Non-Gaussianities of primordial perturbations and tensor sound speed

MASAHIDE YAMAGUCHI

(Tokyo Institute of Technology)

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1403.6065, Toshifumi Noumi & MY

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

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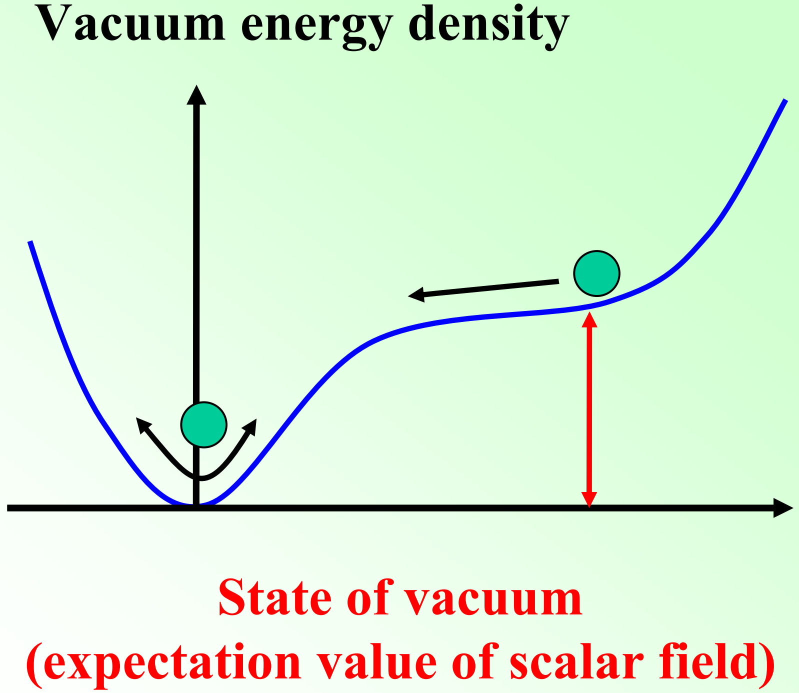
Introduction

Inflation

Starobinsky, Sato, Guth

The Universe rapidly expanded thanks to the vacuum energy density in the early stage.

(The scale factor a(t) expands faster than t)

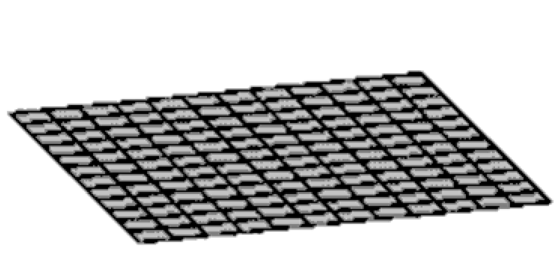


$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \simeq \text{constant.}$$

$$\Rightarrow \quad a(t) \propto e^t$$

Generic predictions of inflation

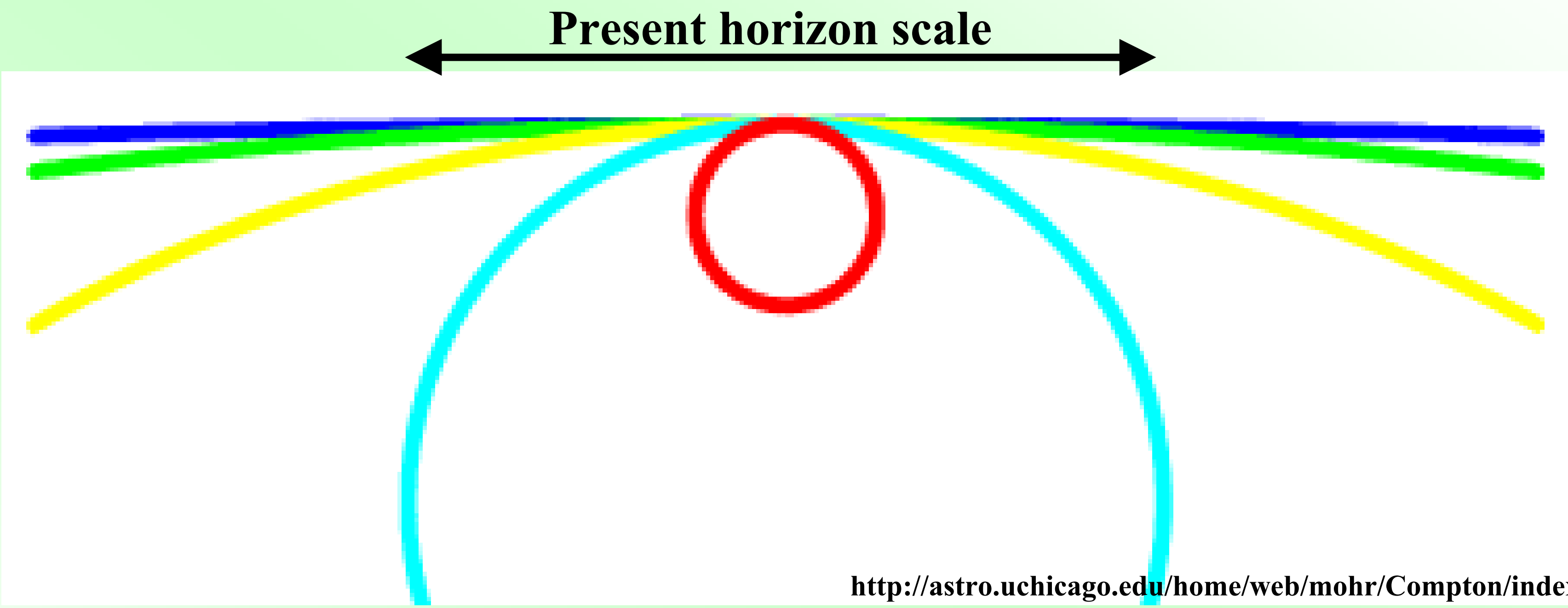
- Spatially flat universe



- Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations
- Almost scale invariant and Gaussian primordial tensor fluctuations

Generates anisotropy of CMBR.

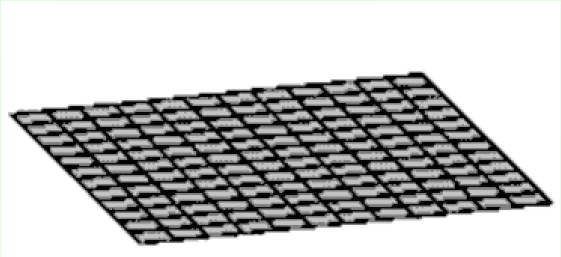
Spatially flat Universe



http://astro.uchicago.edu/home/web/mohr/Compton/index.html

To be flattened due to rapid expansion

Predict (spatially) flat Universe



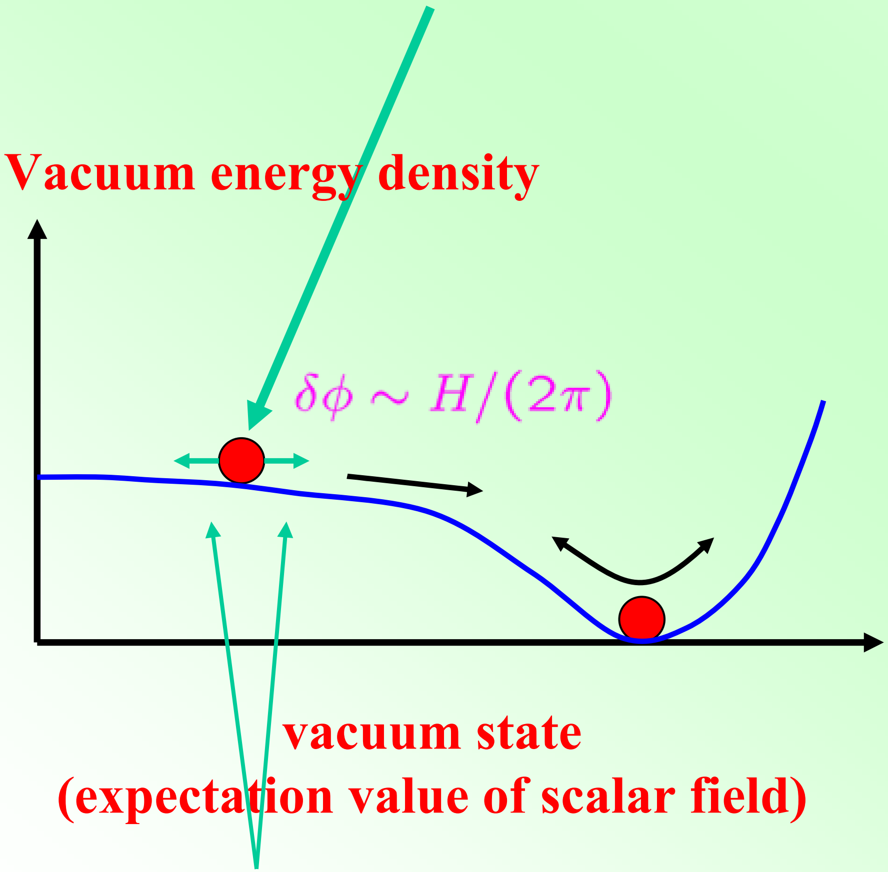
Generation mechanism of primordial perturbations

(e.g, Baumann 0907.5424, Wang 1303.1523 for recent good review)

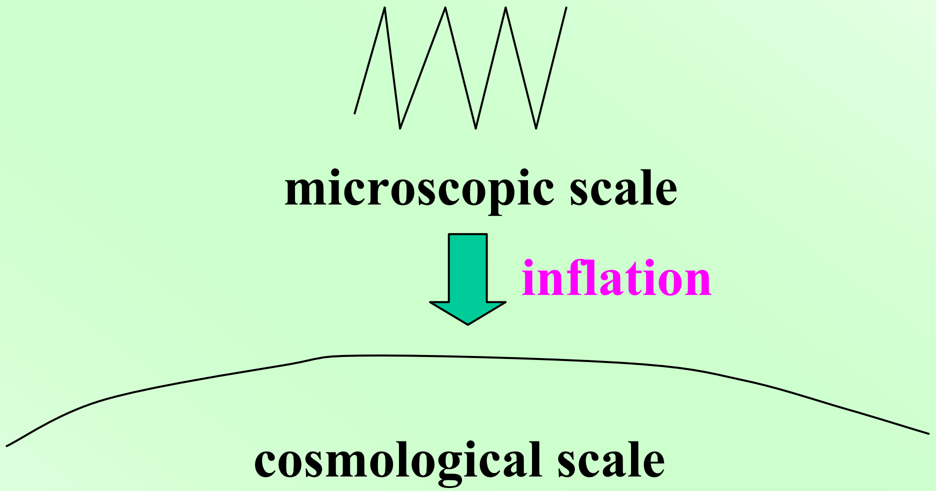
Primordial density fluctuations

Starobinsky, Hawking,
Guth & Pi, 1982

Vacuum fluctuates quantum mechanically.

