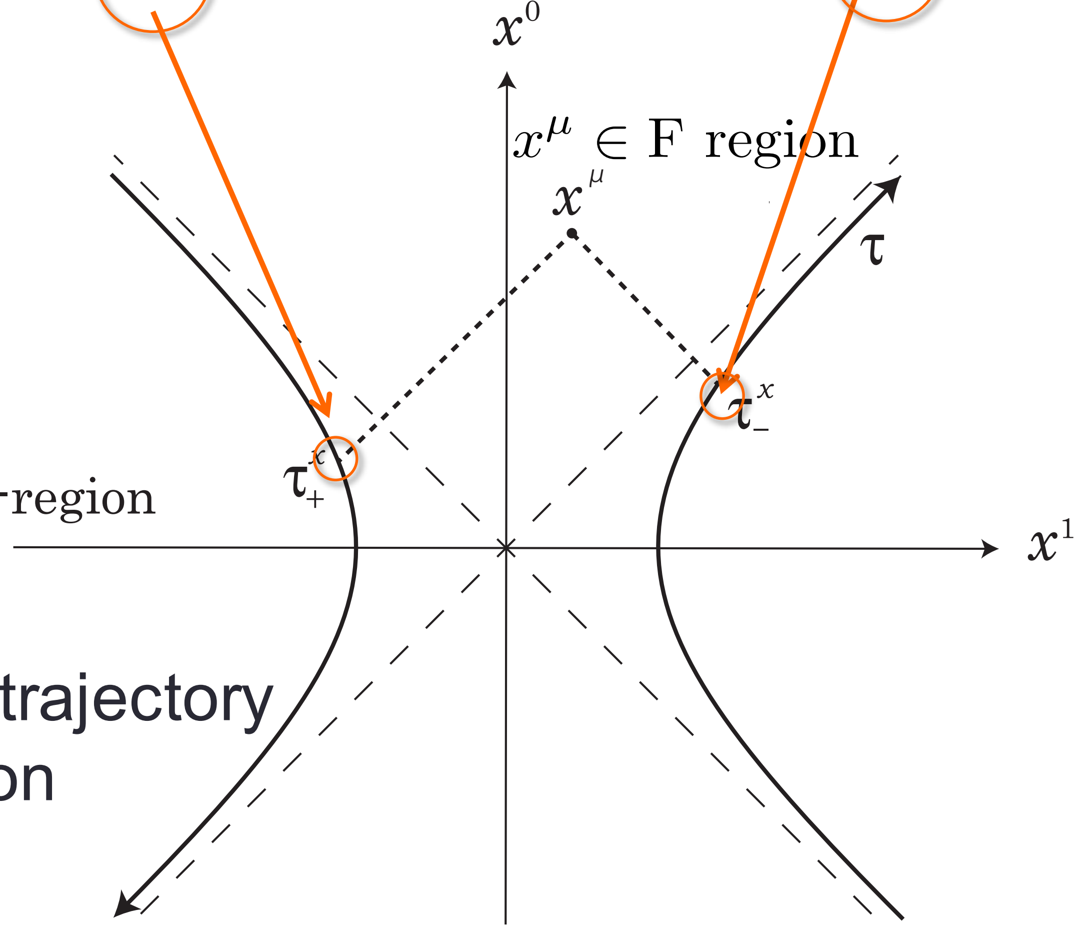


Physical meaning $x^\mu \in \mathrm{F}$ region

τ_-: the point that the light cone of x intersects with the trajectory

τ_+: the point that the light cone of x intersects with the hypothetical trajectory in the L-region

$$\tau = \tau_+ + i\pi + i2\pi n a \qquad \tau = \tau_- - i\epsilon + i2\pi n a,$$

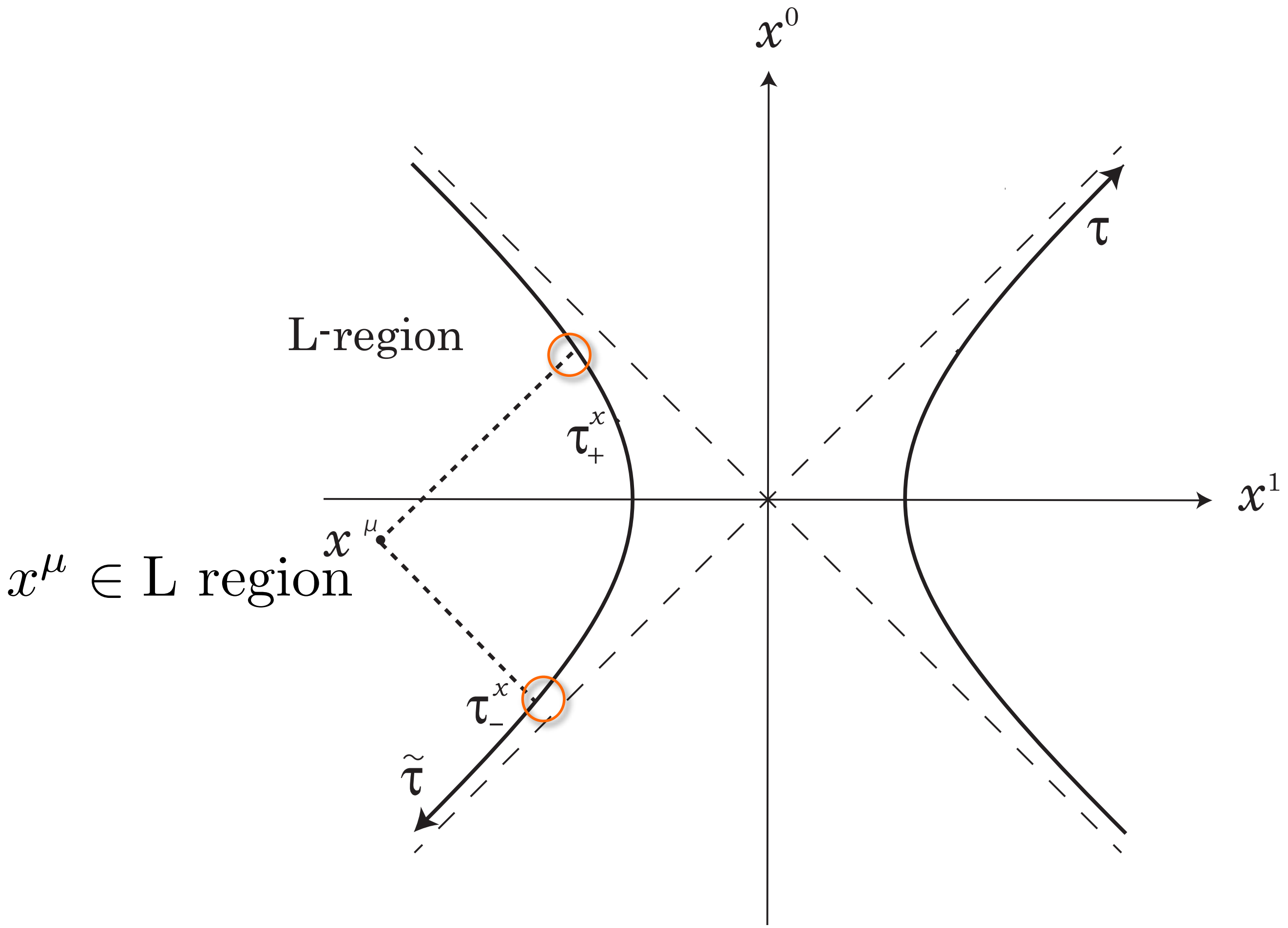


Hypothetical trajectory in the L-region

Physical meaning $x^\mu \in \mathrm{L}$ region

$$\tau = \tau_\pm + i\pi + i2\pi n a$$

$\tau_\pm$ is Rindler coordinate time of the point that the light cone of x intersects with the hypothetical trajectory in the L-region.



$$P(x,\ \omega) \equiv \int d\tau \frac{e^{i\omega\tau}}{(x^0 - z^0(\tau) - i\epsilon)^2 - (x^1 - z^1(\tau))^2 - x_\perp^2}$$

$$P(x,\omega) = \begin{cases} -\dfrac{\pi i}{\rho_0}\dfrac{1}{e^{2\pi\omega/a}-1}\left(\boxed{e^{i\omega\tau_-^x}} - e^{i\omega\tau_+^x}\right) & x^\mu \in \text{R region} \\ -\dfrac{\pi i}{\rho_0}\left(\dfrac{1}{e^{2\pi\omega/a}-1}\boxed{e^{i\omega\tau_-^x}} - \dfrac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}\boxed{e^{i\omega\tau_+^x}}\right) & x^\mu \in \text{F region} \\ -\dfrac{\pi i}{\rho_0}\dfrac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}\left(e^{i\omega\tau_-^x} - \boxed{e^{i\omega\tau_+^x}}\right) & x^\mu \in \text{L region} \end{cases}$$

$$\rho_0 = \frac{a}{2}\sqrt{\left(-x^\mu x_\mu + \frac{1}{a^2}\right)^2 + \frac{4}{a^2}(t^2 - x^2)}$$

Interference term $\quad x, y \in R, F$

$$Z_x(\omega) = \theta(-t + x^1) + e^{\pi\omega/a}\theta(t - x^1)$$

$$h(\omega) = \frac{1}{-m\omega^2 + m\Omega^2 - i\frac{\omega\lambda^2}{4\pi}}$$

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle$$

$$= i\lambda^2 \int \frac{d\omega}{2\pi} \frac{1}{4\pi\rho_0(x)} \frac{1}{4\pi\rho_0(y)} \frac{1}{1 - e^{-2\pi\omega/a}}$$

$$\times \Bigg(\underbrace{\cancel{[h(\omega) - h(-\omega)]e^{-i\omega(\tau_-^x - \tau_-^y)}}}_{\tau_-} + \underbrace{h(-\omega)e^{-i\omega(\tau_+^x - \tau_-^y)} Z_x(-\omega) - h(\omega)e^{-i\omega(\tau_-^x - \tau_+^y)} z_y(-\omega)}_{\tau_+} \Bigg)$$

Inhomogeneous term

$$\cancel{\langle \phi_{inh}(x)\phi_{inh}(y)\rangle = -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{1}{4\pi\rho_0(x)} \frac{1}{4\pi\rho_0(y)} \frac{e^{-i\omega(\tau_-^x - \tau_-^y)}}{1 - e^{-2\pi\omega/a}} [h(\omega) - h(-\omega)]}$$

Inhomogeneous term cancels out by a term of the interference term

→ The naïve radiation cancels out, and there remains some term.

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -\frac{i\lambda^2}{(4\pi)^2\rho_0(x)\rho_0(y)} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} h(\omega) \left[\frac{1}{1 - e^{-2\pi\omega/a}} e^{-i\omega(\tau_-^x - \tau_+^y)} Z_y(-\omega) - \frac{1}{1 - e^{2\pi\omega/a}} e^{-i\omega(\tau_-^y - \tau_+^x)} Z_x(\omega) \right]$$

What does the remaining two point function mean?

