Tax Revenue-Financed Public Abatement and Welfare: A Consumption Tax Case

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Abstract

This paper develops a model of public abatement earmarked by either a pollution tax or a consumption tax and shows that consumption tax revenue-financed public abatement is superior to pollution tax revenue-financed public abatement in terms of welfare when the pollution tax rate rises. It implies that the optimal pollution tax rate is higher under consumption tax revenue-financed public abatement than it is under pollution tax revenue-financed public abatement. These results are policy oriented that policy makers of environmental protection are worth considering.

1. Introduction

An important issue relating to pollution tax revenue is the earmarking of pollution tax revenue for environmental protection. The notable example goes to environmental fund in Central and Eastern Europe such that governments levy a charge on pollution emissions and the accruing revenue is earmarked for the financing environmental protection. In this context, the public sector plays an important role in protecting the environment (i.e., waste collection and disposal, and waste water treatment, etc) by using the pollution tax revenue collected from the private sector. However, there arises a caveat to use pollution tax revenue for pollution abatement activities. Because, there is a trade-off between pollution abatement and pollution tax revenue procurement in the sense that pollution tax revenue declines with reduction in pollution (OECD, 1993). This is a serious bottleneck for environmental protection undertaken by pollution tax revenue-financed public abatement.

On the other hand, there are alternative taxes as a possible source for financing environmental protection demonstrated by the public sector. In particular, there are many cases that product charges applied to consumer products such as vehicles, batteries, fertilizers and car tires are earmarked for the financing of environmental protection although product charges are not imposed directly on pollution emission[1]. For example, revenues from tire product charges were earmarked for the financing of the Central Environmental Protection Fund in Hungary (see Morris *et al.* 1999). Fertilizer charges were imposed in Sweden and the accruing revenue were used to create funds for financing measures to mitigate negative environmental effects of agriculture (see Ribeiro *et al.* 1999). In Bulgaria, one-third of the national environmental fund was generated by a tax on imports of second-hand automobiles (see OECD 1995).

Given these insights, in this paper, we assess the justification for pollution tax revenue-financed public abatement by examining the welfare consequences of public abatement financed by either pollution tax revenue or consumption tax revenue. The main reason for we choose consumption taxes is that consumption taxes have the advantage of an enormous taxation base because total consumption currently represents around 70% of GDP in the riches OECD countries (see Albrecht 2006). Also, the consumption of certain products (i.e., motor fuels) tends to be applied to product charges and the accruing revenue are earmarked for environmental protection, as mentioned above, and therefore it would not be distant from reality.

In previous studies of public abatement issues, most of them are analyzed with the aid of trade theory (e.g., Khan 1995[2], Chao and Yu, 1999 and Hatzipanayotou et al., 2002, 2003, 2005) and assume that public abatement depends on pollution tax revenue and/or lump sum transfers. They have abstracted from the

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[1] Product charges usually address environmental problems in two ways: by taxing good or service that is closely associated with the environmentally damaging activity and by using all or part of that revenue to assist in mitigating environmental damage caused. Consider for instance, motor fuels are subject to the charges in that driving the vehicle creates air pollution, noise, and congestion (see Morris *et al.* 1999).

[2] Khan (1995) shows public abatement provided by a central agency without considering the tax revenue-financed issue.

issue of general tax revenue including consumption tax revenue earmarked for the financing of public abatement. Haibara (2006) addresses tariff revenue-financed public abatement and shows the welfare consequences of a tariff. However, his interests are situated in the effects of a tariff and the welfare effects of a pollution tax are missing. Hence addressing the welfare consequences of a pollution tax under consumption tax revenue-financed public abatement can remedy the deficiencies of the previous studies. Beyond that, this paper would provide an important caveat for countries (i.e., Central and Eastern Europe) that rely heavily on pollution tax revenue for public abatement activities. Also, this paper would provide a guideline for countries that attempt to use consumption tax revenue for public abatement. In this sense, rather than merely filling the gap of the previous studies, this paper is policy oriented that policy makers of environmental protection are worth considering.

2. The model

We assume a perfectly competitive small open economy, which, for simplicity, produces two private goods, x and y. However, the production of good x generates pollution that reduces households' utility, while the production of good y does not. There are two ways of abating pollution: private abatement and public abatement. Under private abatement, a pollution tax is imposed on the private sector producing good x in order to reduce pollution. Under public abatement, the public sector abates pollution generated from the private sector. In this context, public abatement is funded either by a pollution tax or by a consumption tax. The production side of the economy is described by the following revenue function:

$$\overline{R}(p, t, v^p) = \max_{x,y,z}[px + y - tz : (x, z) \in T(v^p)]$$

where p is the relative price of good x in terms of good y , x is the output of good x, z is the amount of pollution generated from the production of good x, t is the pollution tax rate, v^p is the domestic factor used by the private sector and $T(v^p)$ is the production technology set. Manipulation of the revenue function yields the restricted revenue function, $R(p, g, t) = \overline{R}(p, t, v^p)$ [3], where g is the amount of public abatement provided by the public sector. In this context, it is well known that $R_p = x, R_g = -C^g < 0$, where C^g denotes the unit cost of public abatement. In addition, it is conventional to assume $R_{gg} = 0$ (Abe, 1992). From the revenue function, one can obtain

$$R_t = -z.$$
(1)

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[3] Mathematical derivations are provided in appendix A.