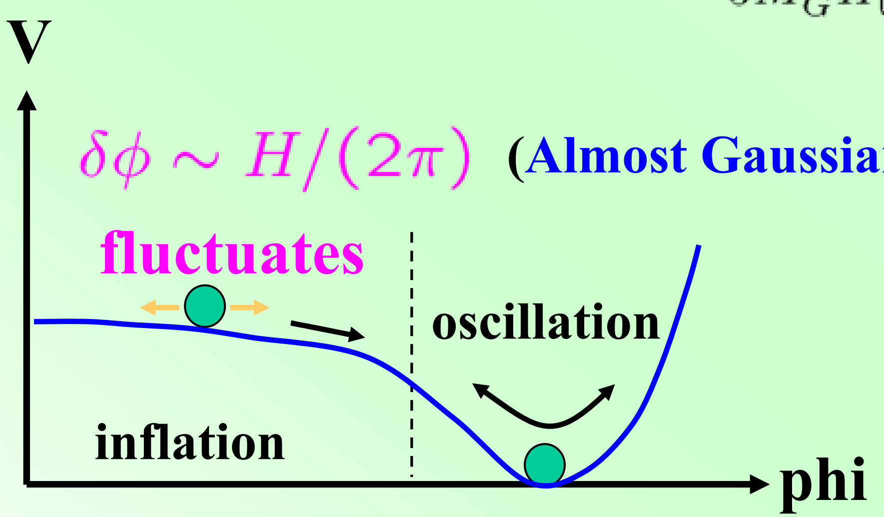


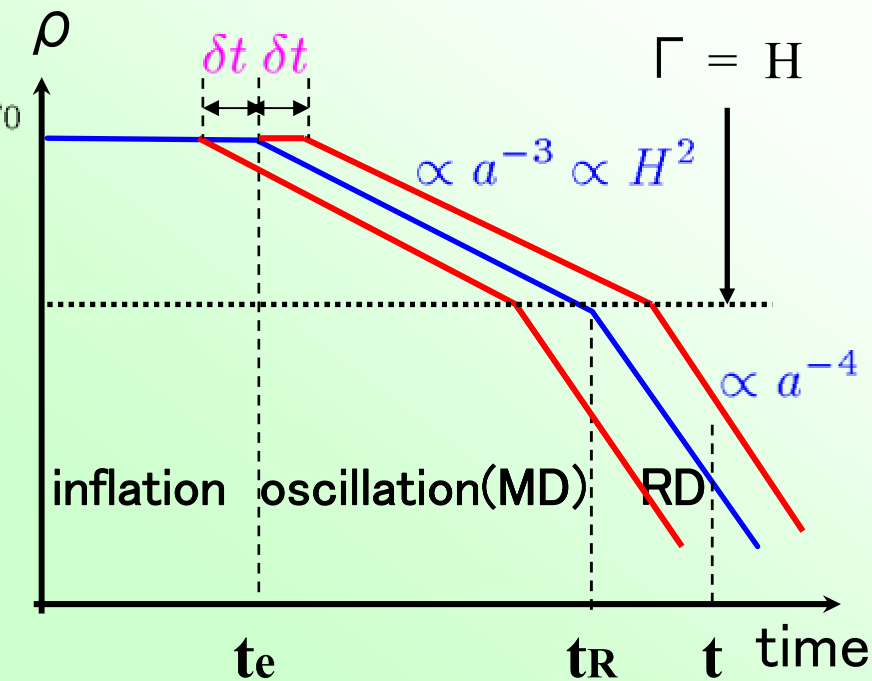
These quantum fluctuations are stretched to cosmological scales thanks to inflationary expansion, and become seeds to produce stars and galaxies.



Due to vacuum fluctuations, density fluctuations are generated.

Primordial density fluctuations II





$$\zeta\Big|_{t_k} \sim \delta\rho/\rho\Big|_{t_k} \propto \delta t/t \sim \delta\phi/(\dot\phi H^{-1}) \sim H^2/\dot\phi\Big|_{t_k^*}$$

curvature perturbation — Density fluctuation — $\left(\delta t(x,t) = \delta\phi(x,t)/\dot\phi_{cl}\right)$

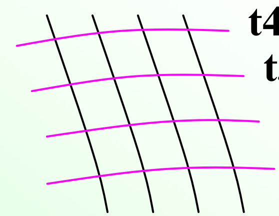
$\mathcal{P}_\zeta(k) \propto k^{n_s-1}$, **ns : spectral index**

Almost scale invariant fluctuations (ns ~ 1) are predicted.

(N.B. density fluctuations are not generated unless time-translational inv. is violated.)

Why do we often call them curvature perturbations instead of density fluctuations ?

It depends on time slicing (equal time surfaces) :

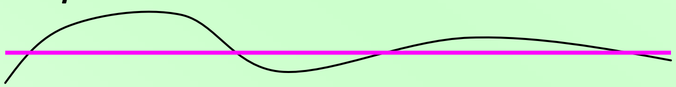


● spatially flat slice

ρ = const

t = const

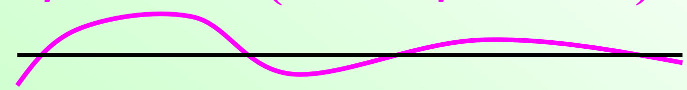
(no spatial curvature)



density fluctuates on this surface

● equal density slice (or comoving slice)

t = ρ = const (or t = ϕ = const)



curvature fluctuates on this surface

$$\boxed{\zeta = -\psi - H\frac{\delta\rho}{\dot\rho}}$$

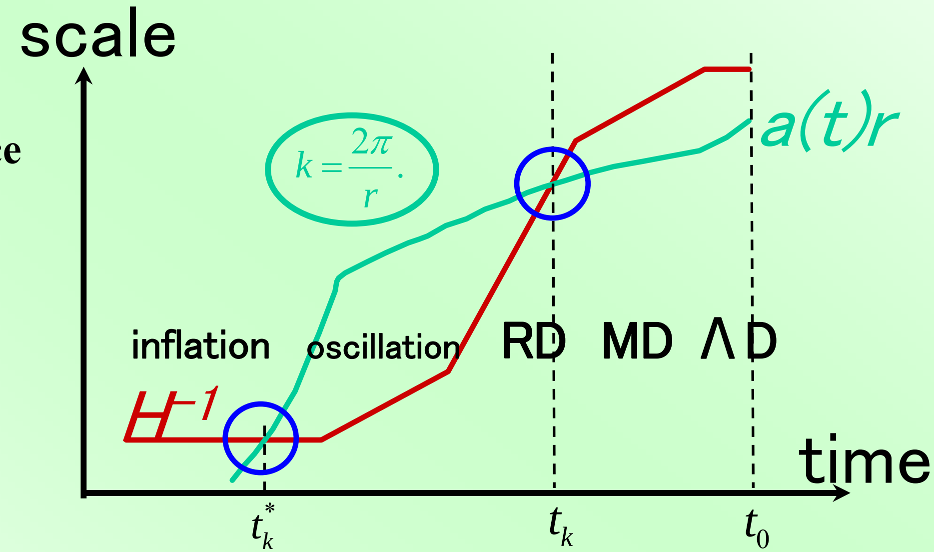
: curvature perturbation on equal density hypersurface (gauge invariant)

$$\left(-\mathcal{R} = -\psi - H\frac{\delta\phi}{\dot\phi}\right)$$

$$ds^2 = -dt^2 + a^2(t)(1-2\psi)d\boldsymbol{x}^2$$

$$\Rightarrow \quad {}^{(3)}R = \frac{4}{a^2}\nabla^2\psi$$

(spatial curvature on t=const surface)

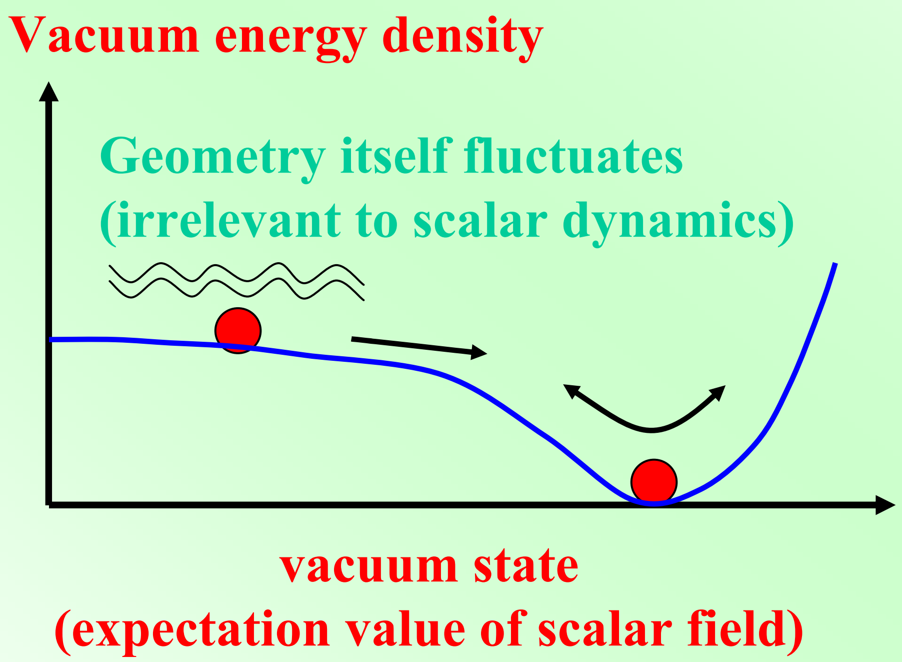


ζ is conserved during superhorizon epoch in case of absence of non-adiabatic pressure perturbations.

Primordial tensor fluctuations (gravitational waves)

(Starobinsky)

Vacuum fluctuates quantum mechanically.



$$\gamma \sim \frac{H}{M_G}$$

(directly probes the energy density of the Universe)

$$\left(H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{\rho}{3M_G^2} \right)$$

Such vacuum fluctuations generate not only density fluctuations, but also ripples of spacetime, i.e. gravitational waves.

k-inflation (Kinetically driven inflation)

(Armendariz-Picon et al., 1999)

$$S_\phi = \int d^4x\sqrt{-g}\,K(\phi, X), \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

(**e.g. DBI inflation:** $S_\phi = \int d^4x\sqrt{-g}\left[-T(\phi)\sqrt{1-2X/T(\phi)} + T(\phi) - V(\phi)\right]$)

$$\Longrightarrow \quad T^\mu_\nu = K_X\partial^\mu\phi\partial_\nu\phi + K\delta^\mu_\nu. \qquad \left(K_X = \frac{\partial K}{\partial X}\right)$$

Flat Friedmann

$$\Longrightarrow \quad \begin{cases} \rho = 2XK_X - K, \\ p = K. \end{cases} \qquad \text{c.f.} \quad \begin{cases} K = X - V(\phi) \\ \Longrightarrow \rho = X + V \end{cases}$$

$$|XK_X| \ll |K| \Longrightarrow \rho \simeq -p \Longrightarrow$$ **Inflation !!**

If we consider a non-trivial kinetic function,
inflation can still be realized even without potential.