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -\frac{i\lambda^2}{(4\pi)^2\rho_0(x)\rho_0(y)}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}h(\omega)\left[\frac{1}{1-e^{-2\pi\omega/a}}e^{-i\omega(\tau_-^x-\tau_+^y)}Z_y(-\omega)-\frac{1}{1-e^{2\pi\omega/a}}e^{-i\omega(\tau_-^y-\tau_+^x)}Z_x(\omega)\right]$$

$$\omega = -i\Omega_\pm$$

$$h(\omega) = \frac{1}{-m\omega^2+m\Omega^2-i\frac{\omega\lambda^2}{4\pi}}$$

$$\Omega_\pm = \gamma \pm \sqrt{\gamma^2-\Omega^2} \qquad \gamma = \frac{\lambda^2}{8\pi m}$$

$$= -\frac{i\lambda^2}{(4\pi)^2\rho_0(x)\rho_0(y)}\left(\frac{I(x,y)}{2m}+(x\leftrightarrow y)\right)$$

$$I(x,y) = -i\theta(\tau_-^y-\tau_+^x)\left[\frac{1}{\Omega_+\Omega_-}\frac{a}{2\pi}+\frac{e^{-\Omega_-(\tau_-^y-\tau_+^x)}}{\Omega_--\Omega_+}\frac{1}{\sin\pi\Omega_-/a}+\frac{e^{-\Omega_+(\tau_-^y-\tau_+^x)}}{\Omega_+-\Omega_-}\frac{1}{\sin\pi\Omega_+/a}\right.$$
$$\left.+\sum_{n=1}^{\infty}\frac{(-1)^n e^{-na(\tau_-^y-\tau_+^x)}}{(\Omega_--na)(\Omega_+-na)}\frac{a}{\pi}\right]+i\theta(\tau_+^x-\tau_-^y)\left[\frac{1}{\Omega_+\Omega_-}\frac{a}{2\pi}+\sum_{n=1}^{\infty}\frac{(-1)^n e^{na(\tau_-^y-\tau_+^x)}}{(\Omega_-+na)(\Omega_++na)}\frac{a}{\pi}\right]$$

$$\gamma > \Omega \qquad x, y \in \text{F region}$$

Energy flux can be derived from the two point function

$$T_{0i} = \lim_{y\to x}\left\langle \frac{\partial\phi(x)}{\partial x^0}\frac{\partial\phi(y)}{\partial y^i}\right\rangle = \lim_{y\to x}\frac{\partial}{\partial x^0}\frac{\partial}{\partial y^i}\left\langle \phi(x)\phi(y)\right\rangle$$

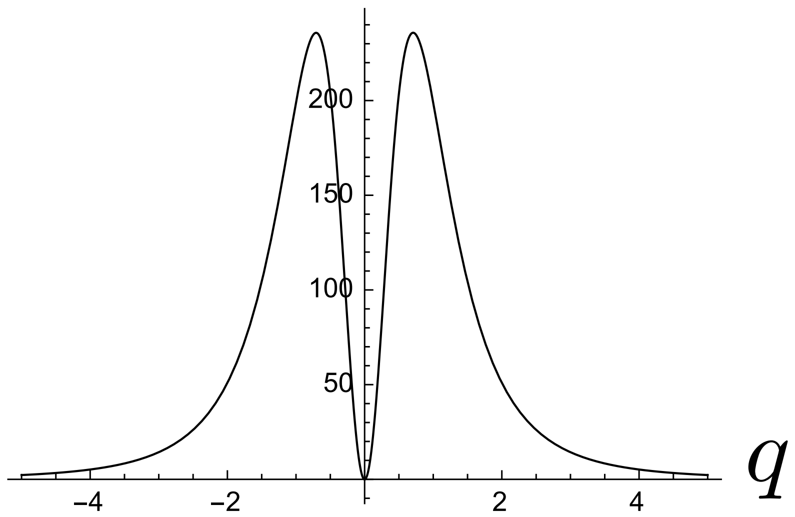
$$f = -\sum_i T_{0i}n^i = \frac{a\lambda^2}{(4\pi)^2 m r^2\sin^4\theta}\mathcal{F}(q)$$

$$n^i = \frac{x^i}{r}$$

$$q = \frac{a}{\sin\theta}\left(t - r - \frac{1}{2a^2 r}\right)$$

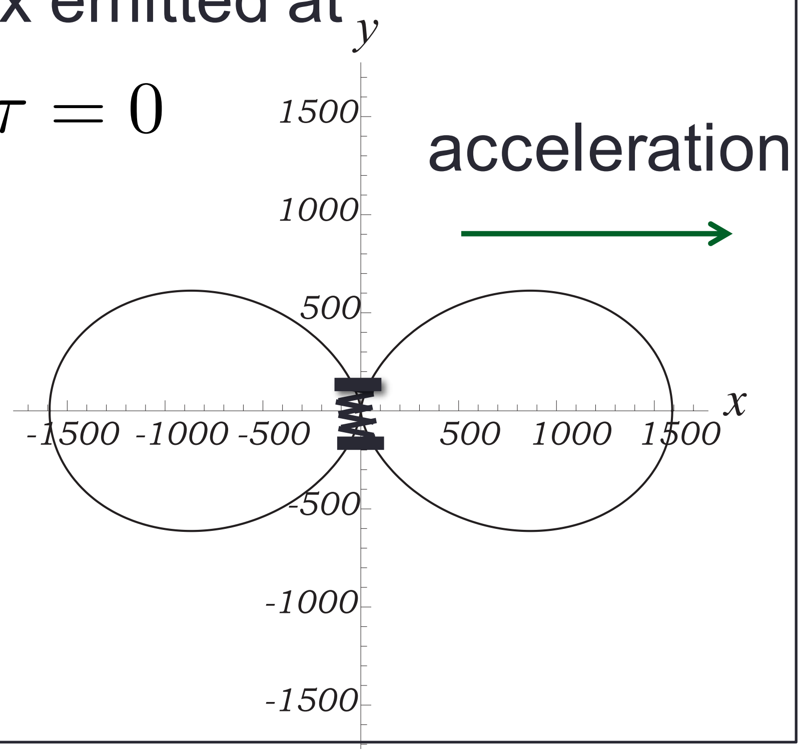
$$\frac{\lambda}{8\pi m a} = 1 \qquad \frac{\Omega}{a} = 0.01$$

$\mathcal{F}(q)$



Polar plot of the radiation flux emitted at

$\tau = 0$



Energy flux can be derived from the two point function

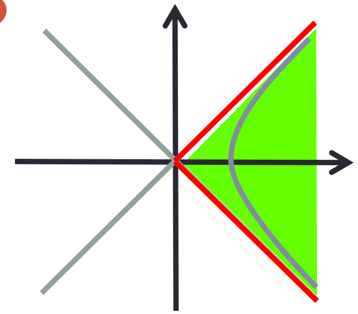
Non-vanishing energy flux!

Energy radiation rate is

$$\frac{dE}{dt} = \lim_{r\to\infty} r^2 \int d\Omega_{(2)} f \sim \frac{a\lambda^2}{4\pi m}\mathcal{F} \sim \frac{a\lambda}{4\pi m}\frac{a^2}{2\pi\Omega^2}$$

- ✓ The radiation energy from the Unruh-deWitt detector, which is consistent with the previous work by Lin and Hu (2006).
- ✓ This is not the radiations naively expected from the thermally excited detector due to the Unruh effect.
- ✓ There is non-vanishing energy flux.
- ✓ The origin of the non-vanishing flux is has not been understood.

Interpretation of the origin of the radiation ?



Remaining two point function $x, y \in \mathrm{R}$ region

$$\langle\phi_{inh}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{inh}(y)\rangle + \langle\phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a} - 1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau)) \underline{G_A}(y, z(\tau')) h(\omega) - \underline{G_A}(x, z(\tau)) G_R(y, z(\tau')) h(-\omega) \Big]$$

Canceled term

$$\langle\phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a} - 1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau)) \underline{G_R}(y, z(\tau')) h(\omega) - \underline{G_R}(x, z(\tau)) G_R(y, z(\tau')) h(-\omega) \Big]$$

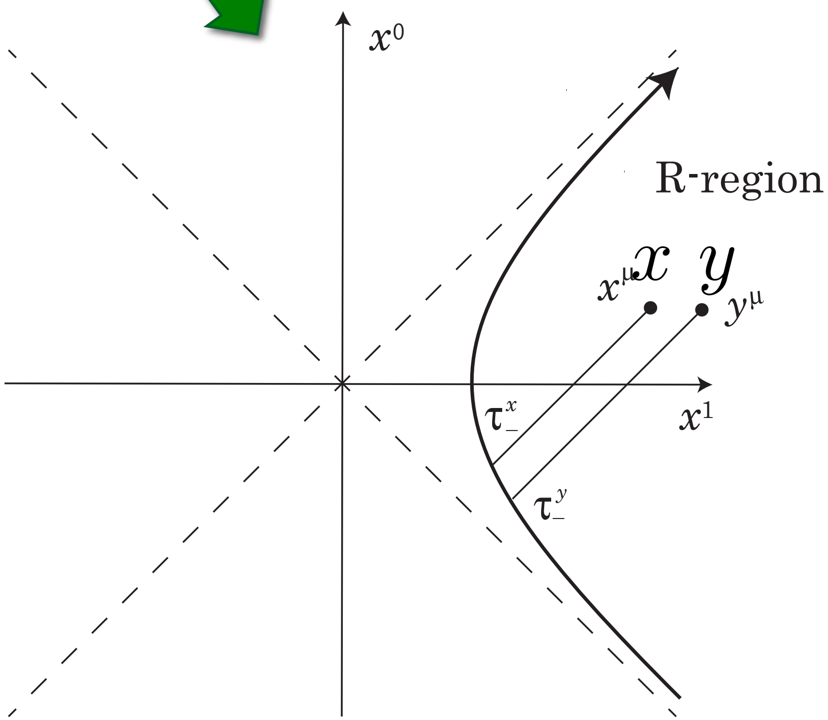
Interpretation of the origin of the radiation ?

Canceled term $x, y \in \mathrm{R}$ region

$$\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

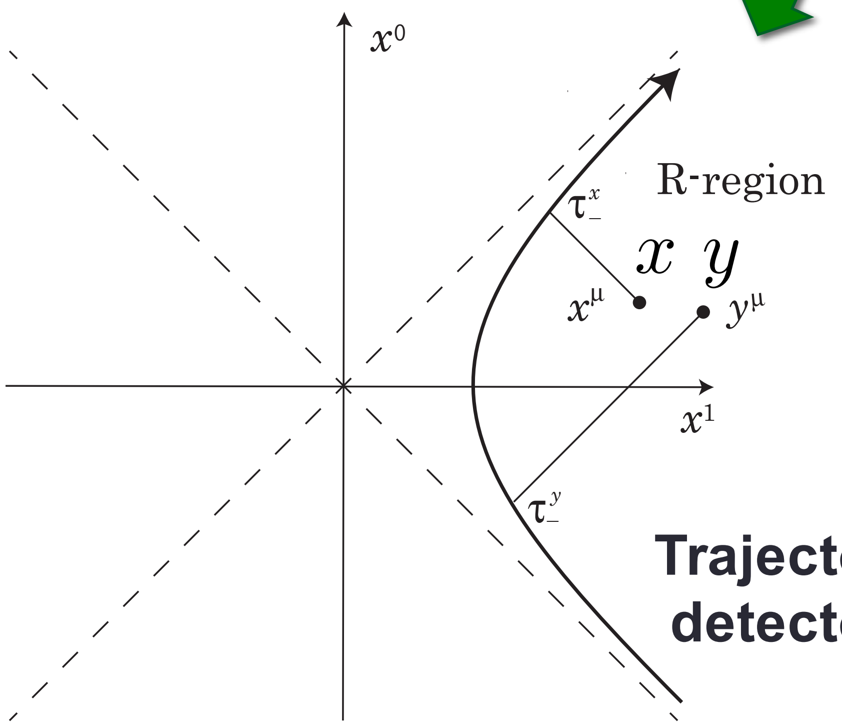
$$\times \Big[G_R(x,z(\tau))G_{\underline{R}}(y,z(\tau'))h(\omega) - G_{\underline{R}}(x,z(\tau))G_R(y,z(\tau'))h(-\omega) \Big]$$



Interpretation of the origin of the radiation ?