Equation (1) indicates the amount of pollution. Regarding this, $R_{tt} = -\partial z / \partial t > 0$ implies private abatement that an increase in the pollution tax rate reduces pollution. The expression $R_{gt} = -\partial C^g / \partial t > 0$ captures public abatement, which says that an increase in the pollution tax rate reduces the unit cost of public abatement, thereby increasing public abatement.

Turning to the consumption side, we define the expenditure function, $E(p + \tau, z - g, u)$, where τ is the consumption tax rate on good x, $z - g$ [4] represents net the amount of pollution received by households, and u is individual utility. The usual property of expenditure function states that E_p is the compensated demand for good x and $E_{pp} < 0$. The reciprocal marginal utility of income is $E_u > 0$ [5]. We assume that good x is a normal good, thereby $E_{pu} > 0$. Since pollution, z, reduces individual utility, expenditure must increase in order to keep a constant utility, and therefore assume $E_z > 0$ (Copeland, 1994).
The economy's budget constraint is

$$E(p + \tau, z - g, u) = R(p, g, t) - gR_g(p, g, t) + (1 - \alpha)tz + (1 - \beta)\tau E_p(p, z - g, u) \ . \tag{2}$$

The first term of the right-hand side of (2) denotes factor income from private goods production and the second term on the right-hand side of it denotes factor income from public abatement. The third and the forth terms represent pollution tax revenue and consumption tax revenue redistributed to households, respectively. In this context, α fraction of pollution tax revenue is earmarked for the financing of public abatement, while β fraction of consumption tax revenue is earmarked for the financing of public abatement activities.

Hence, the government's budget constraint for public abatement is

$$-gR_g = \alpha tz + \beta \tau E_p(p + \tau, z - g, u) \ . \tag{3}$$

The model comprising equations (1), (2) and (3) involves three endogenous variables, z, g, and u. The policy variables are the pollution tax rate t. The next section shows the results of comparative statics.

Consumption tax revenue- financed public abatement when there is a pollution tax
Firstly, we consider the case in which consumption tax revenue is earmarked while all of pollution tax revenue is rebated to households (i.e., $\alpha = 0$, $0 < \beta < 1$). To investigate the welfare effects of an increase in the pollution tax rate, we totally differentiate equations (1), (2), and (3) to yield [6]

$$dg / dt = R_{tt}[\beta \tau E_{pz} - \beta \tau E_{pu}(E_z - t)] / \Omega - (1 - \tau E_{pu})gR_{gt} / \Omega \tag{4}$$

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[4] We assume that $z - g > 0$.

[5] In this paper, we assume that $E_u = 1$.

[6] Mathematical derivations are provided in appendix B.

$$du/dt = -gR_{gt}[(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}]/\Omega$$
$$+ R_{tt}[R_g(E_z - t) - R_g(1 - \beta)\tau E_{pz} + t\beta\tau E_{pz}]/\Omega \tag{5}$$

where $\Omega = (1 - \tau E_{pu})[R_g - \beta\tau E_{pz}(1 + R_{tg})] + \beta\tau E_{pu}[E_z + R_g - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}]$ is the determinant of the coefficient matrix, which must be negative for stability[7].

Equation (4) represents the change in public abatement by a pollution tax under consumption tax revenue-financed public abatement. Regarding the first term of the right-hand side of equation (4), an increase in the pollution tax rate has a positive impact on public abatement if we assume $E_{pz} < 0$. Intuitively, an increase in the pollution tax reduces pollution though private abatement $R_{tt} = -\partial z/\partial t > 0$, and therefore increases the amount of consumption of good x because pollution is a substitute for the consumption of good x $E_{pz} < 0$ [8]. Also, a reduction in pollution by increasing a pollution tax increases the consumption of good x because good x is a normal good $E_{pu} > 0$. These effects of an increase in the consumption of good x have a positive impact on public abatement since consumption tax revenue is earmarked for the financing of public abatement. The second term of the right-hand side of equation (4) $-(1 - \tau E_{pu})gR_{gt}/\Omega > 0$ [9] indicates that an increase in the pollution tax increases public abatement by means of a decrease in the unit cost of public abatement $R_{gt} = -\partial C^g/\partial t > 0$. Overall, an increase in the pollution tax rate increases public abatement if we assume $E_{pz} < 0$.