Formal derivation of primordial tensor fluctuations

$$ds^2 = -dt^2 + a^2(t)\,(e^{\gamma})_{ij}\,dx^i dx^j = -dt^2 + a^2(t)\left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{ik}\gamma_{kj} + \cdots\right) dx^i dx^j,$$

$$\left(\gamma_{ii} = 0,\ \ \gamma_{ij,i} = 0.\right)$$

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2}M_G^2 R - K(\phi, X)\right], \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$\Rightarrow$ $$S_T^{(2)} = \frac{1}{8}\int dt d^3x\, a^3 M_G^2\left(\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}\gamma_{ij,k}\gamma_{ij,k}\right).$$

$\Rightarrow$ **(Standard) quantization of free fields**

$$\mathcal{P}_T = 2\langle\gamma_{ij}\gamma^{ij}\rangle \sim 8 \times 8\pi G\left(\frac{H}{2\pi}\right)^2 = \frac{2}{\pi^2}\left(\frac{H}{M_G}\right)^2.$$

Note that the whole metric is "not" quantized.
The background geometry exists classically and only the perturbations are quantized.

Primordial density fluctuations

Garriga & Mukhanov 1999

Perturbed metric :

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2 e^{2\zeta} d\boldsymbol{x}^2$$

Comoving gauge :

$$\phi = \phi(t), \qquad \delta\phi = 0.$$

Prescription:

- **Expand the action up to the second order**
- **Eliminate α and β by use of the constraint equations**
- **Obtain the quadratic action for ζ**

$$S_S^{(2)} = \int dt d^3x\, a^3\, M_G^2 \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2}{a^2}\zeta_{,k}\zeta_{,k}\right)$$

$$c_s^2 = \frac{K_X}{K_X + 2XK_{XX}}$$

: sound velocities of curvature perturbations

$$\left(K_X = \partial K/\partial X,\ K_{XX} = \partial^2 K/\partial X^2\right)$$

Scalar and tensor perturbations

- **scalar (density, curvature) perturbations ζ :**
  associated with the inflaton perturbations
- **tensor perturbations (gravitational waves) γ_{ij} :**
  degree of freedom intrinsic to gravity

Theoretical predictions :

$$\mathcal{L}_\phi = K(\phi, X) = \underbrace{-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}_{= X} - V(\phi).$$

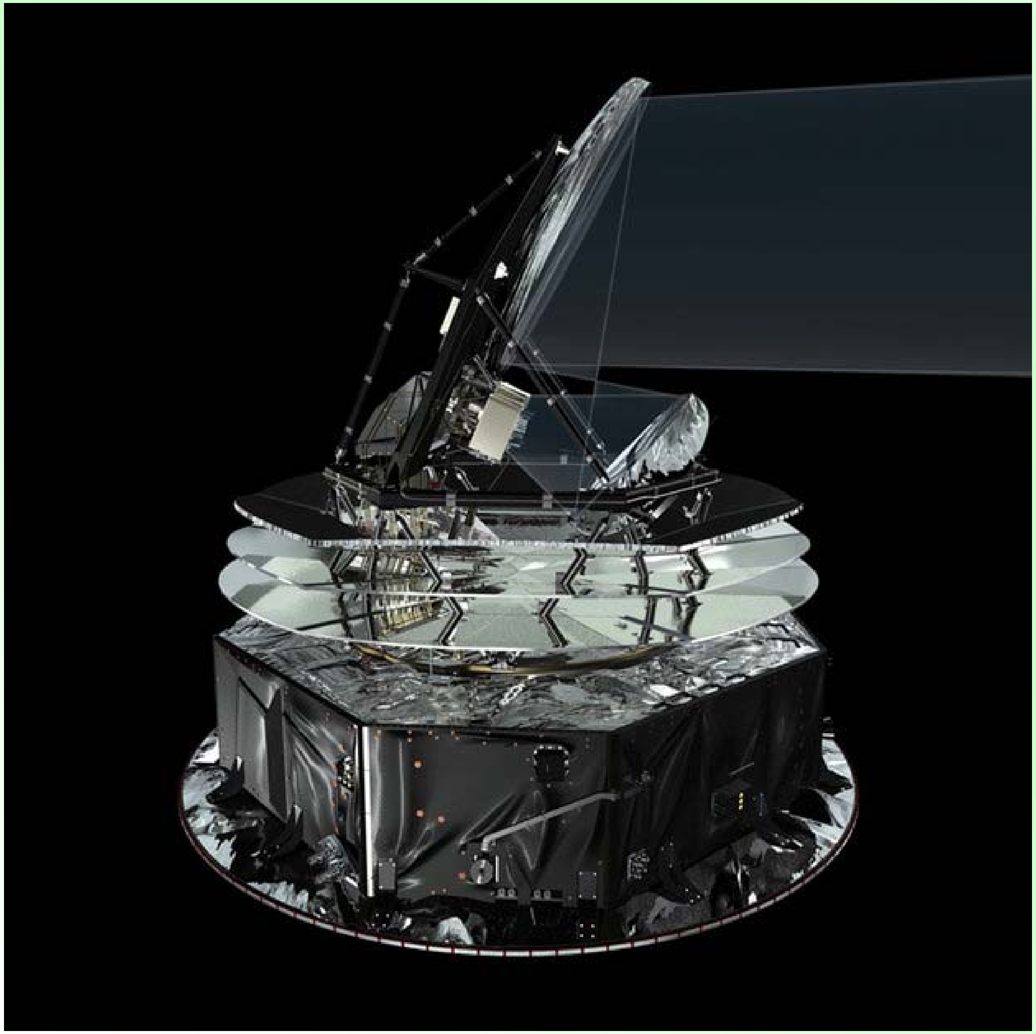
$$
\left\{
\begin{aligned}
\mathcal{P}_\zeta(k) &\simeq \frac{1}{8\pi^2 \epsilon c_s} \left(\frac{H}{M_G}\right)^2, \\
n_s - 1 &= \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k} \simeq -2\epsilon - 2\eta - s, \\
\mathcal{P}_T(k) &\simeq \frac{2}{\pi^2} \left(\frac{H}{M_G}\right)^2, \\
n_T &= \frac{d \ln \mathcal{P}_T(k)}{d \ln k} \simeq -2\epsilon, \\
r &\equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)} \simeq 16 \epsilon c_s \, (= -8 c_s n_T) .
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
c_s^2 &= \frac{K_X}{K_X + 2X K_{XX}}, \\
\epsilon &= -\frac{\dot{H}}{H^2} \ll 1, \\
|\eta &= \frac{\dot{\epsilon}}{H\epsilon}| \ll 1, \\
|s &= \frac{\dot{c}_s}{H c_s}| \ll 1.
\end{aligned}
\right.
\quad \Rightarrow \quad c_s^2 = 1.
$$

$\Leftarrow$ **called consistency relation**

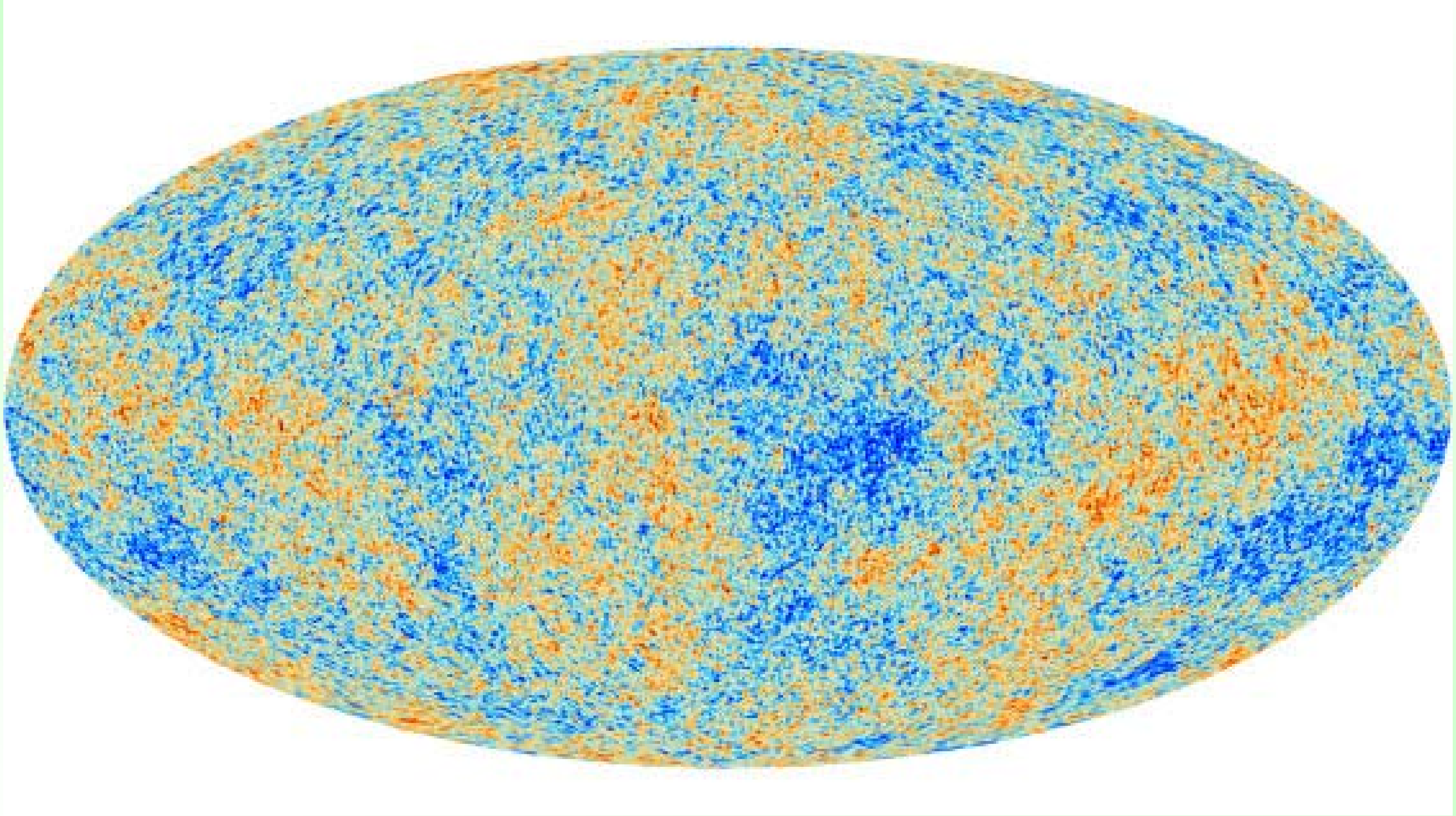
Cosmic Microwave Background (CMB) Anisotropies

Planck satellite



Planck is launched, on an Ariane 5 rocket from ESA's Spaceport in French Guiana, into its planned trajectory towards L2 on May 14 2009.