Remaining term $x, y \in \mathrm{R}$ region

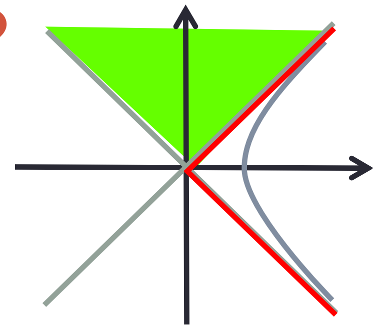
$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$
$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$
$$\times \Big[G_R(x, z(\tau)) \underline{G_A}(y, z(\tau'))h(\omega) - \underline{G_A}(x, z(\tau))G_R(y, z(\tau'))h(-\omega) \Big]$$



Nonlocal feature of the correlation?

Trajectory of detector

Interpretation of the origin of the radiation ?



Remaining two point function $x, y \in \mathrm{F}$ region

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau))\underline{G_R(y, \tilde{z}(\tau'))}h(\omega) - \underline{G_R(x, \tilde{z}(\tau))}G_R(y, z(\tau'))h(-\omega) \Big]$$

Canceled term

$$\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau))\underline{G_R(y, z(\tau'))}h(\omega) - \underline{G_R(x, z(\tau))}G_R(y, z(\tau'))h(-\omega) \Big]$$

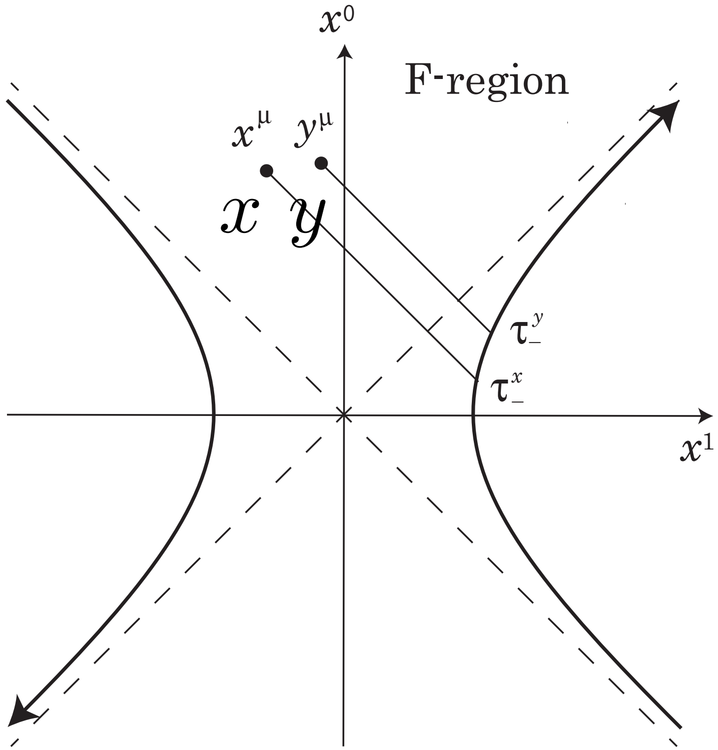
Interpretation of the origin of the radiation ?

Canceled term $\quad x, y \in \mathrm{F}$ region

$$\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x,z(\tau))G_R(y,z(\tau'))h(\omega) - G_R(x,z(\tau))G_R(y,z(\tau'))h(-\omega) \Big]$$

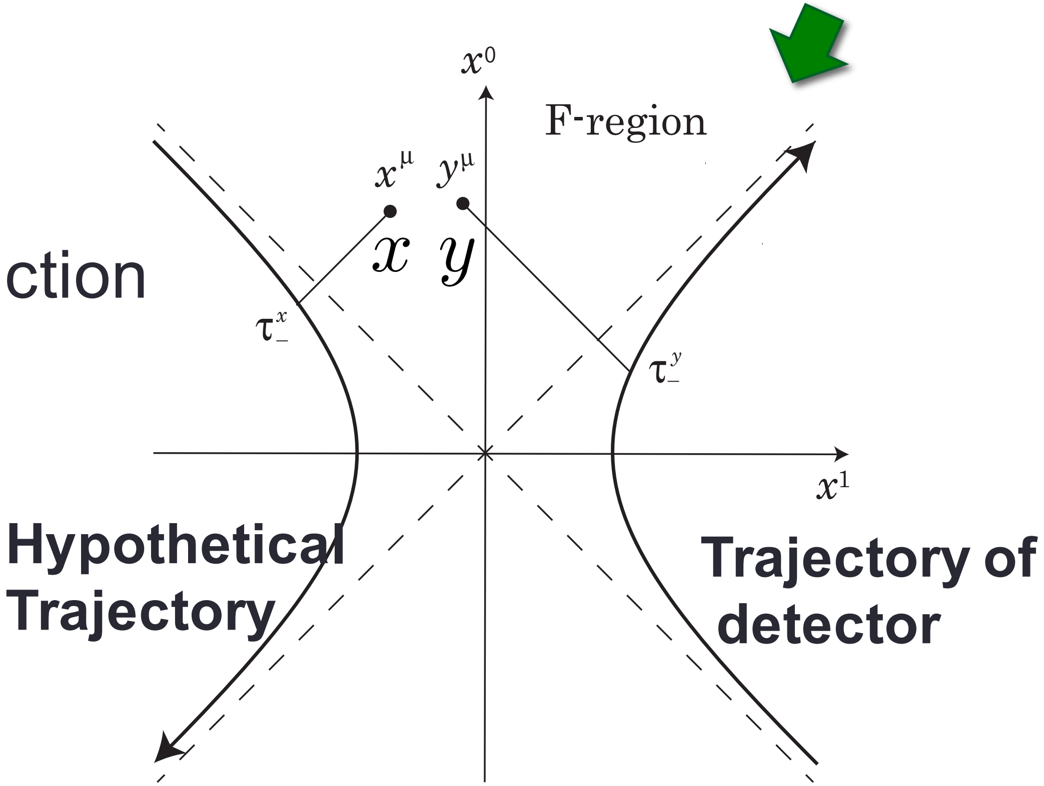


Interpretation of the origin of the radiation ?

Remaining two point function $\qquad x, y \in \mathrm{F}$ region

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$
$$= -i\lambda^2 \int \frac{d\omega}{2\pi}\frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}\int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$
$$\times\Big[G_R(x, z(\tau))G_R(y, \tilde{z}(\tau'))h(\omega) - G_R(x, \tilde{z}(\tau))G_R(y, z(\tau'))h(-\omega)\Big]$$

Nonlocal feature
of correlation function



Interpretation of the origin of the radiation ?

Remaining two point function $x, y \in \mathrm{F}$ region

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau)) \underline{G_R(y, \tilde{z}(\tau'))} h(\omega) - \underline{G_R(x, \tilde{z}(\tau))} G_R(y, z(\tau')) h(-\omega) \Big]$$

Canceled term

$$\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

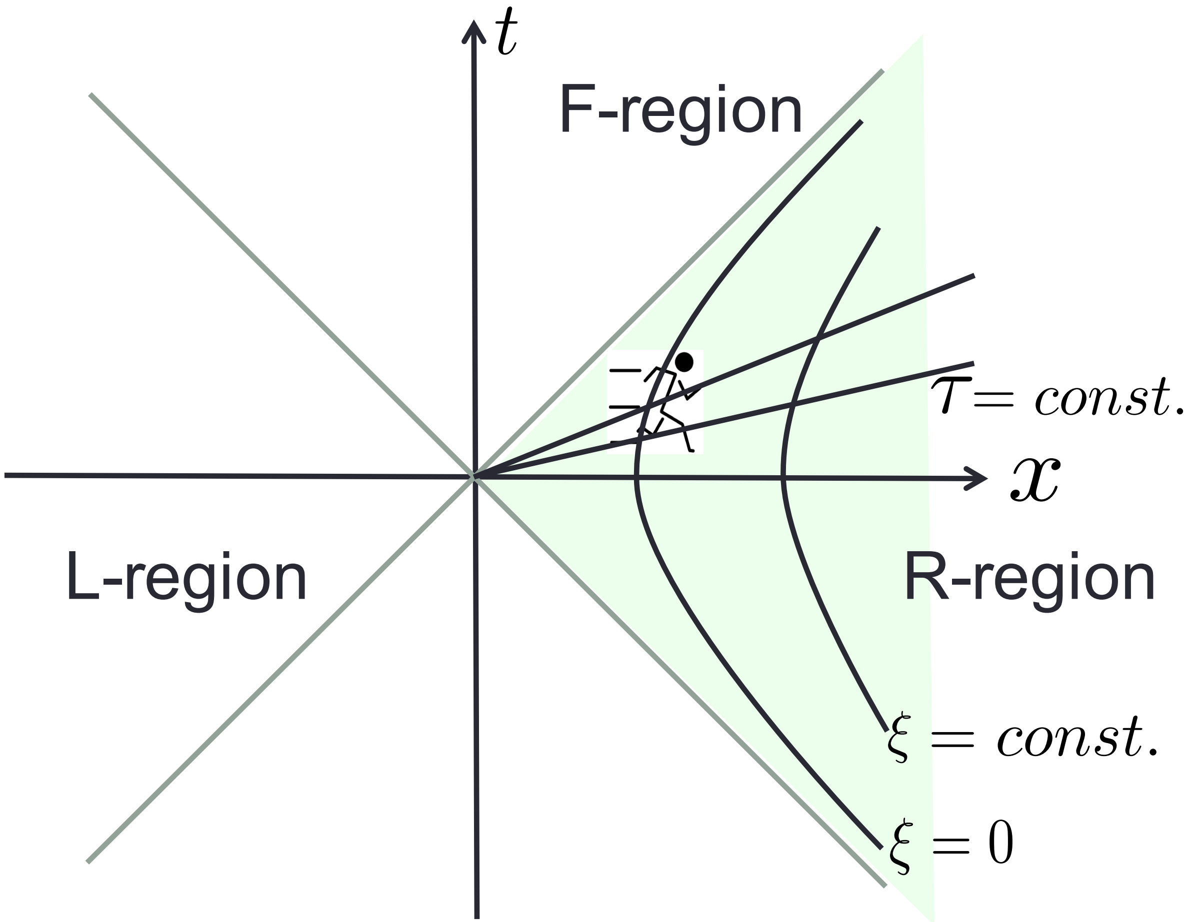
$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau)) \underline{G_R(y, z(\tau'))} h(\omega) - \underline{G_R(x, z(\tau))} G_R(y, z(\tau')) h(-\omega) \Big]$$

Interpretation of the origin of the radiation ?

We infer that the nonlocal correlation of the Minkowski vacuum is the origin of the quantum radiation.

Rindler spacetime is the coordinate of a observer in a uniformly accelerating motion.



$$t = \frac{1}{a}\sinh a\tau + \xi \sinh a\tau$$

$$x = \frac{1}{a}\cosh a\tau + \xi \cosh a\tau$$

Canonical quantization of the massless scalar field.