Equation (5) shows the change in welfare by increasing a pollution tax under consumption tax revenue-financed public abatement. The sign of the right-hand side of equation (4) is determined from the assumptions $E_z + R_g > 0, E_z > t$, and $E_{pz} < 0$. That is, when public abatement is under-provided ($E_z > -R_g$), when the pollution tax rate is small ($E_z > t$) and when the consumption of good x and pollution are substitutes ($E_{pz} < 0$) then an increase in the pollution tax rate can increase welfare. To investigate this, the first term of the right-hand side of equation (4) represents the public abatement effect. Regarding this effect, one should remember the fact that an increase in the pollution tax rate increases public abatement by means of a decrease in the unit cost of public abatement. It has a positive impact on welfare because public abatement is under-provided and the consumption of good x is a substitute for pollution. In particular, if consumption of good x is a substitute for pollution, a reduction in pollution increases consumption tax revenue redistributed to households, thereby increasing welfare. On the other hand, a reduction in pollution reduces pollution tax revenue redistributed to households, thereby reducing welfare. Yet, if the pollution tax rate is small enough to guarantee $E_z > t$ then welfare loss arising from a

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[7] Mathematical derivations are provided in appendix C.

[8] The actual example of $E_{pr} < 0$ is served by Copeland (1994) that the demand of hiking boots rise as the amount of pollution declines in a mountain side.

[9] Regarding the sign of $1 - \tau E_{pu}$, the homogeneity of expenditure function E yields $E_u = E_{1u} + (p + \tau)E_{pu}$, where the price of good y is normalized to unity. We have $1 - \tau E_{pu} = E_{1u} + pE_{pu} > 0$ because E_u is assumed to be unity.

reduction in pollution tax revenue would be small enough to be outweighed by welfare gain arising from pollution abatement.

The second term of the right-hand side of equation (5) captures the private abatement effect. Likewise the public abatement effect, this effect has a positive impact on welfare under the familiar assumptions $E_{pz} < 0$ and $E_z > t$. The explanations are straightforward that a reduction in pollution achieved by increasing a pollution tax (i.e., private abatement) increases the consumption of good x, thereby consumption tax revenue. The increased consumption tax revenue raises welfare by means of an expansion of public abatement and lump-sum redistribution to households. Also, the assumption $E_z > t$ ensures that welfare gain arising from pollution abatement (i.e., private abatement) dominates welfare loss arising from a reduction in pollution tax revenue. Overall welfare can rise by increasing the pollution tax rate under consumption tax revenue-financed public abatement if we assume $E_z + R_g > 0, E_z > t$, and $E_{pz} < 0$. To verify this, we show that the optimal pollution tax rate is positive under consumption tax revenue-financed public abatement. To this end, we set $du/dt = 0$ in equation (5) to yield

$$t_c^{opt} = \frac{\{R_{tt}R_g[E_z - (1-\beta)\tau E_{pz}] - gR_{gt}[E_z + R_g + E_z R_{tg} - \tau E_{pz}(1+R_{tg})]\}}{[(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}]} \tag{6}$$

Regarding equation (6), if we assume $E_z + R_g > 0, E_z > t$, $E_{pz} < 0$ and the stability shown in appendix B, which says that $R_g - \beta\tau E_{pz} < 0$ ensured by a small consumption tax, then the optimal pollution tax rate becomes positive under consumption tax revenue-financed public abatement $t_c^{opt} > 0$. It is consistent with the result shown in equation (5) that an increase in the pollution tax rate increases welfare under consumption tax revenue-financed public abatement. Also, by using equations (6) and (5), one obtains

$$du/dt = (t_c^{opt} - t)A/\Omega \tag{7}$$

where $A = [(R_g - \beta\tau E_{pz})R_{tt} - gR_{tg}R_{tg}] < 0$. Equation (7) implies that welfare rises if the government sets the pollution tax rate t toward the optimal pollution tax rate t_c^{opt}.