Map of CMBR by PLANCK



http://www.rssd.esa.int/index.php?project=planck

Constraints on scalar and tensor perturbations

Observational constraints :

$$\left\{\begin{aligned} \mathcal{P}_\zeta(k_0) &= 2.196^{+0.053}_{-0.059}\times 10^{-9}, \\ n_s &= 0.9603 \pm 0.0073, \\ r &< 0.12, \\ & \qquad k_0 = 0.002\mathrm{Mpc}^{-1}. \end{aligned}\right.$$

Theoretical predictions :

$$\left\{\begin{aligned} \mathcal{P}_\zeta(k) &\simeq \frac{1}{8\pi^2\epsilon c_s}\left(\frac{H}{M_G}\right)^2, \\ n_s - 1 &= \frac{d\ln\mathcal{P}_\zeta(k)}{d\ln k} \simeq -2\epsilon - 2\eta - s, \\ \mathcal{P}_T(k) &\simeq \frac{2}{\pi^2}\left(\frac{H}{M_G}\right)^2, \quad n_T = \frac{d\ln\mathcal{P}_T(k)}{d\ln k} \simeq -2\epsilon, \\ r &\equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon c_s\,(= -8c_s n_T). \end{aligned}\right.$$

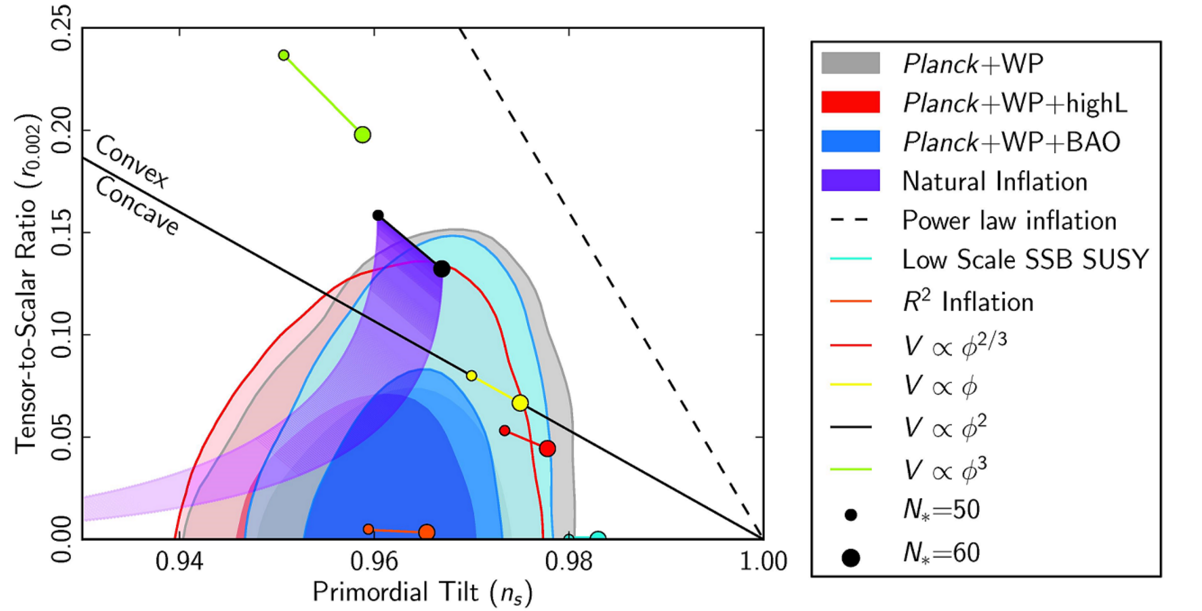
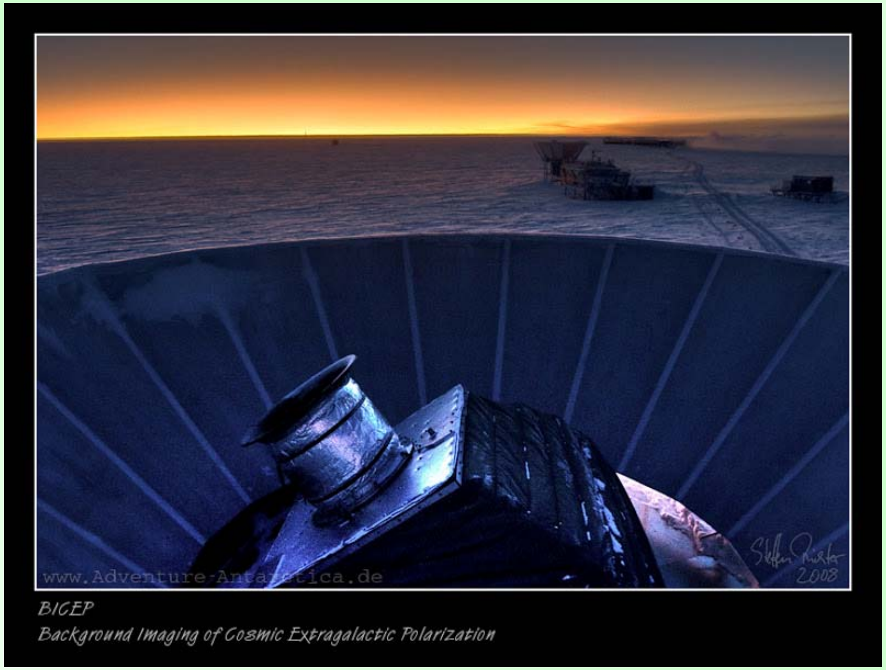


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

This is the situation before the BICEP2 results.

BICEP & BICEP2



BICEP:

Background Imaging of Cosmological Extragalactic Polarization (BICEP) has mapped three seasons of degree-scale CMB polarization from the South Pole, and has recently made public its 2-year data release.

http://www.cfa.harvard.edu/CMB/bicep1/

BICEP2:

BICEP2 extends the BICEP program through the use of monolithic antenna-coupled polarimeter arrays. BICEP2 deployed to the South Pole in November of 2009 and was decommissioned in 2012.

http://www.cfa.harvard.edu/CMB/bicep2/

BICEP2 results

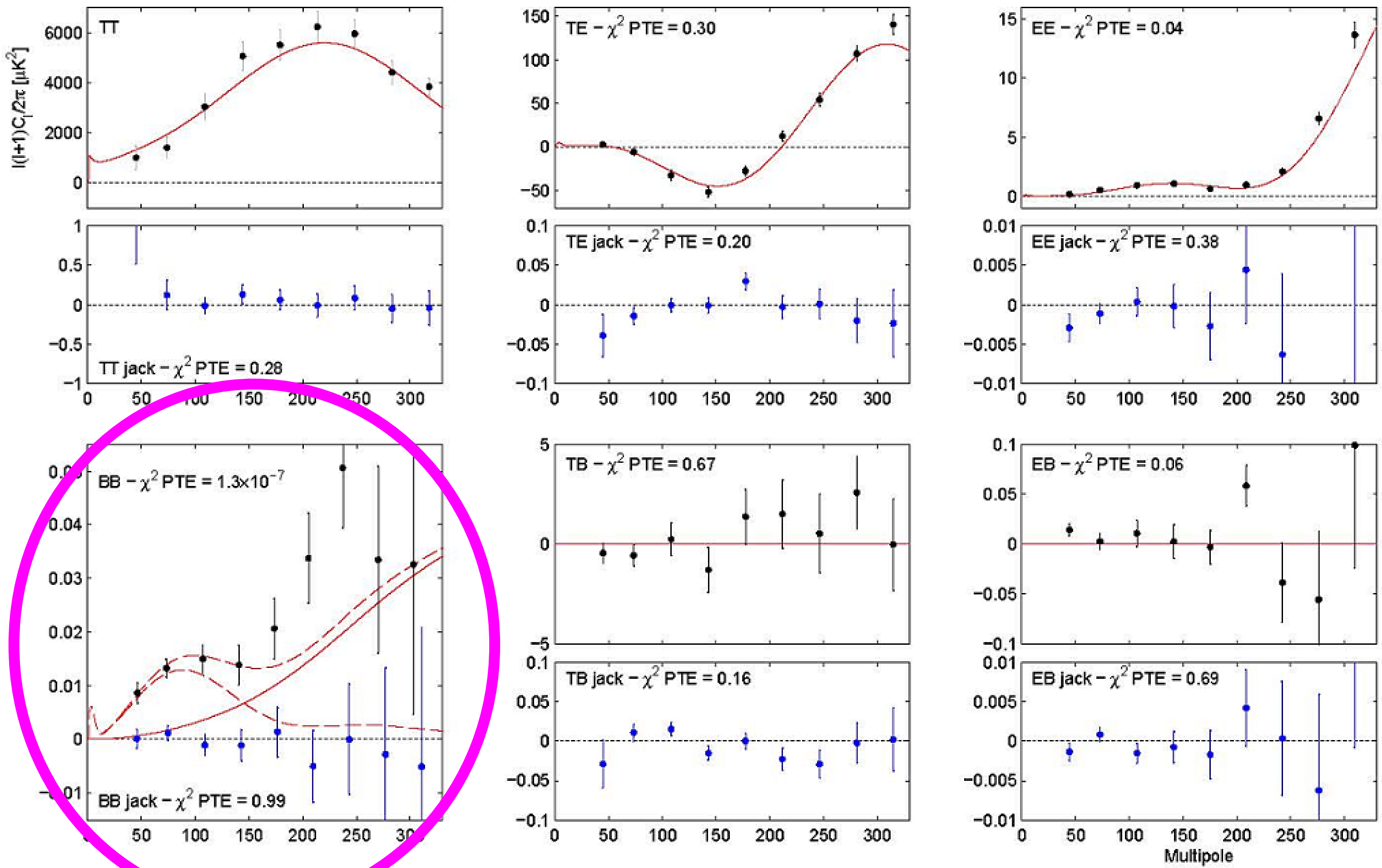


FIG. 2.— BICEP2 power spectrum results for signal (black points) and temporal-split jackknife (blue points). The red curves show the lensed-ΛCDM theory expectations — in the case of *BB* an $r = 0.2$ spectrum is also shown. The error bars are the standard deviations of the lensed-ΛCDM+noise simulations. The probability to exceed (PTE) the observed value of a simple χ^2 statistic is given (as evaluated against the simulations). Note the very different *y*-axis scales for the jackknife spectra (other than *BB*). See the text for additional discussion of the *BB* spectrum.