The Rindler vacuum state

$$|0, R\rangle \longleftrightarrow a_j|0, R\rangle = 0$$

Rindler excited state

$$|n, R\rangle = \frac{1}{\sqrt{n!}}(a_j^\dagger)^n|0, R\rangle$$

Minkowski vacuum is expressed as the entangled state across the R-Rindler region and the L-Rindler region

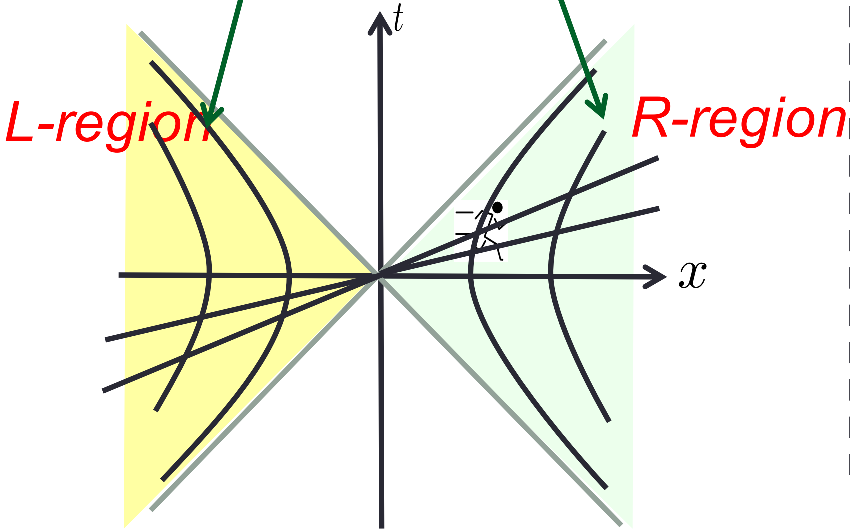
$$|0,\mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j, \mathrm{R}\rangle \otimes |n_j, \mathrm{L}\rangle \right]$$

j : mode

$$N_j = \sqrt{1 - e^{-2\pi\omega_j/a}}$$

$|0, L\rangle$ $\qquad$ $|0, R\rangle$

$b_j|0, L\rangle = 0 \qquad a_j|0, R\rangle = 0$



$$\psi(x_R) = \sum_j (u_j(x_R)a_j + u_j^*(x_R)a_j^\dagger)$$

$$\psi(x_L) = \sum_j (v_j(x_L)b_j + v_j^*(x_L)b_j^\dagger)$$

$$\langle 0, M|\psi(x_L)\psi(y_R)|0, M\rangle$$

$$= \sum_j (v_j(x_L)u_j(x_R) + v_j^*(x_L)u_j^*(x_R)) \times \boxed{\frac{e^{\pi\omega_j/a}}{e^{2\pi\omega_j/a} - 1}}$$

Results of the entanglement

What is the origin of the radiation ?

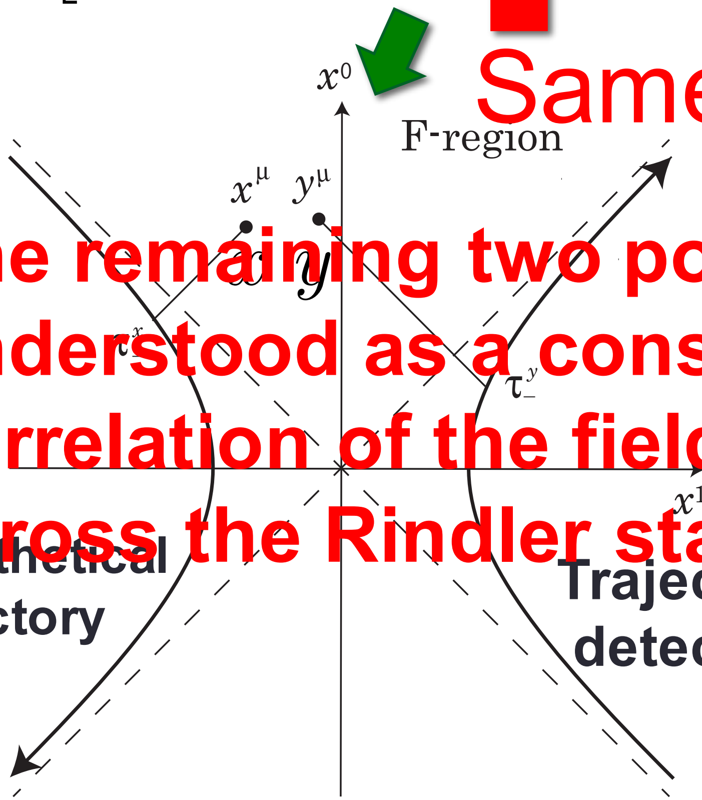
$x, y \in \mathrm{F\ region}$

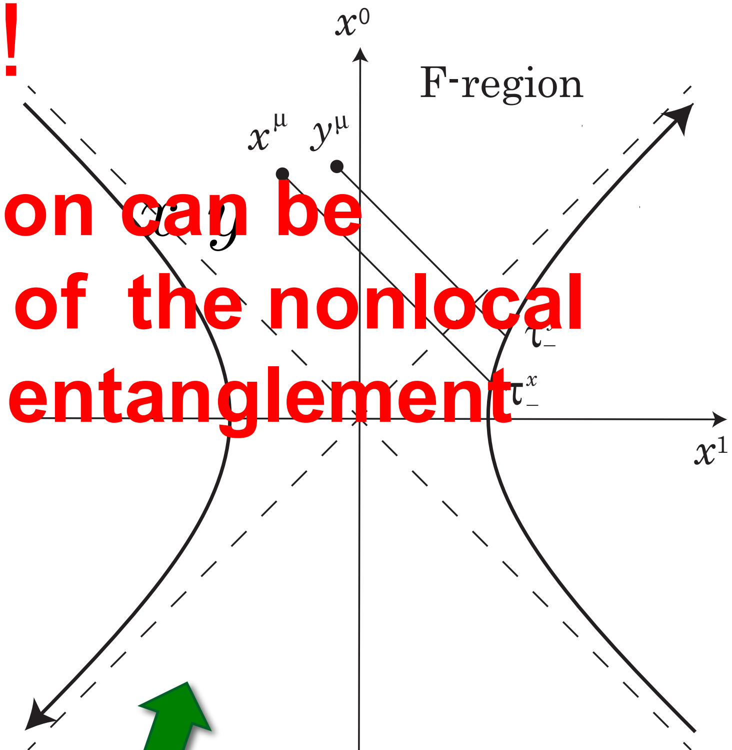
$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle + \langle \phi_{inh}(x)\phi_{inh}(y)\rangle$$

$$= -i\lambda^2 \int \frac{d\omega}{2\pi} \boxed{\frac{e^{\pi\omega/a}}{e^{2\pi\omega/a}-1}} \int d\tau \int d\tau' e^{-i\omega(\tau-\tau')}$$

$$\times \Big[G_R(x, z(\tau)) G_R(y, \tilde{z}(\tau')) h(\omega) - G_R(x, \tilde{z}(\tau)) G_R(y, z(\tau')) h(-\omega) \Big]$$

Same factor !





The remaining two point function can be understood as a consequence of the nonlocal correlation of the field and the entanglement across the Rindler states.

Hypothetical Trajectory

Trajectory of detector

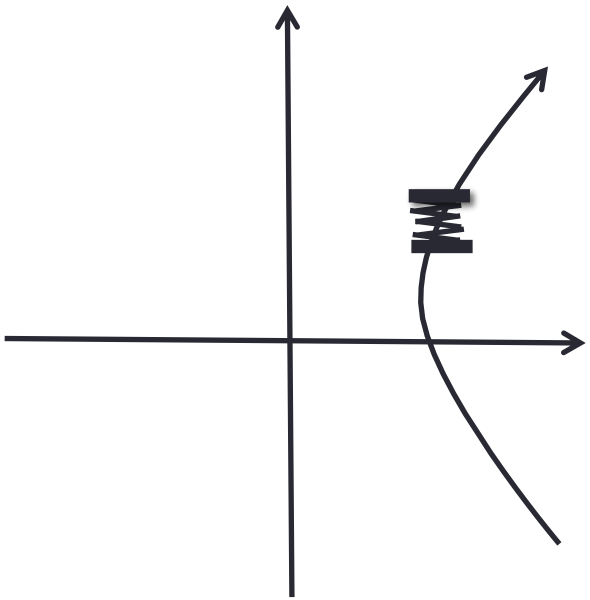
Canceled term $\langle \phi_{inh}(x)\phi_{inh}(y)\rangle$

Results from the detector model

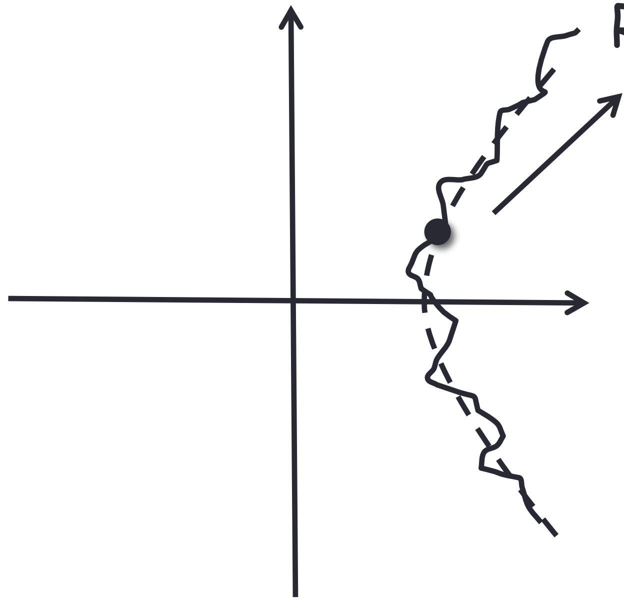
- ✓Non-vanishing quantum radiation produced by the Unruh-de Witt detector in a uniformly accelerating motion, which is consistent with the previous calculation by Lin & Hu (06).
- ✓Origin of the quantum radiation is inferred to be related to the nonlocal correlation of the Minkowski vacuum, and the entanglement of the state between the left and the right Rindler wedges. This is also an aspect of the Unruh effect

4. Unruh-de Witt detector v.s. Particle

Harmonic Oscillator
(Detector)



Particle Iso et al. (2011)



Instead of detector's excitation、we next consider
a particle in a uniformly accelerated motion,
the random motions of a particle appears due to the coupling
to the vacuum fluctuations. We consider radiations from it.

Lin, Hu (2006)
Iso, et al. (2011)
Oshita, YK, Zhang (2015)

2. Particle and Scalar field

$$S[z,\phi] = -m\int d\tau\sqrt{\eta_{\mu\nu}\dot{z}^{\mu}\dot{z}^{\nu}} + \int d^4x\frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi + S_{int}(z,\phi)$$

$$S_{int}(z,\phi) = e\int d\tau d^4x\sqrt{g_{\mu\nu}(x)\dot{z}^{\mu}\dot{z}^{\nu}}\phi(x)\delta^4\left(x - z(\tau)\right)$$
$$= e\int d\tau\sqrt{\eta_{\mu\nu}\dot{z}^{\mu}\dot{z}^{\nu}}\phi(z(\tau))$$

Oshita, YK, Zhang (2016)

3. Particle and Electromagnetic field

$$S[z,A] = -m\int d\tau\sqrt{\eta_{\mu\nu}\dot{z}^{\mu}\dot{z}^{\nu}} - \frac{1}{4}\int d^4x F^{\mu\nu}F_{\mu\nu} + S_{\rm int}(z,A)$$

$$S_{\rm int}(z,A) = -e\int d\tau\int d^4x\delta^4_D\left(x - z(\tau)\right)\dot{z}^{\mu}(\tau)A_{\mu}(x)$$
$$= -e\int d\tau\dot{z}^{\mu}(\tau)A_{\mu}(z(\tau))$$

Particle and scalar field

Iso, et al. (2011)

☑ $$\partial^\mu \partial_\mu \phi(x) = e \int d\tau \sqrt{\eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu}\,\delta^4(x - z(\tau))$$

$$\phi(x) = \phi_{inh}(x) + \phi_h(x)$$

④ two-point function →EM tensor

①

$$\phi_{\rm inh}(x) = \int d^4x' G_R(x,x') e \int d\tau' \sqrt{\eta_{\mu\nu}\dot{z}^\mu \dot{z}^\nu}\,\delta^4(x' - z(\tau'))$$
$$= e \int d\tau' G_R(x - z(\tau'))$$

③

☑ $$m\ddot{z}^\mu = e\left(\ddot{z}^\mu \phi + \dot{z}^\mu \dot{z}^\alpha \frac{\partial \phi}{\partial x^\alpha} - \eta^{\mu\alpha}\frac{\partial \phi}{\partial x^\alpha}\right)\Bigg|_{x=z(\tau)} + F^\mu$$