Pollution tax revenue- financed public abatement when there is a consumption tax

In this section, we examine the welfare effects of a pollution tax when the government earmarks pollution tax revenue to finance public abatement (i.e., $\beta = 0$, $0 < \alpha < 1$). The results obtained by comparative statics show (see appendix B)

$$dg/dt = (1 - \tau E_{pu})[\alpha R_t(1-\varepsilon) - gR_{gt}]/\Delta = [\alpha R_t(1-\varepsilon) - gR_{gt}]/(R_t - gR_{gt}) \tag{8}$$

$$du/dt = R_{tt}(E_z - t - \tau E_{pz})(R_g - \alpha t R_{tg})/\Delta$$
$$[\alpha R_t(1-\varepsilon) - gR_{gt}][(E_z + R_g) - \tau E_{pz}(1+R_{tg}) + (E_z - t)R_{tg}]/\Delta \tag{9}$$

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where $\Delta = (1 - \tau E_{pu})(R_g - \alpha t R_{tg}) < 0$ is the determinant of the coefficient matrix and $\varepsilon = -t R_{tt} / R_t > 0$ represents the elasticity of pollution emission with respect to the pollution tax rate.

Equation (8) shows the change in public abatement by a pollution tax under pollution tax revenue-financed public abatement. Regarding equation (8), if we assume that the elasticity of pollution emission with respect to the pollution tax rate is large enough to guarantee $\varepsilon > 1$ then an increase in the pollution tax rate causes a negative impact on public abatement. As explained in introduction, a reduction in pollution achieved by increasing the pollution tax rate reduces pollution tax revenue earmarked for the financing of public abatement, and as a result, public abatement declines under pollution tax revenue-financed public abatement. If this negative effect on public abatement $\alpha R_t (1 - \varepsilon) / (R_t - g R_{gt}) < 0$ outweighs the positive effect on public abatement $-g R_{gt} / (R_t - g R_{gt}) > 0$ then an increase in a pollution tax decreases public abatement. In contrast, if we assume $\varepsilon < 1$, which implies that the elasticity of pollution emission with respect to the pollution tax rate is small, a reduction in pollution achieved by increasing a pollution tax is not so substantial that the government can procure pollution tax revenue, and as a result, public abatement rises.

Turning to equation (9), we can examine the welfare consequences of a pollution tax under pollution tax revenue-financed public abatement. Regarding the right-hand side of equation (9), the first term $R_{tt}(E_z - t - \tau E_{pz})(R_g - \alpha t R_{tg}) / \Delta = R_{tt}(E_z - t - \tau E_{pz}) / (1 - \tau E_{pu})$ indicates the private abatement effect on welfare. Likewise consumption tax revenue-financed public abatement, private abatement undertaken by increasing a pollution tax raises welfare if we assume $E_{pz} < 0$ and $E_z > t$.

The second term $[\alpha R_t (1 - \varepsilon) - g R_{gt}][(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}] / \Delta$ represents the public abatement effect. Unlike consumption tax revenue-financed public abatement, the welfare effects of the public abatement are ambiguous although we establish the familiar assumptions, such that $E_z + R_g > 0, E_z > t$, and $E_{pz} < 0$. This can be attributed to the fact that the tax base of public abatement declines by increasing a pollution tax under pollution tax revenue-financed public abatement if we assume $\varepsilon > 1$. Regarding this, the decreased public abatement reduces pollution abatement, which implies that the consumption tax revenue returned to households declines under the assumption $E_{pz} < 0$. Also, a reduction in pollution abatement harms utility of households because the households' marginal willingness to pay for pollution abatement is high, such that $E_z + R_g > 0, E_z > t$. These effects have negative impacts on welfare. The relative strengths of these negative impacts compared with positive impacts can determine the welfare consequences of a pollution tax under pollution tax revenue-financed public abatement. Likewise consumption tax revenue-financed public abatement, one obtains the optimal pollution tax rate[10] under pollution tax revenue-financed public abatement effect by setting $du / dt = 0$ in equation (9).

$$t_p^{opt} = \frac{-g R_{gt}[E_z + R_g + E_z R_{tg} - \tau E_{pz}(1 + R_{tg})] + (E_z - \tau E_{pz})(\alpha R_{tg} R_t + R_{tt} R_g) + (E_z + R_g - \tau E_{pz})\alpha R_t}{[\alpha R_{tg} R_t + R_{tt} R_g - g R_{gt} R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}]} \quad (10)$$

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[10] Mathematical derivations are provided in appendix D.

Regarding the right-hand side of equation (10), one can obtain $t_p^{opt} > 0$ if we assume $E_z + R_g > 0, E_z > t$, and $E_{pz} < 0$. It implies that an increase in the pollution tax rate can increase welfare under pollution tax revenue-financed public abatement. In this context, if the government sets the pollution tax rate toward the optimal pollution tax rate then welfare improves under pollution tax revenue-financed public abatement. To see this, by using equations (9) and (10)[11], one obtains

$$du/dt = (t_p^{opt} - t)B/\Delta \tag{11}$$

where $B = [\alpha R_{tg} R_t - gR_{gt} R_{tg} + R_{tt} R_g - \alpha(E_z + R_g - \tau E_{pz})R_{tt}] < 0$, which implies $du/dt > 0$, if $t_p^{opt} > t$.