1403.3985

BICEP2 results III

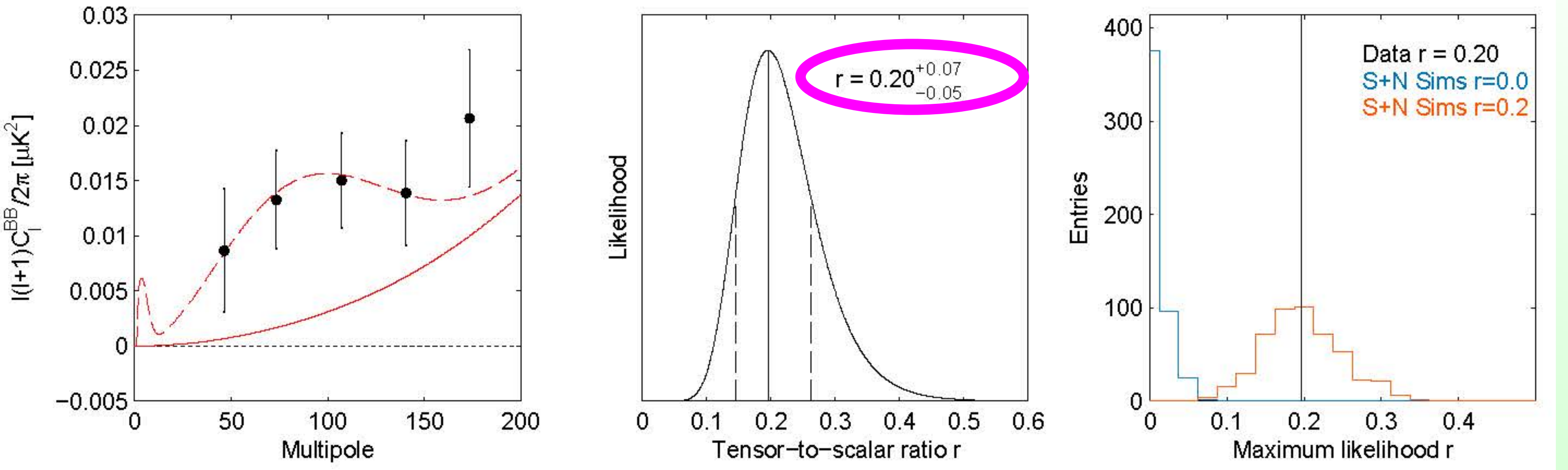


FIG. 10.— *Left:* The BICEP2 bandpowers plotted with the maximum likelihood lensed-ΛCDM+$r = 0.20$ model. The uncertainties are taken from that model and hence include sample variance on the r contribution. *Middle:* The constraint on the tensor-to-scalar ratio r. The maximum likelihood and $\pm 1\,\sigma$ interval is $r = 0.20^{+0.07}_{-0.05}$, as indicated by the vertical lines. *Right:* Histograms of the maximum likelihood values of r derived from lensed-ΛCDM+noise simulations with $r = 0$ (blue) and adding $r = 0.2$ (red). The maximum likelihood value of r for the real data is shown by the vertical line.

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Tension between PLANCK and BICEP2

PLANCK

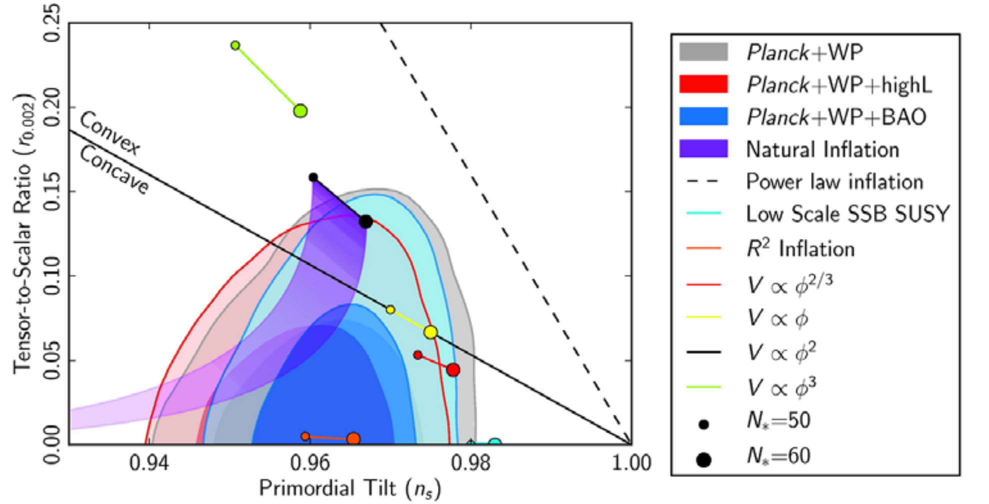
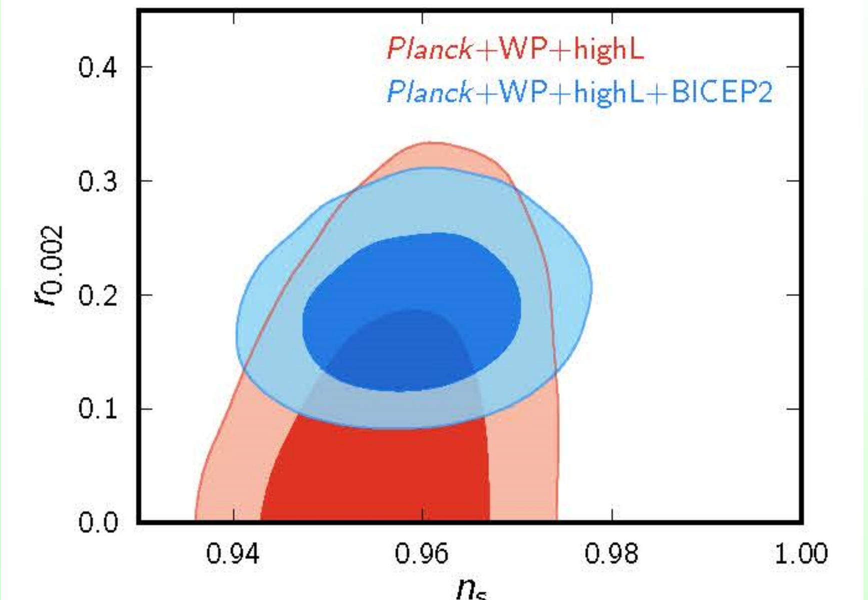


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Planck 2013 results. XXII

BICEP2



($r < 0.12$) **($r \sim 0.2$)**

FIG. 13.— Indirect constraints on r from CMB temperature spectrum measurements relax in the context of various model extensions. Shown here is one example, following Planck Collaboration XVI (2013) Figure 23, where tensors and running of the scalar spectral index are added to the base ΛCDM model. The contours show the resulting 68% and 95% confidence regions for r and the scalar spectral index n_s when also allowing running. The red contours are for the "Planck+WP+highL" data combination, which for this model extension gives a 95% bound $r < 0.26$ (Planck Collaboration XVI 2013). The blue contours add the BICEP2 constraint on r shown in the center panel of Figure 10. See the text for further details.

1403.3985

We look forward to the Planck polarization data, which will be released this winter.

$$r \equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon c_s \, (= -8 c_s n_T)\,.$$

(parameter degeneracies)

Non-Gaussianities of curvature perturbations

Non-Gaussianity

Generally speaking, the inflation predicts
almost Gaussian density and tensor perturbations.

Scalar non-Gaussianity (bispectrum):

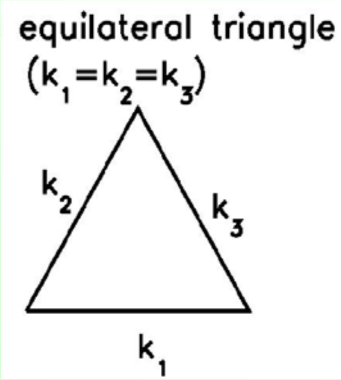
$$\frac{\langle\zeta\zeta\zeta\rangle^2}{\langle\zeta\zeta\rangle^3} = f_{NL}^2\langle\zeta\zeta\rangle \sim f_{NL}^2 10^{-10}$$

- ζ **is exactly Gaussian** ➔ $f_{NL} = 0$
- **standard inflation** ➔ $f_{NL} \sim 0.01$
  (canonical kinetic + potential)
- **non-canonical kinetic term** ➔ **large** f_{NL} **(5 ~ 100)**

Shapes of bispectrum

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B(k_1,k_2,k_3).$$

Length of three sides !! ⟸ **triangle**



● Equilateral type :

$$B_{\rm equil}(k_1,k_2,k_3) = 6\mathcal{P}^2 f_{\rm NL}^{\rm equil}\left\{-\frac{1}{k_1^3k_2^3}-\frac{1}{k_2^3k_3^3}-\frac{1}{k_3^3k_1^3}-\frac{2}{(k_1k_2k_3)^2}+\left[\frac{1}{k_1k_2^2k_3^3}+(5\ \text{perm.})\right]\right\}.$$

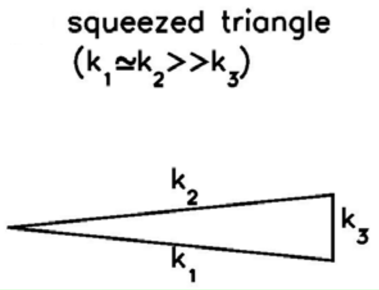
(P : normalization of powerspectrum)

ζ originates from an inflaton and the non-G is generated at the horizon exit.

● Local (squeezed) type :

$$\zeta(\boldsymbol{x}) = \zeta_G(\boldsymbol{x}) + \frac{3}{5} f_{NL}\zeta_G^2(\boldsymbol{x}).$$

$$B_{\rm local}(k_1,k_2,k_3) = 2\mathcal{P}^2 f_{\rm NL}^{\rm local}\left[\frac{1}{k_1^3k_2^3}+(2\ \text{perm.})\right],$$



ζ originates from a light field other than inflaton and the non-G is generated during superhorizon evolution.