②

$$m\ddot{z}^\mu = \frac{e^2}{12\pi}\left(\dddot{z}^\mu + \dot{z}^\mu\left(\ddot{z}\right)^2\right) + e\left(\ddot{z}^\mu \phi_{\rm h} + \dot{z}^\mu \dot{z}^\alpha \frac{\partial \phi_{\rm h}}{\partial x^\alpha} - \eta^{\mu\alpha}\frac{\partial \phi_{\rm h}}{\partial x^\alpha}\right)\Bigg|_{x=z(\tau)} + F^\mu$$

Radiation reaction   Random force

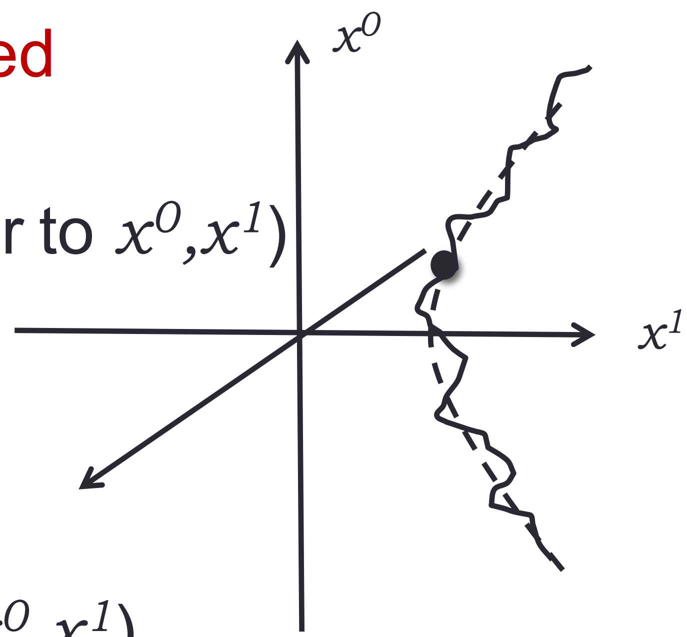
② Particle's equation of motion can be solved as perturbations

$$m\ddot{z}^\mu = \frac{e^2}{12\pi}\left(\dddot{z}^\mu + \dot{z}^\mu\left(\ddot{z}\right)^2\right) \quad + e\left(\ddot{z}^\mu\phi_\mathrm{h} + \dot{z}^\mu\dot{z}^\alpha\frac{\partial\phi_\mathrm{h}}{\partial x^\alpha} - \eta^{\mu\alpha}\frac{\partial\phi_\mathrm{h}}{\partial x^\alpha}\right)\Bigg|_{x=z(\tau)} + F^\mu$$

Radiation reaction　　　　Random force

$$z^\mu(\tau) = z^\mu_\mathrm{cl}(\tau) + \delta z^\mu(\tau) \qquad F^\mu = ma(\dot{z}^1, \dot{z}^0, 0, 0)$$

δz^μ Linearized perturbed equation



Transverse component (perpendicular to x^0,x^1)

$$v^i = \delta\dot{z}^i$$

$$m\dot{v}^i = \frac{e^2}{12\pi}(\ddot{v}^i - a^2 v^i) + e\frac{\partial\phi_h}{\partial x^i}\Big|_{x=z(\tau)}$$

Longitudinal component (parallel to x^0,x^1)

$$m(\delta\ddot{\xi} - a^2\delta\xi) = \frac{e^2}{12\pi}\left(\delta\dddot{\xi} - a^2\delta\dot{\xi}\right) + e\left[a + \partial/\partial\xi\right]\phi_h\big|_{x=z(\tau)}$$

Random motion in the transverse direction

Under the influence of the Unruh effect

$$v^i = \delta \dot{z}^i \qquad m\dot{v}^i = \frac{e^2}{12\pi}(\ddot{v}^i - a^2 v^i) + e\frac{\partial \phi_h}{\partial x^i}\Big|_{x=z(\tau)}$$

$$v^i(\tau) = v^i(\tau_0)e^{-a\sigma(\tau-\tau_0)} + \frac{e}{m}\int_{\tau_0}^{\tau} d\tau' \partial_i \phi_h(z(\tau'))e^{-a\sigma(\tau-\tau')}$$

Initial condition

$$\tau = \tau_0 \qquad v^i = v^i(\tau_0) \qquad \sigma = \frac{e^2 a}{12\pi m} \ll 1$$

$$\left\langle v^i(\tau)v^j(\tau')\right\rangle = e^{-\sigma a(\tau+\tau'-2\tau_0)}v^i(\tau_0)v^j(\tau_0)$$

$$+\frac{e^2}{m^2}\int_{\tau_0}^{\tau}\int_{\tau_0}^{\tau'} d\tau'' d\tau''' e^{-a\sigma(\tau-\tau'')}e^{-a\sigma(\tau'-\tau''')}\langle \partial_i\phi_h(z(\tau''))\partial_j\phi_h(z(\tau'''))\rangle$$

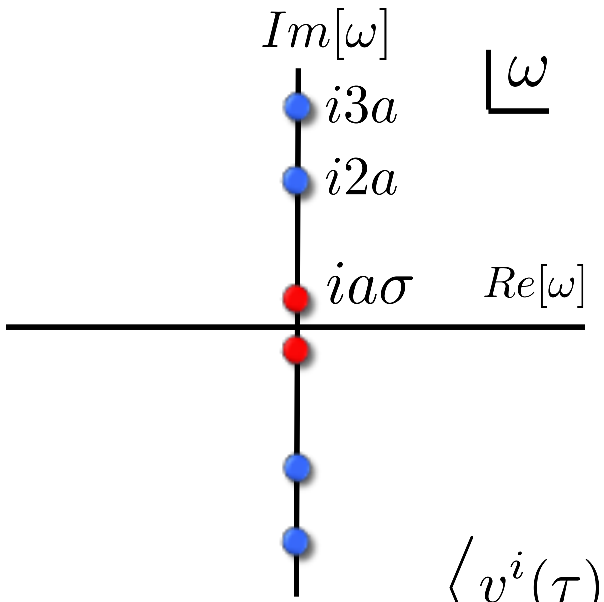
Minkowski vacuum state

$$\left\langle \phi_h(x)\phi_h(x')\right\rangle = -\frac{1}{4\pi^2}\frac{1}{(t-t'-i\delta)^2 - |\mathbf{x}-\mathbf{x}'|^2}$$

$$\left\langle \partial_i\phi_h(z(\tau))\partial_j\phi_h(z(\tau'))\right\rangle = \frac{a^4}{32\pi^2}\frac{\delta_{ij}}{\sinh^4\left(\frac{a(\tau-\tau'-i\delta)}{2}\right)}$$

$$\tau_0 \to -\infty \qquad \left\langle v^i(\tau) v^j(\tau') \right\rangle_S = \frac{e^2 \delta_{ij}}{24\pi^2 m^2} \int d\omega\omega \frac{\omega^2 + a^2}{a^2\sigma^2 + \omega^2} \coth(\pi\omega/a) e^{i\omega(\tau-\tau')}$$

$$= \frac{e^2 \delta_{ij}}{24\pi^2 m^2} \left\{ \pi a^2 (1-\sigma^2) \cot \pi\sigma e^{-a\sigma|\tau-\tau'|} - 2a^2 \sum_{n=2}^{\infty} \frac{n(n^2-1)}{n^2-\sigma^2} e^{-na|\tau-\tau'|} \right\}$$



$$\sigma = \frac{e^2 a}{12\pi m} \ll 1 \qquad a|\tau - \tau'| \ll 1$$

$$\left\langle v^i(\tau) v^j(\tau') \right\rangle_S \simeq \delta_{ij} \frac{a}{2\pi m} \underbrace{- \delta_{ij} \frac{a^2 e^2}{12\pi^2 m^2} \left\{ \frac{1}{(a|\tau-\tau'|)^2} + \log|a(\tau-\tau')| \right\}}_{O\left(a^2/m^2\right)}$$

$$\frac{1}{2m} \left\langle v^i v^j \right\rangle = \delta_{ij} \frac{T_U}{2} \qquad T_U = \frac{a}{2\pi}$$

➡ Energy equi-partition relation

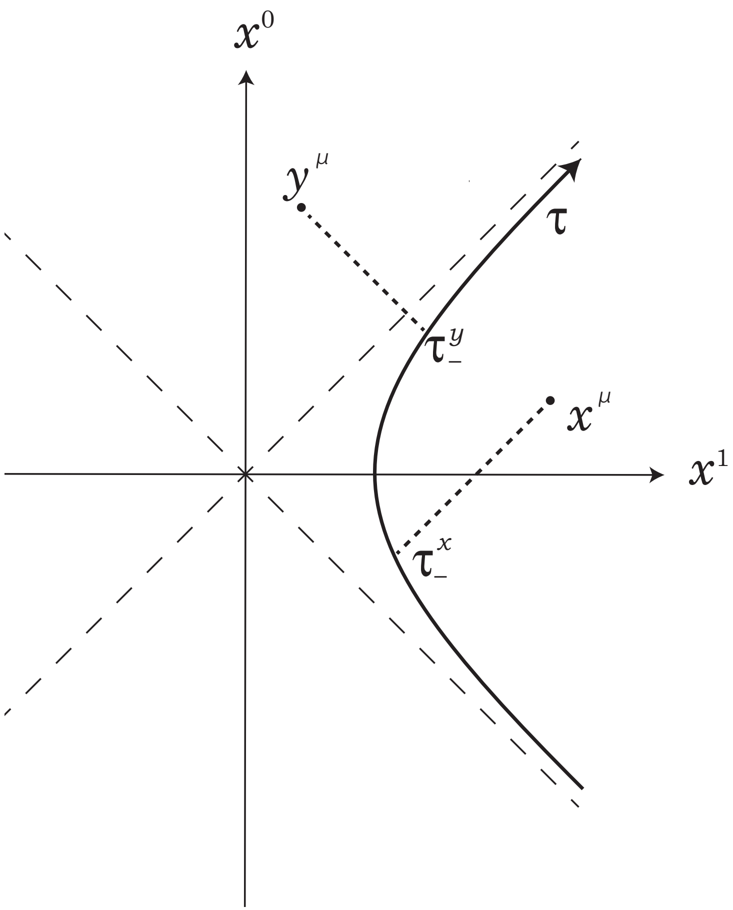
② Solved the particle's motion (We focus on the transverse fluctuations)

Solution of the field (two-point function – EM tensor)

$$\phi(x) = \phi_{\rm h}(x) + \phi_{\rm inh}(x) \qquad G_R(x - z(\tau')) = \theta(t - z^0(\tau'))\delta((x - z(\tau'))^2)$$

$$\phi_{\rm inh}(x) = e\int d\tau' G_R(x - z(\tau')) = \frac{e}{4\pi\rho(x)} = \frac{e}{4\pi\dot{z}(\tau_-)\cdot(x - z(\tau))}$$

$$\rho(x) = \dot{z}(\tau_-)\cdot(x - z(\tau_-))$$



$$z^\mu = \bar{z}^\mu + \delta z^\mu + \cdots$$

$$\rho(x) = \rho_0(x) + \delta\rho(x) + \cdots$$

$$\rho_0(x) = \dot{\bar{z}}(\tau_-)\cdot(x - \bar{z}(\tau_-))$$

$$\delta\rho(x) = \delta\dot{z}(\tau_-)\cdot(x - \bar{z}(\tau_-)) + \delta z(\tau_-)$$

$$\delta\dot{z}^i(\tau) = v^i(\tau)$$ Solved already

$$\phi_{inh}(x) \cong \frac{e}{4\pi\rho_0(x)}\left(1 - \frac{\delta\rho(x)}{\rho_0(x)}\right)$$