Welfare comparison

Welfare levels under pollution tax revenue-financed public abatement and consumption tax revenue-financed public abatement can be compared by the right-hand side of equation (9) with the right-hand side of equation (5). With regard to the denominators of equation (5) and equation (9), one obtains

$$\Delta - \Omega = (1 - \tau E_{pu})[\beta\tau E_{pz}(1 + R_{tg}) - \alpha t R_{tg}] - \beta\tau E_{pu}[E_z + R_g - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}] \tag{12}$$

If we assume $E_z + R_g > 0, E_z > t$, and $E_{pz} < 0$ then the right-hand side of equation (12) is negative, which implies that $|\Delta| > |\Omega|$ since $\Delta < 0$ and $\Omega < 0$. In the case of the numerator, we denote the numerators of equations (5) and (9) by A_t and B_t, respectively, as follows.

$$A_t = -gR_{gt}[(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}] + R_{tt}[R_g(E_z - t) - R_g(1 - \beta)\tau E_{pz} + t\beta\tau E_{pz}]$$

$$B_t = R_{tt}(E_z - t - \tau E_{pz})(R_g - \alpha t R_{tg}) + [\alpha R_t(1 - \varepsilon) - gR_{gt}][(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}]$$

$$A_t - B_t = -\alpha R_t(E_z + R_g - \tau E_{pz})(1 + R_{tg} - \varepsilon) + (R_g + t)(\beta\tau E_{pz}R_{tt} + \alpha R_t R_{tg}) \tag{13}$$

Regarding equation (13), if we assume $E_z + R_g > 0$, $E_{pz} < 0$, $E_z - t = 0$[12], and $\varepsilon > 1 + R_{tg}$ then one obtains $A_t - B_t < 0$, which implies $|A_t| > |B_t|$ since $A_t < 0$. By using the result $|\Delta| > |\Omega|$, one obtains $du/dt|_{(5)} > du/dt|_{(9)}$. The reason behind this outcome is that if the elasticity of pollution with

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[11] Mathematical derivations are provided in appendix E.

[12] The assumption $E_z - t = 0$ does not undermine the result that an increase in a pollution tax increases welfare under consumption tax revenue-financed public abatement.

respect to the pollution tax is substantial to guarantee $\varepsilon > 1 + R_{tg}$ then pollution emission is quite elastic with respect to the pollution tax and as a result, an increase in the pollution tax would reduce pollution tax revenue as a source of public abatement under pollution tax revenue-financed public abatement. In contrast, the tax base of public abatement rises under consumption tax revenue-financed public abatement under the assumption $E_{pz} < 0$. In these circumstances, provided that public abatement is under-provided, the magnitude of welfare improvement as a result of an increase in a pollution tax is higher under consumption tax revenue-financed public abatement than it is under pollution tax revenue-financed public abatement.

Proposition. *Suppose that public abatement is earmarked by either pollution tax revenue or consumption tax revenue. Then, following an increase in the pollution tax rate, consumption tax revenue-financed public abatement raises welfare by more than does pollution tax revenue-financed public abatement provided that a) public abatement is under provided* $E_z + R_g > 0$ *;b) the pollution tax rate is equal to the marginal damage caused by pollution* $E_z = t$ *; c) the consumption of good x and pollution are substitutes* $E_{pz} < 0$ *; and d) the elasticity of pollution emission with respect the pollution tax rate* ε *is substantial to guarantee* $\varepsilon > 1 + R_{tg}$ *.*

To demonstrate public abatement, revenue from a pollution tax tends to be earmarked for the financing of public abatement (OECD, 1995). However, the above proposition contradicts this view. The proposition recognizes that pollution tax revenue-financed public abatement might undermine public abatement by reducing the tax base of public abatement (i.e., earmarked pollution tax revenue) as a result of an increase in private abatement. In this case, when public abatement is under-provided, when marginal damage of pollution is equal to the pollution tax rate, the elasticity of pollution emission with respect to the pollution tax rate is substantial and when good x is a substitute for pollution, it is better to finance public abatement with consumption tax revenue, rather than with pollution tax revenue. This result implies that the optimal pollution tax rate is greater under consumption tax revenue-financed public abatement than it is under pollution tax revenue-financed public abatement. To see this, one compares the right-hand side of equation (10) with that of equation (6) as[13]

$$(t_p^{opt} - t_c^{opt})\Phi\Theta = \beta\tau E_{pz} R_{tt}(E_z + R_g - \tau E_{pz})[gR_{gt}(1 + R_{tg}) - (\alpha R_t R_{tt} + \alpha R_t R_{tg} + R_{tt} R_g)]$$

$$-gR_{gt}\alpha R_{tt}(E_z + R_g - \tau E_{pz})[E_z + R_g + E_z R_{tg} - \tau E_{pz}(1 + R_{tg})]$$
$$+\alpha R_t R_{tt} R_g(E_z + R_g - \tau E_{pz})\{1 - \varepsilon'[E_z - (1 - \beta)\tau E_{pz}]\} \qquad (14)$$

where

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[13] Mathematical derivations are provided in appendix F.