Though there are other types such as orthogonal and folded …

Local and equilateral shapes

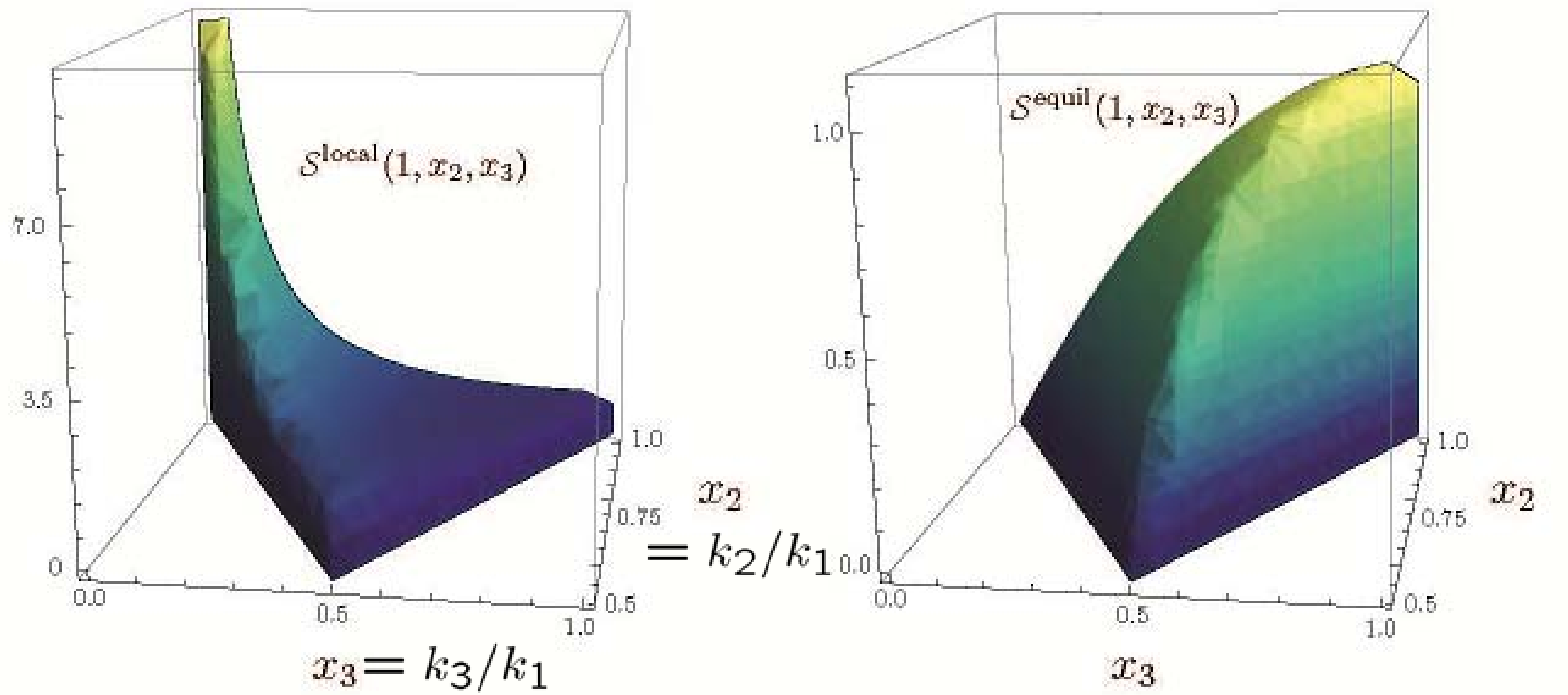


Figure 29: 3D plots of the *local* and *equilateral* bispectra. The coordinates x_2 and x_3 are the rescaled momenta k_2/k_1 and k_3/k_1, respectively. Momenta are order such that $x_3 < x_2 < 1$ and satsify the triangle inequality $x_2 + x_3 > 1$.

Baumann 0907.5424

$$k_3 < k_2 < k_1, \qquad k_2 + k_3 > k_1.$$

Planck constraints on non-Gaussianity

$f_{NL}^{\text{local}} = 2.7 \pm 5.8$	$f_{NL}^{\text{equil}} = -42 \pm 75$	$f_{NL}^{\text{ortho}} = -25 \pm 39$
(1 σ)	**(1 σ)**	**(1 σ)**
$-8.9 < f_{NL}^{\text{local}} < 14.3$	$-192 < f_{NL}^{\text{equil}} < 108$	$-103 < f_{NL}^{\text{ortho}} < 53$
(2 σ)	**(2 σ)**	**(2 σ)**
WMAP 9yr	**WMAP 9yr**	**WMAP 9yr**
$-3 < f_{NL}^{\text{local}} < 77$	$-221 < f_{NL}^{\text{equil}} < 323$	$-445 < f_{NL}^{\text{ortho}} < -45$
(2 σ)	**(2 σ)**	**(2 σ)**

No significant non-Gaussianities are observed.

Equilateral type bispectrum

Bispectrum of curvature perturbations of k-inflation

Maldacena 2003.
Seery & Lidsey 2005.
Chen et al. 2007.

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_G^2 R - K(\phi, X)\right], \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

ADM formalism :

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t)e^{2\zeta}\delta_{ij}, \qquad N = 1 + \alpha_1 + \alpha_2 + \cdots,$$

$$N_i = a^2\partial_i(\beta_1 + \beta_2 + \cdots) + \tilde{N}_{1i} + \cdots], \qquad \partial^i \tilde{N}_{ni} = 0.$$

Unitary gauge : $\phi = \phi(t), \quad \delta\phi = 0.$

Prescription:

- Expand the action up to the third order
- Eliminate α and β by use of the constraint equations
- Obtain the cubic action for ξ
- Calculate 3-point functions by using the in-in formalism.

Cubic action

$$S_3 = M_G^2 \int dt d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot\zeta^3 + \mathcal{C}_2 \zeta\dot\zeta^2 + \mathcal{C}_3 H \zeta^2 \dot\zeta + \frac{\mathcal{C}_4}{a^4}\partial^2\chi(\partial\zeta\cdot\partial\chi) + \frac{\mathcal{C}_5}{a^4}\partial^2\zeta(\partial\chi)^2 + \frac{\mathcal{C}_6}{a^2}\zeta(\partial\zeta)^2 + \frac{\mathcal{C}_7}{a^2}\dot\zeta(\partial\zeta\cdot\partial\chi) + \frac{2}{a^3} f(\zeta) \left.\frac{\delta L}{\delta\zeta}\right|_1\right],$$

$$\begin{cases}
\mathcal{C}_1 = -\frac{1}{H^2}\left[\Sigma\left(1-\frac{1}{c_s^2}\right)+2\lambda\right], \\
\mathcal{C}_2 = \frac{\epsilon}{c_s^4}\left(\epsilon - 3 - 3c_s^2\right), \\
\mathcal{C}_3 = \frac{\epsilon}{2c_s^2 H}\frac{d}{dt}\left(\frac{\eta}{c_s^2}\right), \\
\mathcal{C}_4 = \frac{\epsilon}{2}, \\
\mathcal{C}_5 = \frac{\epsilon}{4}, \\
\mathcal{C}_6 = \frac{\epsilon}{c_s^2}\left(\epsilon - 2s + 1 - c_s^2\right), \\
\mathcal{C}_7 = -2\frac{\epsilon}{c_s^2}.
\end{cases}$$

$$\begin{cases}
\Sigma = XK_X + 2X^2 K_{XX} = \frac{H^2\epsilon}{c_s^2}, \\
\lambda = X^2 K_{XX} + \frac{2}{3}X^3 K_{XXX}. \\
\chi = \partial^{-2}\Lambda, \quad \Lambda = \frac{a^2\epsilon}{c_s^2}\dot\zeta. \\
\left.\frac{\delta L}{\delta\zeta}\right|_1 = a\left[\frac{d\Lambda}{dt} + H\Lambda - \epsilon\partial^2\zeta\right]. \\
f(\zeta) = \frac{\eta}{4c_s^2}\zeta^2 + \frac{1}{c_s^2 H}\zeta\dot\zeta + \frac{1}{4a^2H^2}\left[-(\partial\zeta)^2 + \partial^{-2}\partial^i\partial^j(\partial_i\zeta\partial_j\zeta)\right] + \frac{1}{2a^2H}\left[\partial\chi\cdot\partial\zeta - \partial^{-2}\partial^i\partial^j(\partial_i\zeta\partial_j\chi)\right].
\end{cases}$$

In-in formalism

What we want is the expectation value instead of transition amplitude. $\Rightarrow$ **In-in formalism**

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = -i\int_{t_0}^{t} dt' \langle \left[\zeta(\mathbf{k}_1,t)\zeta(\mathbf{k}_2,t)\zeta(\mathbf{k}_3,t), H_{\rm int}(t')\right]\rangle,$$

t_0 is some early time when the fluctuation is well inside the horizon,
t is a several e-folds time after the horizon exit.

Cubic action S3 $\Rightarrow$ $H_{\rm int}(t) = -\int d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot\zeta^3 + \mathcal{C}_2\zeta\dot\zeta^2 + \cdots\right].$

$\Rightarrow$ $$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^7\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}_\zeta^2\frac{\mathcal{A}}{k_1^3k_2^3k_3^3}.$$

$$\begin{aligned}\mathcal{A} \;=\; & \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)\frac{3k_1^2k_2^2k_3^2}{2K^3} + \left(\frac{1}{c_s^2}-1\right)\left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{8}\sum_i k_i^3\right)\\ & + \frac{\epsilon}{c_s^2}\left(-\frac{1}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j}k_ik_j^2 + \frac{1}{K}\sum_{i>j}k_i^2k_j^2\right) + \frac{\eta}{c_s^2}\left(\frac{1}{8}\sum_i k_i^3\right)\\ & + \frac{s}{c_s^2}\left(-\frac{1}{4}\sum_i k_i^3 - \frac{1}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right). \qquad (K = k_1+k_2+k_3)\end{aligned}$$

$c_s^2 \ll 1$ $\Rightarrow$ Large non-Gaussianity is produced.

f_{NL}

f_{NL} is defined to characterize the size of the three-point correlation function,

$$f_{\rm NL} = \frac{5}{18}\frac{B(k,k,k)}{P_\zeta^2(k)} = \frac{10}{9}\frac{\mathcal{A}_{k_1=k_2=k_3=k}}{k^3}.$$

$$\left[\begin{array}{rcl} \langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle & = & (2\pi)^3\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B(k_1,k_2,k_3) \\ & = & (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}_\zeta^2\frac{\mathcal{A}}{k_1^3k_2^3k_3^3}. \end{array}\right]$$

$$\Rightarrow \quad f_{\rm NL} \simeq \frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right) - \frac{35}{108}\left(\frac{1}{c_s^2}-1\right).$$

$$\Sigma = XK_X + 2X^2K_{XX} = \frac{H^2\epsilon}{c_s^2},$$
$$\lambda = X^2K_{XX} + \frac{2}{3}X^3K_{XXX}.$$

$c_s \ll 1$ ➔ large non-Gaussianity can be easily generated !!

(The Planck results give the constraint, $c_s \geq 0.02 (95\%\ \mathrm{CL})$.)

How can we understand the relation between c_s & f_{NL} ?

Effective field theory of inflation

Basic idea of effective field theory of inflation

(Cheung et al. 2008)

Inflation must end to be followed by hot big bang Universe.

$\Rightarrow$ **spontaneously breaks time diffeomorphism inv.**

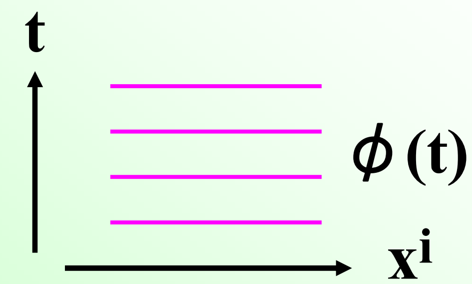
$\Leftrightarrow$ **Time-dependent spatial diffeo is unbroken.**

$$\delta x^i = \epsilon^i(t, x^i)$$

$\Rightarrow$ **In the low energy effective theory, any term respecting the unbroken symmetry is allowed.**

We can investigate the properties of perturbations generated during inflation without resort to a particular Lagrangian.