Two point function

$$T_{\mu\nu} = \langle \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}\eta_{\mu\nu}\partial^\alpha \phi \partial_\alpha \phi \rangle$$
$$= \lim_{y\to x} \langle \partial_\mu \phi(x) \partial_\nu \phi(y) - \frac{1}{2}\eta_{\mu\nu}\partial^\alpha \phi(x) \partial_\alpha \phi(y) \rangle$$

$$\phi(x) = \phi_{\mathrm{h}}(x) + \phi_{\mathrm{inh}}(x)$$

$$\langle \phi(x)\phi(y)\rangle = \langle \phi_{\mathrm{h}}(x)\phi_{\mathrm{h}}(y)\rangle + \langle \phi_{\mathrm{h}}(x)\phi_{\mathrm{inh}}(y)\rangle + \langle \phi_{\mathrm{inh}}(x)\phi_{\mathrm{h}}(y)\rangle + \langle \phi_{\mathrm{inh}}(x)\phi_{\mathrm{inh}}(y)\rangle$$

$$\phi_{inh}(x) = \frac{e}{4\pi\rho_0(x)}\left(1 - \frac{\delta\rho(x)}{\rho_0(x)}\right)$$

$$\delta\rho(x) \simeq \delta\dot{z}^i(\tau_-^x)x_i$$

$$\sigma = \frac{e^2 a}{12\pi m} \ll 1$$

$$\delta\dot{z}^i(\tau) = \frac{e}{m}\int_{-\infty}^{\tau} d\tau' \partial_i \phi_h(\bar{z}(\tau'))e^{-a\sigma(\tau-\tau')}$$
$$= e\int \frac{d\omega}{2\pi} h(\omega)\partial_i\varphi(\omega)$$

$$h(\omega) = \frac{1}{m(-i\omega + a\sigma)}$$

$$\partial_i\phi_h(z(\tau)) = \int \frac{d\omega}{2\pi}\partial_i\varphi(\omega)e^{-i\omega\tau}$$

$$\langle \phi_h(x)\phi_{\mathrm{inh}}(y)\rangle = -\frac{e}{4\pi\rho_0^2(y)}\langle \phi_h(x)\delta\rho(y)\rangle = -\frac{e y_i}{4\pi\rho_0^2(y)}\langle \phi_h(x)\delta\dot{z}^i(\tau_-^y)\rangle$$

Two point function

$$\langle\phi(x)\phi(y)\rangle = \langle\phi_{\rm h}(x)\phi_{\rm h}(y)\rangle + \langle\phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle + \langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle$$

$$\langle\phi_h(x)\phi_{\rm inh}(y)\rangle = -\frac{e}{4\pi\rho_0^2(y)}\langle\phi_h(x)\delta\rho(y)\rangle = -\frac{ey_i}{4\pi\rho_0^2(y)}\langle\phi_h(x)\delta\dot{z}^i(\tau_-^y)\rangle$$

$$\langle\phi_h(x)\delta\dot{z}^i(\tau_-^y)\rangle = e\int\frac{d\omega}{2\pi}h(\omega)\langle\phi_h(x)\partial_i\varphi(\omega)\rangle$$

$$\langle\phi_h(x)\partial_i\varphi(\omega)\rangle = \int d\tau e^{i\omega\tau}\left(\frac{\partial}{\partial y^i}\langle\phi_h(x)\phi_h(y)\rangle\right)_{y=z(\tau)} = \frac{1}{4\pi^2}\frac{\partial}{\partial x^i}P(x,\ \omega)$$

$$P(x,\ \omega) \equiv \int d\tau\frac{e^{i\omega\tau}}{(x^0-z^0(\tau)-i\epsilon)^2-(x^1-z^1(\tau))^2-x_\perp^2}$$

$$\langle\phi_h(x)\phi_h(z(\tau))\rangle = -\frac{1}{4\pi^2}\frac{1}{\eta_{\mu\nu}(x^\mu-z^\mu(\tau))(x^\nu-z^\nu(\tau))}$$

$$(x^0-z^0(\tau))^2-(x^1-z^1(\tau))^2-x_\perp^2 = 0 \quad \Leftrightarrow \quad \tau_\pm^x$$

Correlation of the homogeneous solution and inhomogeneous solution

$$Z_x(w) = e^{\pi\omega/a}\theta(x^0 - x^1) + \theta(-x^0 + x^1)$$

$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle = \frac{-iae^2}{m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(x)}\Bigg[$$

$$+\int \frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\left(\left(\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_-^y-\tau_-^x)} + \left(-\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_+^y-\tau_-^x)}Z_y(-\omega)\right)\frac{1}{a\sigma - i\omega}$$

$$-\int \frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\Bigg(\underbrace{\left(\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_-^x-\tau_-^y)}}_{\tau_-} + \underbrace{\left(-\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_+^x-\tau_-^y)}Z_x(-\omega)}_{\tau_+}\Bigg)\frac{1}{a\sigma + i\omega}\Bigg]$$

Correlation of the homogeneous solution and inhomogeneous solution

$$Z_x(w) = e^{\pi\omega/a}\theta(x^0 - x^1) + \theta(-x^0 + x^1)$$

$$\langle\phi_{inh}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{inh}(y)\rangle = \frac{-iae^2}{m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(x)}\Bigg[$$

$$+\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\left(\left(\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_-^y-\tau_-^x)}+\left(-\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_+^y-\tau_-^x)}Z_y(-\omega)\right)\frac{1}{a\sigma - i\omega}$$

$$-\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\Bigg(\underbrace{\left(\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_-^x-\tau_-^y)}}_{\tau_-}+\underbrace{\left(-\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_+^x-\tau_-^y)}Z_x(-\omega)}_{\tau_+}\Bigg)\frac{1}{a\sigma + i\omega}\Bigg]$$

Correlation of the inhomogeneous solution and inhomogeneous solution

$$\langle\phi_{inh}(x)\phi_{inh}(y)\rangle = \frac{e^2}{(4\pi)^2}\frac{1}{\rho_0(x)\rho_0(y)}$$

$$-\frac{iae^2}{m(4\pi)^2}\frac{x^iy^i}{\rho_0^2(x)\rho_0^2(y)}\overbrace{\left[\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\frac{i\omega}{a}\left(-e^{i\omega(\tau_-^y-\tau_-^x)}-e^{-i\omega(\tau_-^x-\tau_-^y)}\right)\right]}^{\tau_-}$$

Correlation of the homogeneous solution and inhomogeneous solution

quantum radiation

$$\langle\phi_{inh}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{inh}(y)\rangle = \frac{-iae^2}{m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(x)}\Bigg[$$

$$+\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\left(\left(\frac{aL_y^2}{2\rho_0(y)}+\cancel{\frac{i\omega}{a}}\right)e^{i\omega(\tau_-^y-\tau_-^x)}+\left(-\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_+^y-\tau_-^x)}Z_y(-\omega)\right)\frac{1}{a\sigma-i\omega}$$

$$-\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\Bigg(\underbrace{\left(\frac{aL_x^2}{2\rho_0(x)}-\cancel{\frac{i\omega}{a}}\right)e^{-i\omega(\tau_-^x-\tau_-^y)}}_{\tau_-}+\underbrace{\left(-\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_+^x-\tau_-^y)}Z_x(-\omega)}_{\tau_+}\Bigg)\frac{1}{a\sigma+i\omega}\Bigg]$$

Correlation of the inhomogeneous solution and inhomogeneous solution

$$\langle\phi_{inh}(x)\phi_{inh}(y)\rangle = \frac{e^2}{(4\pi)^2}\frac{1}{\rho_0(x)\rho_0(y)}$$

$\leftarrow$classical radiation

$$-\frac{iae^2}{m(4\pi)^2}\frac{x^iy^i}{\rho_0^2(x)\rho_0^2(y)}\left[\int\frac{d\omega}{2\pi}\frac{1}{e^{-2\pi\omega/a}-1}\frac{i\omega}{a}\overbrace{\left(-\cancel{e^{i\omega(\tau_-^y-\tau_-^x)}}-\cancel{e^{-i\omega(\tau_-^x-\tau_-^y)}}\right)}^{\tau_-}\right]$$

$\langle\phi_{inh}(x)\phi_{inh}(y)\rangle$ is canceled out.

Iso, et al. (2011)

Chen & Tajima (99)' naïve radiation cancels out.

$$\langle \phi(x)\phi(y)\rangle - \langle \phi_{\rm h}(x)\phi_{\rm h}(y)\rangle = \langle \phi_{\rm inh}(x)\phi_{\rm h}(y)\rangle + \langle \phi_{\rm h}(x)\phi_{\rm inh}(y)\rangle + \langle \phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle$$

$$= \frac{e^2}{(4\pi)^2\rho_0(x)\rho_0(y)} - \frac{iae^2}{2m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(y)}K(x,y)$$

Classical Quantum radiation

$$K(x,y) = +\int \frac{d\omega}{2\pi}\frac{h(\omega)}{e^{-2\pi\omega/a}-1}\left(\frac{aL_y^2}{2\rho_0(y)}e^{i\omega(\tau_-^y-\tau_-^x)} + \left(-\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\right)e^{i\omega(\tau_+^y-\tau_-^x)}Z_y(-\omega)\right)$$

$$-\int \frac{d\omega}{2\pi}\frac{h(-\omega)}{e^{-2\pi\omega/a}-1}\left(\underbrace{\frac{aL_x^2}{2\rho_0(x)}e^{-i\omega(\tau_-^x-\tau_-^y)}}_{\tau_-} + \underbrace{\left(-\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\right)e^{-i\omega(\tau_+^x-\tau_-^y)}Z_x(-\omega)}_{\tau_+}\right)\Bigg]$$

$$h(\omega) = \frac{1}{m}\frac{1}{a\sigma - i\omega} \qquad L^2 = -x^\mu x_\mu + \frac{1}{a^2} \qquad Z_x(w) = e^{\pi\omega/a}\theta(x^0-x^1) + \theta(-x^0+x^1)$$

Quantum radiation is originated from the interference term

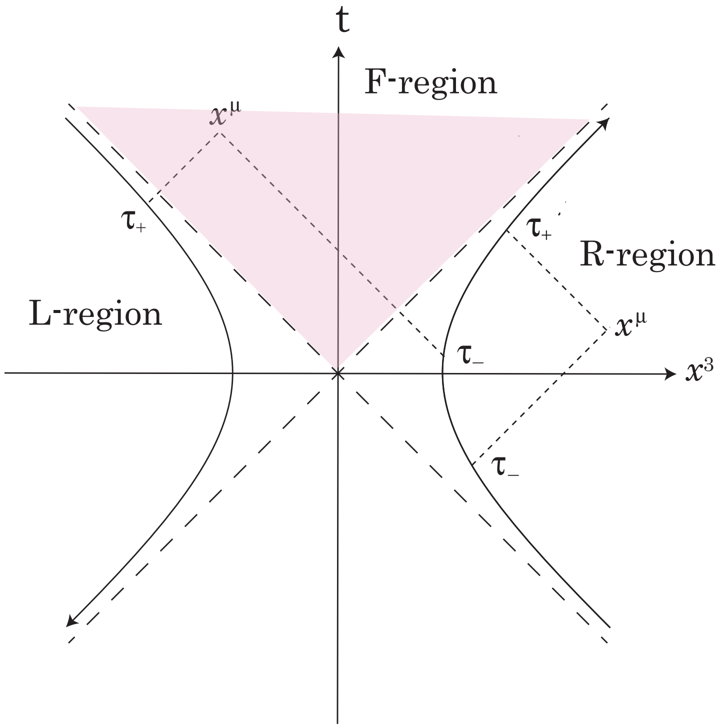
$$\langle \phi_{inh}(x)\phi_h(y)\rangle + \langle \phi_h(x)\phi_{inh}(y)\rangle$$