$$\Phi = [(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}], \Theta = [\alpha R_{tg}R_t + R_{tt}R_g - gR_{gt}R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}], \quad \text{and}$$

$\varepsilon' = -R_{tt}/R_t$. If we assume $E_z + R_g > 0$, $E_{pz} < 0$, and the degree of private abatement $\varepsilon' = -R_{tt}/R_t$ is substantial, one obtains $t_p^{opt} < t_c^{opt}$. It is consistent with the result shown in proposition, which says that consumption tax revenue-financed public abatement is a welfare superior policy compared with pollution tax revenue-financed public abatement when the government increases the pollution tax rate.

3. Concluding Remarks

In this paper, we have analyzed public abatement financed by tax revenue and have compared the welfare effects of a pollution tax under pollution tax revenue-financed public abatement and it is under consumption tax revenue-financed public abatement. The main finding of this paper casts doubt on whether the revenue from a pollution tax should be used for environmental protection. This finding requires the following: (1) the elasticity of pollution with respect to the pollution tax is substantial, (2) public abatement is under-provided, (3) the initial pollution tax rate does not exceed (or equal to) the marginal damage of pollution, and (4) pollution and consumption are substitutes. In these circumstances, this paper shows that consumption tax revenue-financed public abatement is a welfare superior policy compared with pollution tax revenue-financed public abatement when the government increase the pollution tax rate. These results provide an important caveat for countries that attempt to use pollution tax revenue for public abatement and also provide an idea that general revenue (e.g., consumption tax revenue) could be possible sources of environmental protection demonstrated by public abatement.

Finally, as readers may find that this paper has abstracted from the welfare consequences of a consumption tax when either pollution tax revenue or consumption tax revenue is earmarked for the financing of public abatement. The main reason for this view is that it would be hard to justify changing a consumption tax for the purpose of environmental protection when all of consumption tax revenue is returned to households. In these circumstances, however, if we assume that pollution is generated from both consumption and production activities, it would be justified to examine a change in a consumption even though all of consumption tax revenue is lump-sum redistributed to households, and obtain a welfare superior policy between pollution tax revenue-financed public abatement and consumption tax revenue-financed public abatement. This task would be left to future research.

Appendix A
In appendix A, we obtain the restricted revenue function $R(p, g, t)$ by using the revenue function $\overline{R}(p, t, v^p)$. The full employment condition requires that

$$v^p + v^g = v \tag{A.1}$$

where v^p, v^g are the factors of production employed by the private sectors and public sectors, respectively, v is the vector of fixed factor endowments.

The equilibrium of factor markets is established when

$$w = \overline{R}_v(p, t, v^p) \tag{A.2}$$

where w is the vector of factor prices and its function is homogeneous degree one in w. Then we define the unit cost of public abatement function as $C^g(w)$.

The demand for factors of production of public sector is expressed as (see Abe (1992))

$$v^g = gC^g(w) \tag{A.3}$$

where g denotes the amount of public abatement provision. Substituting equation (A.2) into (A.3) and using (A.1) yields

$$v^p + gC^g(\overline{R}_v(p, t, v^p)) = v \tag{A.4}$$

From equation (A.4), one can recognize v^p as a function of p, g, t and v. However, since v does not vary, we omit v. Hence, v^p can be written as

$$v^p = v^p(p, g, t) \tag{A.5}$$

Finally, by using the revenue function $\overline{R}(p, t, v^p)$, we can obtain the restricted revenue function $R(p, g, t)$ as

$$R(p, g, t) = \overline{R}(p, t, v^p(p, g, t)) \tag{A.6}$$

Appendix B

Totally differentiating equations (1), (2) and (3) yields

$$dz = -R_{tg}dg - R_{tt}dt , \tag{B.1}$$

$$[1 - (1-\beta)\tau E_{pu}]du + [E_z - (1-\alpha)t - (1-\beta)\tau E_{pz}]dz - [E_z - (1-\beta)\tau E_{pz}]dg = (\alpha R_t - gR_{gt})dt \tag{B.2}$$

$$\beta\tau E_{pu}du + (\alpha t + \tau E_{pz})dz + (R_g - \beta\tau E_{pz})dg = (\alpha R_t - gR_{gt})dt . \tag{B.3}$$