Unitary (comoving) gauge



Unitary(Comoving) gauge :

Time slice (t = const. hypersurface) coincides with ϕ = const. hypersurface.

$$\Leftrightarrow \quad \phi(t, \boldsymbol{x}) = \phi_0(t) + \underbrace{\cancel{\delta\phi(t, \boldsymbol{x})}}_{=0}.$$

$\Rightarrow$ **The scalar field perturbation is eaten by the metric.**

(That is, the scalar perturbations correspond to the NG bosons)

$\Rightarrow$ **The graviton (metric) has three degree of freedom:**

$$\begin{cases} \text{Curvature perturbation } \zeta \\ \text{Tensor perturbations } \gamma_{ij} \end{cases}$$

Action in unitary gauge

Any quantities respecting the time-dependent diffeo. inv.

- **4-dim scalar**
- **generic function of t, f(t)**
- $\partial_\mu t = \delta_\mu^0$ **in unitary gauge, which allows any tensor with 0 upper index** (g^{00} , R^{00}, …)
- **Extrinsic curvature :** $K_{\mu\nu} = h_\mu^\sigma \nabla_\sigma n_\nu .$

(All covariant derivatives of n$_\mu$ can be written using K$_{\mu\nu}$ and derivatives of g^{00})

$$n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}}$$: unit vector orthogonal to t =const.

$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$: projection tensor to t =const.

Note that ${}^{(3)}R_{\alpha\beta\gamma\delta} = h_\alpha^\mu h_\beta^\nu h_\gamma^\rho h_\delta^\sigma R_{\mu\nu\rho\sigma} - K_{\alpha\gamma}K_{\beta\delta} + K_{\beta\gamma}K_{\alpha\delta}.$ **${}^{(3)}$R is redundant.**

$\Rightarrow$ $$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, g_{\mu\nu}, K_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu, \delta_\mu^0, t)$$

(all indices are contracted)

Expanding around FLRW background

Fluctuations around FLRW background:

$$\begin{cases} \delta g^{00} = g^{00} + 1, \\ \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu}, \\ \delta R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - 2(H^2 + k/a^2) h_{\mu[\rho} h_{\sigma]\nu} + (\dot{H} + H^2)(h_{\mu\rho}\delta^0_\nu \delta^0_\sigma + (3\,\mathrm{perms})). \end{cases}$$

- **0th and 1st order in fluctuations:**

kinetic(g^{00} dot{ϕ}^{2}) & potential energy of the background scalar field

$$\Rightarrow \quad S^{(0)+(1)} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\mathrm{pl}}^2 R - c(t) g^{00} - \Lambda(t) \right].$$

(Terms such as $\partial^0 g^{00}, K^\mu_\mu, R^{00}$ can be absorbed into the above terms by integration by parts)

Variation w.r.t. g^{00} & g^{ij}

$$\Rightarrow \quad \begin{cases} 3 M_{\mathrm{pl}}^2 H^2 = c(t) + \Lambda(t), \\ \dot{H} M_{\mathrm{pl}}^2 = -c(t). \end{cases}$$

$$\Rightarrow \quad S^{(0)+(1)} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\mathrm{pl}}^2 R + M_{\mathrm{pl}}^2 \dot{H} g^{00} - M_{\mathrm{pl}}^2 (3H^2 + \dot{H}) \right].$$

Expanding around FLRW background II

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H}(t) g^{00} - M_{\rm pl}^2 (3H^2(t) + \dot{H}(t)) \right.$$
$$\left. + F^{(2)+(3)+\cdots}\left(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \delta^0_\mu, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, t\right)\right].$$

$$F^{(2)+(3)+\cdots} = \frac{1}{2}M_2(t)^4\left(\delta g^{00}\right)^2 + \frac{1}{3!}M_3(t)^4\left(\delta g^{00}\right)^3$$
$$- \frac{1}{2}\bar{M}_1(t)^3 \delta g^{00}\delta K^\mu_\mu - \frac{1}{2}\bar{M}_2(t)^3 \delta K^\mu_\mu K^\nu_\nu - \frac{1}{2}\bar{M}_3(t)^3 \delta K^\mu_\nu K^\nu_\mu + \cdots.$$

This is the most general action in unitary gauge ($\delta\phi(t, \boldsymbol{x}) = 0$), which satisfies the time-dependent spatial diffeo. invariance.

Note, however, that time diffeo. :
$t \longrightarrow \tilde{t} = t + \xi^0(x), \;\; \boldsymbol{x} \longrightarrow \tilde{\boldsymbol{x}}$ **is broken.**

Stuckelberg trick

In order to (apparently) recover broken time diffeo. :

$$t \longrightarrow \tilde{t} = t + \xi^0(x), \;\; \boldsymbol{x} \longrightarrow \tilde{\boldsymbol{x}},$$

we introduce the Stuckelberg field π, which corresponds to the Goldstone boson and transforms as

$$\pi(x) \longrightarrow \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x).$$

$\Rightarrow$ t + π (x) is invariant under this time diffeo.

We have only to make the following replacements :

$$\begin{cases} f(t) \longrightarrow f(t+\pi), \\ \delta^0_\mu \longrightarrow \partial_\mu(t+\pi) = \delta^0_\mu(1+\dot{\pi}) + \delta^i_\mu \partial_i \pi, \\ g^{00} \longrightarrow g^{\mu\nu}\dfrac{\partial(t+\pi)}{\partial x^\mu}\dfrac{\partial(t+\pi)}{\partial x^\nu} = (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi}) g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi, \\ g^{o\nu} \longrightarrow g^{\mu\nu}\dfrac{\partial(t+\pi)}{\partial x^\mu} = (1+\dot{\pi}) g^{0\nu} + g^{i\nu}\partial_i\pi, \\ \ldots \end{cases}$$

Relation between π and ζ

Flat gauge :

$$dl^2 = a^2(t)\delta_{ij}dx^i dx^j ,$$

Unitary gauge (π =0) :

$$dl^2 = a^2(t)(1 + 2\zeta)\delta_{ij}dx^i dx^j .$$

$$a^2(t) = a^2(\tilde{t} - \pi) = a^2(\tilde{t})(1 - 2H\pi)$$

gauge transformation

$$\xi^0(x) = \pi(x)$$

$$\begin{cases} t \longrightarrow \tilde{t} = t + \xi^0(x), \\ \pi(x) \longrightarrow \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x). \end{cases}$$

$$\Rightarrow \quad \zeta = -H\pi$$

We can easily evaluate observable quantities like powerspectrum.

Mixing terms

$$S=\int d^4x\sqrt{-g}\Big\{\frac{1}{2}M_{\rm pl}^2R-M_{\rm pl}^2\left[3H^2(t+\pi)+\dot H(t+\pi)\right]$$
$$+M_{\rm pl}^2\dot H(t+\pi)\left[(1+\dot\pi)^2g^{00}+2(1+\dot\pi)g^{0i}\partial_i\pi+g^{ij}\partial_i\pi\partial_j\pi\right]$$
$$+\frac{1}{2}M_2^4(t+\pi)\left[(1+\dot\pi)^2g^{00}+2(1+\dot\pi)g^{0i}\partial_i\pi+g^{ij}\partial_i\pi\partial_j\pi+1\right]^2$$
$$+\frac{1}{3!}M_3^4(t+\pi)\left[(1+\dot\pi)^2g^{00}+2(1+\dot\pi)g^{0i}\partial_i\pi+g^{ij}\partial_i\pi\partial_j\pi+1\right]^3+\cdots\Big\}.$$

($\pi = 0$ recovers the action in unitary gauge)

Mixing terms between π & g have fewer derivatives :

e.g. $M_{\rm pl}^2\dot H\cdot(1+\dot\pi)^2g^{00}\ni M_{\rm pl}^2\dot H\left(-\dot\pi^2+2\dot\pi\delta g^{00}\right)$

Canonical normalization
$(\pi_c\sim M_{\rm pl}(-\dot H)^{1/2}\pi,\ \delta g_c^{00}\sim M_{\rm pl}\delta^{00})$

$\Rightarrow$ $-\dot\pi_c^2+2\epsilon^{1/2}H\dot\pi_c\delta g_c^{00}$

The Goldstone boson π decouples from graviton g (lapse & shift) at the energy scale E >> $E_{mix} = \varepsilon^{1/2}$ H.

E >> E_{mix} $\Leftrightarrow$ ε << 1 (inflation)

Decoupling limit

(DL limit)

$$S \to \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R - M_{\rm pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + \frac{1}{2}M_2^4\left(\dot{\pi}^2 + 2\dot{\pi} - \frac{(\partial_i\pi)^2}{a^2}\right)^2 - \frac{1}{6}M_3^4\left(\dot{\pi}^2 + 2\dot{\pi} - \frac{(\partial_i\pi)^2}{a^2}\right)^3 + \cdots\right]$$

$$= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R - M_{\rm pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \cdots\right]$$

$$= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R - M_{\rm pl}^2\dot{H}c_\pi^{-2}\left(\dot{\pi}^2 - c_\pi^2\frac{(\partial_i\pi)^2}{a^2}\right) - M_{\rm pl}^2\dot{H}(c_\pi^{-2}-1)\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \cdots\right]$$

$$c_\pi^2 \equiv \frac{-M_{\rm pl}^2\dot{H}}{-M_{\rm pl}^2\dot{H} + 2M_2^4} \longleftarrow \textbf{Coefficient of } (\delta g^{00})^2 \qquad \left(\frac{1}{2}M_2(t)^4\left(\delta g^{00}\right)^2\right)$$

c.f. $c_s^2 = \dfrac{K_X}{K_X + 2XK_{XX}}, \quad X^2 = \left(\dfrac{1}{2}\left(1+\delta g^{00}\right)\dot{\phi}^2\right)^2$

- **It is manifest which term leads to a non-trivial sound velocity.**
- **The (δ g^{00})^2 term leads to not only non-trivial sound velocity but also non-trivial cubic interactions, which leads to large non-Gaussianities for c$_\pi$ << 1 .**
- **Thus, the relation between each term (physics) and observable quantities is clear in EFT.**

Tensor perturbations

Sound speed of tensor perturbations

Sound speed of tensor perturbations

$$S=\int d^4x\sqrt{-g}\left[\boxed{\frac{1}{2}M_{\rm pl}^2R+M_{\rm pl}^2\dot{H}(t)g^{00}-M_{\rm pl}^2(3H^2(t)+\dot{H}(t))}\right.$$ **= S0**

$$\left.+F^{(2)+(3)+\cdots}\left(\delta g^{00},\delta K_{\mu\nu},\delta R_{\mu\nu\rho\sigma};\delta^0_\mu,g_{\mu\nu},g^{\mu\nu},\nabla_\mu,t\right)\right].$$

Ingredients of higher perturbations terms : $\delta g^{00},\delta K,\delta K^\nu_\mu,\delta R,\delta R_{\mu\nu},\cdots$

$$F^{(2)+(3)+\cdots}=\frac{1}{2}M_2^4(t)\left(\delta g^{00}\right)^2-\frac{1}{2}\bar{M}_1(t)^3\delta g^{00}\delta K-\frac{1}{2}\bar{M}_2(t)^3\left(\delta K\right)^2-\frac{1}{2}\bar{M}_3(t)^3\delta K^\mu_\nu K^\nu_\mu+\cdots$$

$$ds^2=-dt^2+h_{ij}dx^idx^j=-dt^2+a^2(t)\left(e^\gamma\right)_{ij}dx^idx^j,\qquad(\gamma_{ii}=0,\ \gamma_{ij,i}=0.)$$