Oshita, YK, Zhang (2015)

$$[\langle\phi(x)\phi(y)\rangle - \langle\phi_{\mathrm{h}}(x)\phi_{\mathrm{h}}(y)\rangle]_S = [\langle\phi_{\mathrm{inh}}(x)\phi_{\mathrm{h}}(y)\rangle + \langle\phi_{\mathrm{h}}(x)\phi_{\mathrm{inh}}(y)\rangle + \langle\phi_{\mathrm{inh}}(x)\phi_{\mathrm{inh}}(y)\rangle]_S$$

$$= \frac{e^2}{(4\pi)^2\rho_0(x)\rho_0(y)} - \frac{iae^2}{2m(4\pi)^2}\frac{x^i}{\rho_0^2(x)}\frac{y^i}{\rho_0^2(y)}\left[\frac{aL_x^2}{2\rho_0(x)}(I_3(x,y) - I_1(x,y)) + \frac{i}{a}I_2(x,y)\right]$$

$x, y \in \mathrm{F}$ region



Perturbative expansion introducing

$$\boxed{\sigma = \frac{e^2 a}{12\pi m}(\ll 1) \text{ as the expansion parameter}}$$

$$I_1(x,y) = -\frac{i}{2\pi\sigma} + \frac{i}{\pi}\log(1 + e^{-a|\tau_-^y - \tau_+^x|}) + \frac{1}{\pi}a(\tau_-^y - \tau_+^x)\theta(\tau_-^y - \tau_+^x) + \mathcal{O}(\sigma)$$

$$I_2(x,y) = -\frac{a}{\pi}\frac{1}{e^{a(\tau_+^x - \tau_-^y)} + 1} + \mathcal{O}(\sigma)$$

$$I_3(x,y) = -\frac{i}{2\pi\sigma} + \frac{i}{\pi}\log(1 - e^{-a|\tau_-^y - \tau_-^x|}) + \frac{1}{\pi}a(\tau_-^y - \tau_-^x)\theta(\tau_-^y - \tau_-^x) + \mathcal{O}(\sigma)$$

Energy momentum tensor

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi$$

Energy flux f is computed from the two point function

$$T_{0i} = \lim_{y \to x} \left\langle \frac{\partial \phi(x)}{\partial x^0} \frac{\partial \phi(y)}{\partial y^i} \right\rangle = \lim_{y \to x} \frac{\partial}{\partial x^0} \frac{\partial}{\partial y^i} \left\langle \phi(x) \phi(y) \right\rangle$$

$$f = -\sum_i T_{0i} n^i \qquad n^i = \frac{x^i}{r}$$

$$= f^C + f^Q$$

=Classical component (Larmor radiation)+ Quantum component

Energy momentum tensor component

$$x \in \mathrm{F} \text{ region}$$

$$T^C_{0\mu} = \frac{a^2 e^2}{(4\pi)^2} \frac{x_0 x_\mu}{\rho_0^4(x)} P^2$$

$$T^Q_{0\mu} = \frac{2a^3 e^2}{m(4\pi)^3} \frac{\mathbf{x}_\perp^2 x_0 x_\mu}{\rho_0^6(x)} \Bigg[-4P(3P^2-1)\Big\{ \log a\epsilon - \log\left(1 - e^{-a|\tau_- - \tau_+|}\right) - a(\tau_- - \tau_+)\theta(\tau_- - \tau_+) \Big\}$$

$$- \frac{2(9P^2-1)}{e^{a(\tau_+ - \tau_-)}+1} + (P^2-1) - P\left\{ \frac{2}{(a\epsilon)^2} - \frac{5}{2} \frac{1}{\cosh^2(a(\tau_+ - \tau_-)/2)} \right\} - \frac{1}{2} \frac{\tanh(a(\tau_+ - \tau_-)/2)}{\cosh^2(a(\tau_+ - \tau_-)/2)} \Bigg]$$

$$P = \frac{-aL^2}{2\rho_0(x)}$$

$$L^2 = -(t^2 - r^2) + \frac{1}{a^2}$$

$$\rho_0(x) = \sqrt{\left(\frac{a}{2}(t^2 - r^2) - \frac{1}{2a} \right)^2 + t^2 - r^2 \cos^2\theta}$$

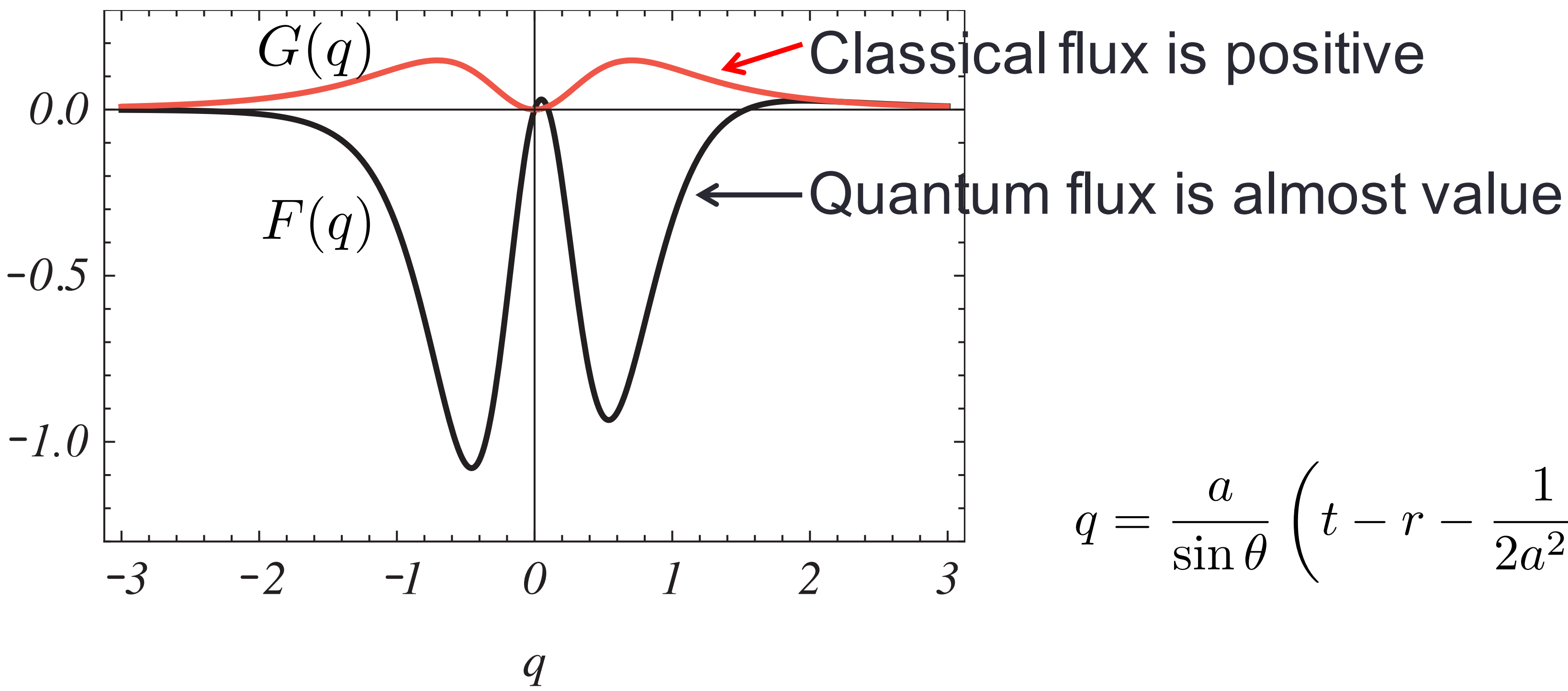
$$\tau_+ - \tau - = \log \left[\frac{+L^2 + \sqrt{L^4 + \frac{4}{a^2}(t^2 - r^2\cos^2\theta)}}{-L^2 + \sqrt{L^4 + \frac{4}{a^2}(t^2 - r^2\cos^2\theta)}} \right]$$

$$f = -\sum_i T_{0i} n^i = f^C + f^Q$$

Oshita, YK, Zhang (2015)

$$f^C = \left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2} \frac{G(q)}{\sin^4\theta}$$

$$f^Q = \frac{a}{2\pi m} \left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2} \frac{F(q)}{\sin^4\theta}$$

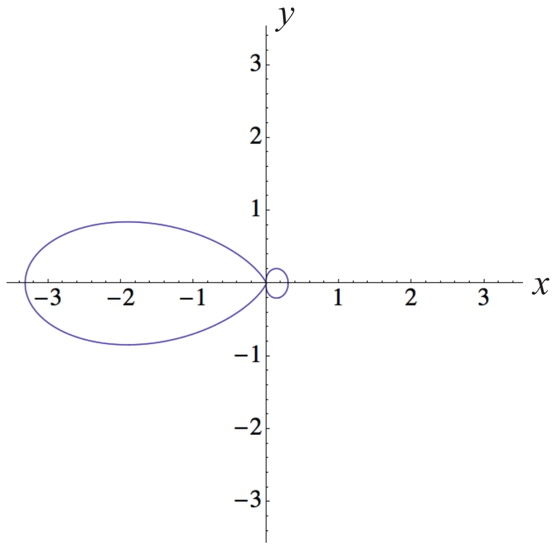


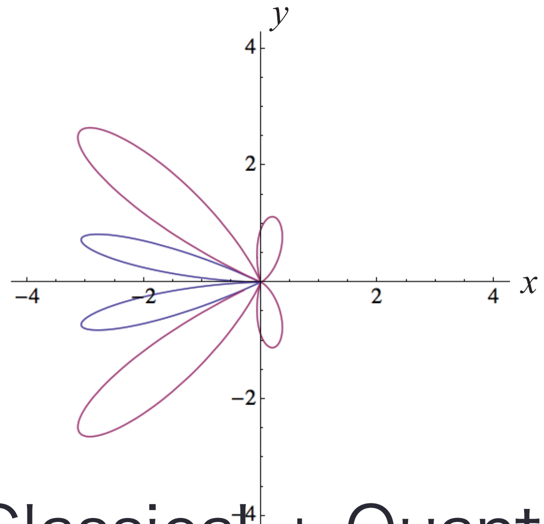
$$q = \frac{a}{\sin\theta}\left(t - r - \frac{1}{2a^2 r}\right)$$

Angular plot of the radiation

$f^C(\tau_-,\theta)$ Classical $\qquad$ $f^Q(\tau_-,\theta)$ quantum

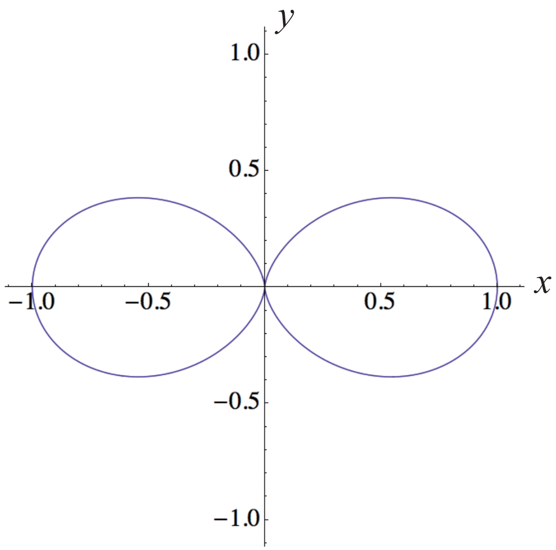
$\tau_- = -0.3/a$

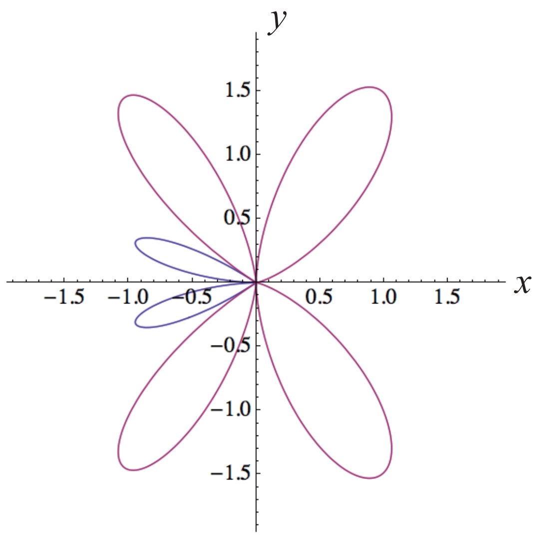




Classical + Quantum > 0

$\tau_- = 0$





$\tau_- = 0.3/a$

$$f = -\sum_i T_{0i} n^i = f^C + f^Q$$

$$f^C = \left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2} \frac{G(q)}{\sin^4\theta}$$