Substituting (B.1) into (B.2) and (B.3) yields

$$\begin{bmatrix} [1-(1-\beta)\tau E_{pu}] & -\{[E_z - (1-\alpha)t - (1-\beta)\tau E_{pz}]R_{tg} + [E_z - (1-\beta)\tau E_{pz}]\} \\ \beta\tau E_{pu} & [R_g - \beta\tau E_{pz} - (\alpha t + \beta\tau E_{pz})R_{tg}] \end{bmatrix} \begin{bmatrix} du \\ dg \end{bmatrix}$$

$$= \begin{bmatrix} \{[E_z - (1-\alpha)t - (1-\beta)\tau E_{pz}]R_{tt} + \alpha R_t - gR_{gt}\} \\ [(\alpha t + \beta\tau E_{pz})R_{tt} + \alpha R_t - gR_{gt}] \end{bmatrix} dt \tag{B.4}$$

Appendix C

To determine the sign of Ω, I apply the stability condition proposed by Abe (1992):

$$g = \beta\tau E_p(p, z - g, u) + gR_g(p, g, t) \tag{C.1}$$

If the equilibrium is locally stable, dg/dg<0. Hence, total differentiation of (C.1) yields

$$dg/dg = \beta\tau E_{pu}(du/dg) + \beta\tau E_{pz}(dz/dg) + (R_g - \tau E_{pz}). \tag{C.2}$$

Substituting $du/dg = \{[E_z - t - (1-\beta)\tau E_{pz}]R_{tg} + [E_z - (1-\beta)\tau E_{pz}]\}/[1 - (1-\beta)\tau E_{pu}]$ and $dz/dg = -R_{tg}$ into (C.2) yields

$$dg/dg = \Omega/[1 - (1-\beta)\tau E_{pu}]$$

where $\Omega = (1 - \tau E_{pu})[R_g - \beta\tau E_{pz}(1 + R_{tg})] + \beta\tau E_{pu}[E_z + R_g - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}]$. The sufficient condition to guarantee dg/dg<0 is $\Omega < 0$ because $[1 - (1-\beta)\tau E_{pu}] > 0$.

Appendix D

$$du/dt = R_{tt}(E_z - t - \tau E_{pz})(R_g - \alpha t R_{tg})/\Delta$$
$$[\alpha R_t(1-\varepsilon) - gR_{gt}][(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}]/\Delta \tag{9}$$

By setting $du/dt = 0$ in equation (9) yields

$$R_{tt}(E_z - t - \tau E_{pz})(R_g - \alpha t R_{tg}) + [\alpha R_t(1-\varepsilon) - gR_{gt}][(E_z + R_g) - \tau E_{pz}(1 + R_{tg}) + (E_z - t)R_{tg}] = 0 \text{ It}$$

can be rewritten as

$$[\alpha(R_t + tR_{tt}) - gR_{gt}][(E_z - t - \tau E_{pz})R_{tg} + E_z + R_g - \tau E_{pz}] + (E_z - t - \tau E_{pz})R_{tt}(R_g - \alpha t R_{tg}) = 0 \tag{D.1}$$

Collecting the terms of equation (D.1) yields

$$(E_z - t - \tau E_{pz})[\alpha R_{tg}R_t - gR_{gt}R_{tg} + R_{tt}R_g] + [\alpha(R_t + tR_{tt}) - gR_{gt}](E_z + R_g - \tau E_{pz}) = 0 \tag{D.2}$$

By solving (D.2) with respect to the pollution tax rate t, one obtains the optimal pollution tax rate under

pollution tax revenue-financed public abatement t_p^{opt} as

$$t_p^{opt} = \frac{-gR_{gt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})] + (E_z - \tau E_{pz})(\alpha R_{tg}R_t + R_{tt}R_g) + (E_z + R_g - \tau E_{pz})\alpha R_t}{[\alpha R_{tg}R_t + R_{tt}R_g - gR_{gt}R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}]}$$

Appendix E

Equation (9) can be rewritten as

$$du/dt = [\alpha(R_t + tR_{tt}) - gR_{gt}][(E_z - t - \tau E_{pz})R_{tg} + E_z + R_g - \tau E_{pz}]$$
$$+(E_z - t - \tau E_{pz})R_{tt}(R_g - \alpha tR_{tg}) \tag{E.1}$$

By using equation (10), one obtains

$$t_p^{opt}[\alpha R_{tg}R_t - gR_{gt}R_{tg} + R_{tt}R_g - \alpha(E_z + R_g - \tau E_{pz})]$$

$$= (E_z - \tau E_{pz})(\alpha R_{tg}R_t - gR_{gt}R_{tg} + R_{tt}R_g) + (\alpha R_t - gR_{gt})(E_z + R_g - \tau E_{pz}) \tag{E.2}$$

Substituting equation (E.1) into (E.2) yields

$$du/dt = (t_p^{opt} - t)B/\Delta \tag{E.3}$$

where $B = [\alpha R_{tg}R_t - gR_{gt}R_{tg} + R_{tt}R_g - \alpha(E_z + R_g - \tau E_{pz})] < 0$ under the familiar assumptions $E_{pz} < 0$ and $E_z + R_g > 0$.