$\delta K^j_i\ni\frac{1}{2}\dot\gamma_{ij}$ **at linear order**

$$\delta K^\nu_\mu\delta K^\mu_\nu\ni\frac{1}{4}\left(\dot\gamma_{ij}\right)^2$$

$$S_T^{(2)}=\frac{1}{8}\int dtd^3x\,a^3M_G^2c_\gamma^{-2}\left(\dot\gamma_{ij}\dot\gamma_{ij}-\frac{c_\gamma^2}{a^2}\gamma_{ij,k}\gamma_{ij,k}\right).$$

$c_\gamma^2=\frac{M_G^2}{M_G^2-\bar{M}_3^2}$ **: sound speed of tensor perturbations**

Formal derivation of primordial tensor fluctuations

$$\begin{cases} ds^2 = -dt^2 + a^2(t)\,(e^\gamma)_{ij}\,dx^i dx^j = -dt^2 + a^2(t)\left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{ik}\gamma_{kj} + \cdots\right) dx^i dx^j, \\ \left(\gamma_{ii} = 0,\ \ \gamma_{ij,i} = 0.\right) \\ S_0 = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H}(t) g^{00} - M_{\rm pl}^2(3H^2(t) + \dot{H}(t))\right]. \end{cases}$$

$$\Rightarrow \quad S_T^{(2)} = \frac{1}{8}\int dt d^3x\, a^3 M_G^2 \left(\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}\gamma_{ij,k}\gamma_{ij,k}\right).$$

$\Rightarrow$ **(Standard) quantization of free fields**

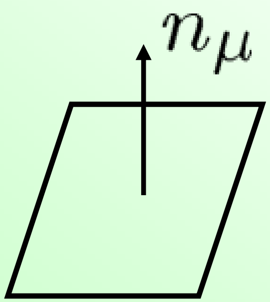
Note that the whole metric is "not" quantized.

The background geometry exists classically and only the perturbations are quantized.

Higher derivative term

e. g. $$f(\phi)\left(\nabla_\mu\nabla_\nu\phi\right)^2 \ni f(\bar{\phi})\dot{\bar{\phi}}^2 K^\nu_\mu K^\mu_\nu$$

$$\because \quad n_\mu \propto \partial_\mu\phi$$



ϕ = const
(t = const)

$$\Rightarrow \quad n_\mu = -\gamma\nabla_\mu\phi, \quad \gamma = \frac{1}{\sqrt{-X}}.$$

$$\left(h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu,\ K_{\mu\nu} = h^\sigma_\mu \nabla_\sigma n_\nu\right)$$

$$\Rightarrow \quad \begin{aligned} \nabla_\mu\nabla_\nu\phi &= -\nabla_\nu(\gamma^{-1}n_\mu) \\ &= -\gamma^{-1}(K_{\mu\nu} - \dot{n}_\mu n_\nu) + \frac{\gamma}{2} n_\mu \nabla_\nu X. \end{aligned}$$

This kind of term appears in Horndeski's theory (Generalized Galileon):

$$G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \ni G_{4,X}(\bar{\phi}, g^{00}\dot{\bar{\phi}}^2/2)\left(K^2 - K^\nu_\mu K^\mu_\nu\right).$$

Effects of change of sound speed of tensor perturbations

$$\delta K^\nu_\mu \delta K^\mu_\nu \ni \frac{1}{4}\left(\dot\gamma_{ij}\right)^2 + \mathcal{O}(\gamma^4)$$

- **The normalization of powerspectrum is changed.**

$$S_T^{(2)} = \frac{1}{8}\int dt d^3x\, a^3 M_G^2 c_\gamma^{-2}\left(\dot\gamma_{ij}\dot\gamma_{ij} - \frac{c_\gamma^2}{a^2}\gamma_{ij,k}\gamma_{ij,k}\right).$$

$$\Rightarrow \left\{\begin{aligned} \mathcal{P}_T(k) &\simeq \frac{2}{\pi^2 c_\gamma}\left(\frac{H}{M_G}\right)^2, \\ n_T &= \frac{d\ln \mathcal{P}_T(k)}{d\ln k} \simeq -2\epsilon - T, \quad T = \frac{\dot c_\gamma}{H c_\gamma}. \\ r &\equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon c_s \underline{c_\gamma^{-1}} \,(\neq -8c_s n_T)\,. \end{aligned}\right.$$

(parameter degeneracies)

- **No cubic interaction of γ associated with this interaction !!**
  (different from the case of curvature perturbations)

$\Rightarrow$ **c_γ has nothing to do with the auto-bispectrum of tensor perturbations.**

(N.B. $\delta K^\nu_\mu \delta K^\sigma_\nu \delta K^\mu_\sigma$ can generate large tensor bispectrum)

How can we probe the tensor sound speed ?

Importance of cross bispectum

By using the Stuckelberg trick to recover the interaction of π,

$$\delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu} \longrightarrow \boxed{\frac{(\partial^2\pi)^2}{a^4} + \frac{1}{4}\dot{\gamma}_{ij}^2} \boxed{-\frac{1}{2}\dot{\gamma}_{ij}\frac{\partial_k\gamma_{ij}\partial_k\pi}{a^2}} \quad (\gamma\,\gamma\,\pi)$$

$$\boxed{-2\gamma_{ij}\frac{\partial_i\partial_j\pi\partial_k^2\pi}{a^4} - \frac{1}{2}(\ddot{\gamma}_{ij} + H\dot{\gamma}_{ij} - \frac{\partial_k^2\gamma_{ij}}{a^2})\frac{\partial_i\pi\partial_j\pi}{a^2}} \quad (\gamma\,\pi\,\pi)$$

$$\boxed{+\dot{\pi}\frac{4\partial^2\partial_i\pi\partial_i\pi - 2(\partial_i\partial_j\pi)^2 + 4(\partial^2\pi)^2}{a^4} + \ldots .} \quad (\pi\,\pi\,\pi)$$

operator	$\dot{\pi}^2$	$\frac{(\partial_i\pi)^2}{a^2}$	$\frac{(\partial_i^2\pi)^2}{a^4}$	$(\dot{\gamma}_{ij})^2$	$\frac{(\partial_k\gamma_{ij})^2}{a^2}$	γ^3	$\gamma^2\pi$	$\gamma\pi^2$	π^3
S_0	✓	✓		✓	✓	✓		✓	
$(\delta g^{00})^2$	✓								✓
$\delta g^{00}\delta K$		✓						✓	✓
$(\delta K)^2$			✓					✓	✓
$\delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu}$			✓	✓			✓	✓	✓

TABLE I: Operators relevant to dispersion relations of primordial perturbations and the induced cubic interactions in the decoupling limit.

$$\left(S_0 = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2R + M_{\rm pl}^2\dot{H}(t)g^{00} - M_{\rm pl}^2(3H^2(t) + \dot{H}(t))\right].\right)$$

The scalar-tensor-tensor interaction, the $\gamma\,\gamma\,\pi$-type interaction, arises only from the operator $\delta K^{\nu}_{\mu}\delta K^{\mu}_{\nu}$.

Scalar-tensor-tensor bispectrum

$$\int d^4x\, a^3 \frac{\bar{M}_3^2}{4} \dot{\gamma}_{ij} \frac{\partial_k \gamma_{ij} \partial_k \pi}{a^2} \,.$$

In-in formalism

$$\langle \zeta_{\mathbf{k}_1} \gamma^{s_2}_{\mathbf{k}_2} \gamma^{s_3}_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3 \Big(\sum_i \mathbf{k}_i\Big) \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{k_1^2 k_2^2 k_3^2} S_{s_2,s_3}(k_1,k_2,k_3)\,,$$

$$S_{s_2,s_3}(k_1,k_2,k_3) = \epsilon^{s_2}_{ij}(\mathbf{k}_2)\epsilon^{s_3}_{ij}(\mathbf{k}_3)\tilde{S}(k_1,k_2,k_3)\,.$$

polarization tensor

$$\tilde{S}(k_1,k_2,k_3) = \epsilon\left(c_\gamma^{-2} - 1\right) \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2}{2k_1 k_2 k_3} \left(\frac{1}{K} + \frac{k_1 + c_\gamma k_2}{K^2} + \frac{2k_1 k_2}{K^3}\right) + (\mathbf{k}_2 \leftrightarrow \mathbf{k}_3)\,,$$

$$(K = k_1 + c_\gamma k_2 + c_\gamma k_3.)$$

One can observe the c_γ dependence on this cross bispectrum.

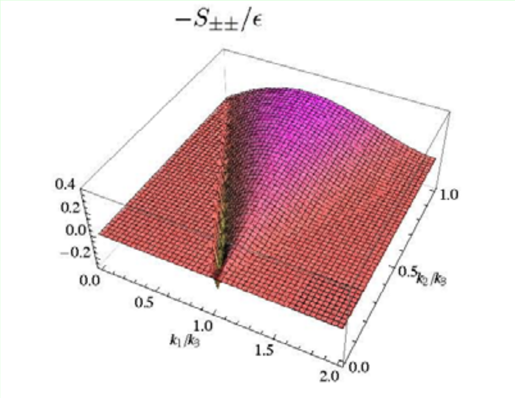


FIG. 1: Shape function $S_{\pm\pm}(k_1,k_2,k_3)$ for $c_\gamma = 0.8$

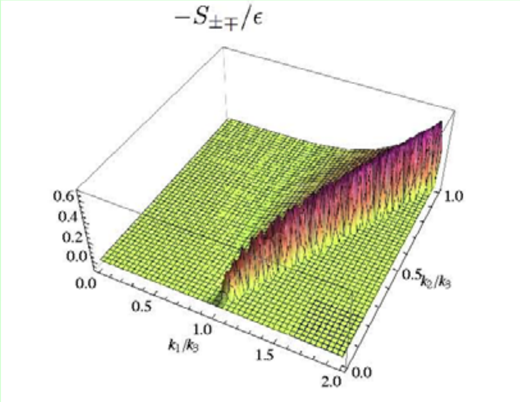


FIG. 2: Shape function $S_{\pm\mp}(k_1,k_2,k_3)$ for $c_\gamma = 0.8$

Summary

- If the BICEP2 results are correct, we now observe
  the powerspectrum of scalar and tensor perturbations.
- Non-Gaussianity (bispectrum) gives additional information.
- For example, the sound speed of the curvature perturbations are strongly correlated with the their auto-bispectrum.
- On the other hand, the tensor sound speed has nothing to do with their auto-bispectrum.
- Rather, it is related to the cross-bispectra, in particular,
  the scalar-tensor-tensor bispectrum.
- The relevant CMB bispectra of two B-modes and one temperature (or one E-mode) anisotropies become a powerful tool to probe the tensor sound speed.