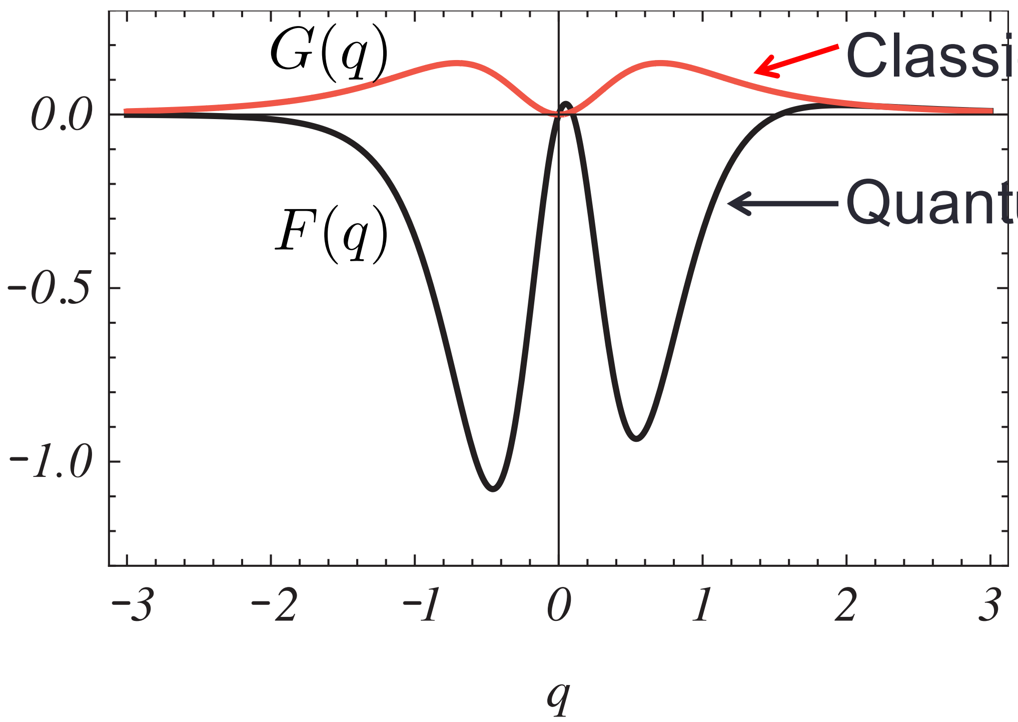
$$f^Q = \frac{a}{2\pi m} \left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2} \frac{F(q)}{\sin^4\theta}$$

Oshita, YK, Zhang (2015)

$$f^C + f^Q > 0$$

Classical + quantum > 0

Quantum radiation suppresses the total radiation flux



Classical flux is positive

Quantum flux is almost negative

$$q = \frac{a}{\sin\theta}\left(t - r - \frac{1}{2a^2 r}\right)$$

Oshita, YK, Zhang (2016)

3. Particle and electromagnetic field model

$$S[z, A] = -m \int d\tau \sqrt{\eta_{\mu\nu} \dot{z}^\mu \dot{z}^\nu} - \frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} + S_{\rm int}(z, A)$$

$$S_{\rm int}(z, A) = -e \int d\tau \int d^4x \delta_D^4 (x - z(\tau)) \dot{z}^\mu(\tau) A_\mu(x)$$
$$= -e \int d\tau \dot{z}^\mu(\tau) A_\mu(z(\tau))$$

☑ $$\partial_\mu \partial^\mu A^\nu = e \int d\tau \dot{z}^\nu(\tau) \delta_D^4(x - z(\tau)) \qquad \partial_\mu A^\mu = 0$$

☑ $$m\ddot{z}_\mu = e(\partial_\mu A_\nu - \partial_\nu A_\mu)\dot{z}^\nu + f_\mu$$

$$z^\mu = \bar{z}^\mu + \delta z^\mu \qquad A^\mu(x) = A_{\rm h}^\mu(x) + e \int d\tau G_R(x, z(\tau)) \dot{z}^\mu(\tau)$$

$$m\delta\ddot{z}_i(\tau) = \frac{e^2}{6\pi}(\delta\dddot{z}_i - a^2 \delta\dot{z}_i) + e(\eta_{i\nu}\dot{\bar{z}}_\alpha - \eta_{i\alpha}\dot{\bar{z}}_\nu)\partial^\nu A_{\rm h}^\alpha(x)\big|_{x=z(\tau)}$$

Random motions in the transverse direction satisfy the energy equi-partition relation due to the Unruh effect.

$$A^{\mu}_{\text{inh}}(x) = \frac{e}{4\pi\rho_0(x)}\left(\dot{z}^{\mu}(\tau^x_-) - E^{\mu}_{(-)i}(x)\delta\dot{z}^i(\tau^x_-)\right) \qquad E^{\mu i}_{(\mp)}(x) = \eta^{\mu i} - \frac{\dot{z}^{\mu}(\tau^x_{\mp})x^i}{\rho_0(x)}$$

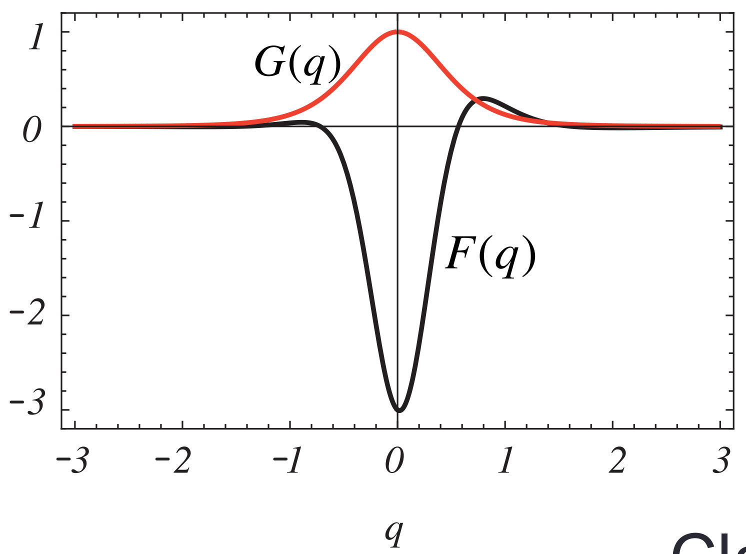
$$[\langle A^{\alpha}_{\text{h}}(x)A^{\beta}_{\text{inh}}(y)\rangle + \langle A^{\alpha}_{\text{inh}}(x)A^{\beta}_{\text{h}}(y)\rangle + \langle A^{\alpha}_{\text{inh}}(x)A^{\beta}_{\text{inh}}(y)\rangle]_S$$

This term cancels out by a term of interference

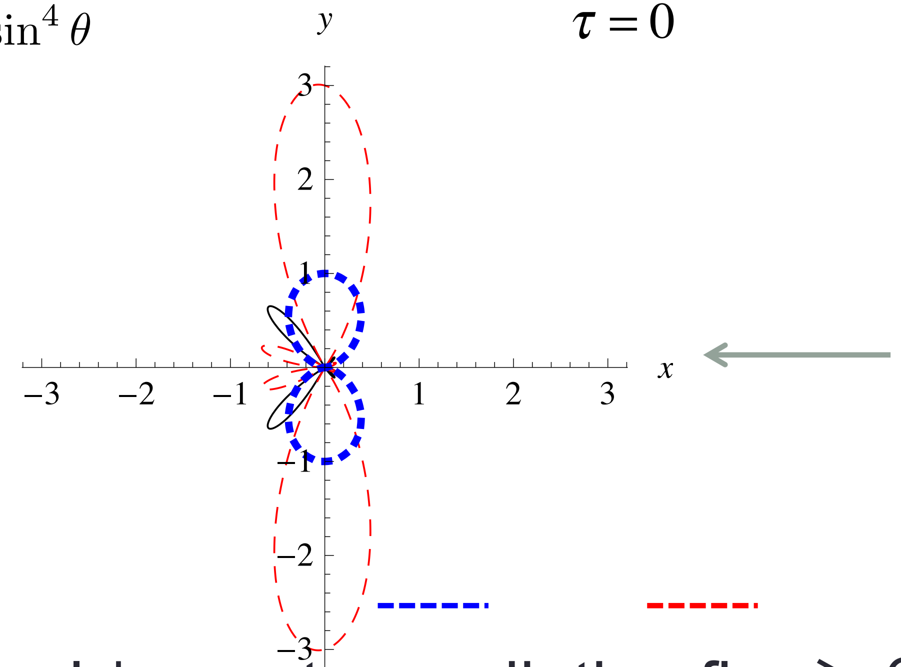
$$T_{0\mu} = -(A_{\alpha,0} - A_{0,\alpha})(A^{\alpha}{}_{,\mu} - A_{\mu}{}^{,\alpha})$$

$$f = -\sum_i T_{0i}n^i = f^C + f^Q$$

$$f^C = \left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2}\frac{G(q)}{\sin^4\theta} \qquad f^Q = \frac{a}{2\pi m}\left(\frac{e}{4\pi}\right)^2 \frac{a^2}{r^2}\frac{F(q)}{\sin^4\theta}$$



Angular plot of radiation



Classical + quantum radiation flux > 0

Energy momentum tensor

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi$$

Energy flux f is computed from the two point function

$$T_{0i} = \lim_{y\to x}\left\langle \frac{\partial\phi(x)}{\partial x^0}\frac{\partial\phi(y)}{\partial y^i}\right\rangle = \lim_{y\to x}\frac{\partial}{\partial x^0}\frac{\partial}{\partial y^i}\left\langle \phi(x)\phi(y)\right\rangle$$

$$f = -\sum_i T_{0i}n^i \qquad n^i = \frac{x^i}{r}$$

$$= f^C + f^Q$$

Energy radiation rate

$$\frac{dE}{dt} = \lim_{r\to\infty} r^2 \int d\Omega f = \frac{dE^C}{dt} + \frac{dE^Q}{dt}$$

Classical $\dfrac{dE^C}{dt} \sim \dfrac{e^2a^2}{(4\pi)^2}$

quantum $\dfrac{dE^Q}{dt} \sim -\dfrac{a}{m}\dfrac{e^2a^2}{(4\pi)^2} \sim -\dfrac{a}{m}\dfrac{dE^C}{dt}$

Quantum effect suppresses the total radiation

Results of the particle models

Random motion in the transverse direction satisfies the energy equi-partition relation (thermal property)

Naive radiation component cancels out.
The interference term makes contribution to radiation flux, but makes negative energy flux of the order

$$\left.\frac{dE}{dt}\right|_{\text{Quantum}} \sim -\frac{a}{m}\left.\frac{dE}{dt}\right|_{\text{Classical}}$$

Quantum effect suppresses the total Larmor radiation.

This quantum effect from the remaining interference term will be related to the nonlocal correlation of the Minkowski vacuum.

5. Summary and conclusions

- ✓ There exists the non-vanishing quantum effect in the radiation produced by the Unruh-de Witt detector and a particle in a uniformly accelerating motion.

- ✓ For the particle models, the quantum effect suppresses the total Larmor radiation.

- ✓ Origin of these effects will be related to the nonlocal correlation of the Minkowski vacuum, which is understood by the entanglement of the Rindler states.

Thank you.