Appendix F

$$t_c^{opt} = \Phi^{-1}\{R_{tt}R_g[E_z - (1-\beta)\tau E_{pz}] - gR_{gt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})]\} \tag{6}$$

$$t_p^{opt} = \Theta^{-1}\{(E_z - \tau E_{pz})(\alpha R_{tg}R_t + R_{tt}R_g) + (E_z + R_g - \tau E_{pz})\alpha R_t\}$$

$$- \Theta^{-1}\{gR_{gt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})]\} \tag{10}$$

Subtracting equation (6) from equation (10) yields

$$(t_p^{opt} - t_c^{opt})\Phi\Theta = -gR_{gt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})][(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}]$$

$$+(E_z - \tau E_{pz})(\alpha R_{tg}R_t + R_{tt}R_g)[(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}]$$
$$+(E_z + R_g - \tau E_{pz})\alpha R_t[(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}]$$
$$+gR_{gt}[\alpha R_{tg}R_t + R_{tt}R_g - gR_{gt}R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}][E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})]$$
$$-[\alpha R_{tg}R_t + R_{tt}R_g - gR_{gt}R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}][E_z - (1-\beta)\tau E_{pz}]R_{tt}R_g$$
$$= gR_{gt}(E_z + R_g - \tau E_{pz})\{\beta\tau E_{pz}R_{tt}(1 + R_{tg}) - \alpha R_{tt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})]\}$$
$$+\alpha R_t(E_z + R_g - \tau E_{pz})\{(R_g - \beta\tau E_{pz})R_{tt} - R_{tt}\varepsilon'[E_z - (1-\beta)\tau E_{pz}]R_g\}$$
$$-\beta\tau E_{pz}R_{tt}(\alpha R_{tg}R_t + R_{tt}R_g)(E_z + R_g - \tau E_{pz})$$
$$= gR_{gt}(E_z + R_g - \tau E_{pz})\{\beta\tau E_{pz}R_{tt}(1 + R_{tg}) - \alpha R_{tt}[E_z + R_g + E_zR_{tg} - \tau E_{pz}(1 + R_{tg})]\}$$
$$+\alpha R_tR_{tt}(E_z + R_g - \tau E_{pz})\{R_g[1 - \varepsilon'[E_z - (1-\beta)\tau E_{pz}] - \beta\tau E_{pz}\}$$

$$-\beta\tau E_{pz}R_{tt}(\alpha R_{tg}R_t + R_{tt}R_g)(E_z + R_g - \tau E_{pz})$$

$$= gR_{gt}(E_z + R_g - \tau E_{pz})\{\beta\tau E_{pz}R_{tt}(1 + R_{tg}) - \alpha R_{tt}[E_z + R_g + E_z R_{tg} - \tau E_{pz}(1 + R_{tg})]\}$$

$$+ \alpha R_t R_{tt}R_g(E_z + R_g - \tau E_{pz})\{[1 - \varepsilon'[E_z - (1 - \beta)\tau E_{pz}]\}$$

$$-\beta\tau E_{pz}R_{tt}(\alpha R_{tg}R_t + \alpha R_t R_{tt} + R_{tt}R_g)(E_z + R_g - \tau E_{pz})$$

$$= \beta\tau E_{pz}R_{tt}(E_z + R_g - \tau E_{pz})[gR_{gt}(1 + R_{tg}) - (\alpha R_t R_{tt} + \alpha R_t R_{tg} + R_{tt}R_g)]$$

$$- gR_{gt}\alpha R_{tt}(E_z + R_g - \tau E_{pz})[E_z + R_g + E_z R_{tg} - \tau E_{pz}(1 + R_{tg})]$$

$$+ \alpha R_t R_{tt}R_g(E_z + R_g - \tau E_{pz})\{1 - \varepsilon'[E_z - (1 - \beta)\tau E_{pz}]\}$$

where

$$\Phi = [(R_g - \beta\tau E_{pz})R_{tt} - gR_{gt}R_{tg}], \Theta = [\alpha R_{tg}R_t + R_{tt}R_g - gR_{gt}R_{tg} - \alpha(E_z + R_g - \tau E_{pz})R_{tt}], \quad \text{and}$$

$$\varepsilon' = -R_{tt}/R_t.$$

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