Black holes as particle accelerators: a brief review

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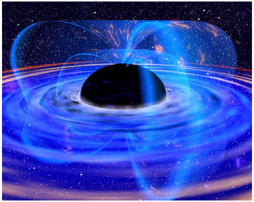
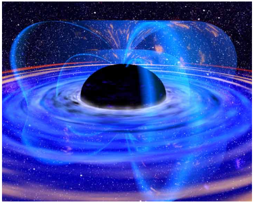
Based on arXiv:1409.7502 with Masashi Kimura (Cambridge)

Particle accelerators

 as 

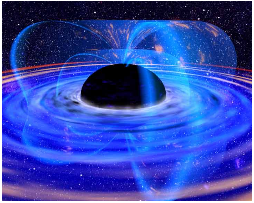
- Terrestrial particle accelerators
  Charged particles are accelerated by electromagnetic force. The centre-of-mass (CM) energy $E_{\rm cm} \simeq 14$ TeV for proton-proton collision.
- Gravitational particle accelerators
  Neutral particles are also accelerated by gravitational force. However, $E_{\rm cm} \simeq 2\sqrt{5}mc^2$ at most for the Schwarzschild BH.

Rotating BHs as particle accelerators

 as 

- Kerr BHs as particle accelerators
  (Bañados, Silk and West 2009, Piran, Shaham and Katz 1975)
  Kerr BHs act as particle accelerators. It is based on classical GR. The CM energy of colliding particles can be very high.
- Not only microscopic particles but also macroscopic objects, such as BHs and compact stars, are accelerated.

Rotating BHs as particle accelerators



as

- Kerr BHs as particle accelerators
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  Kerr BHs act as particle accelerators. It is based on classical GR. The CM energy of colliding particles can be very high.
- Not only microscopic particles but also macroscopic objects, such as BHs and compact stars, are accelerated.
- Don't be confused with "BH production in particle accelerators".
  (Giddings and Thomas 2002, ...)

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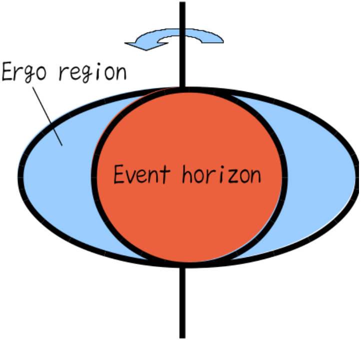
Rotating BHs

- Rotating BHs are uniquely described by a Kerr metric, which is parametrised by the mass M and the spin a.
- $0 < |a| \leq M$: BH
- Spin angular momentum: $J = Ma$.
- Nondimensional spin parameter

$$a_* = \frac{J}{M^2} = \frac{a}{M}.$$



Kerr BHs

- Kerr metric

$$\begin{aligned} ds^2 &= -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ &\quad + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2, \end{aligned}$$

  where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$.
- We assume $0 \le a^2 \le M^2$. Δ vanishes at $r = r_\pm = M \pm \sqrt{M^2 - a^2}$. The horizon radius is $r_H = r_+$.
- Ergosphere: $r_E = M + \sqrt{M^2 - a^2\cos^2\theta}$
- Two commuting Killing vectors: $\xi^a = (\partial/\partial t)^a$ and $\psi^a = (\partial/\partial\phi)^a$.
- Horizon null generator: $\chi^a = \xi^a + \Omega_H \psi^a$

$$\Omega_H = \frac{a}{r_H^2 + a^2}.$$

Geodesic particle in the equatorial plane

- Conserved quantities:

$$m^2 = -p_a p^a, \; E = -p_t = -\xi^a p_a, \; L = p_\phi = \psi^a p_a,$$

  where p^a is the four-momentum.

- The geodesic equations are reduced to the first-order form

$$\begin{aligned} r^2 \dot{t} &= \frac{(r^2+a^2)[E(r^2+a^2)-aL]}{\Delta} - a(aE-L), \\ r^2 \dot{\phi} &= \frac{a[E(r^2+a^2)-aL]}{\Delta} - (aE-L), \end{aligned}$$

  where the dot is the derivative w.r.t. the affine parameter.

Forward-in-time condition and critical particle

- The equatorial geodesic equations are reduced to a 1D potential problem with an effective potential

$$\frac{1}{2}\dot{r}^2 + V(r) = 0,$$

$$V(r) = -\frac{m^2 M}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{1}{2}(E^2 - m^2).$$

- Forward-in-time condition: $\dot{t} > 0$
- This condition near the horizon is reduced to

$$E - \Omega_H L \geq 0$$

- Critical particles: particles which satisfy $E - \Omega_H L = 0$.

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Centre-of-mass (CM) energy

- Suppose particles 1 and 2 are at the same spacetime point.
- The sum of the two momenta

$$p^a_{\rm tot} = p^a_1 + p^a_2.$$

- Total energy observed in the centre-of-mass frame

$$E^2_{\rm cm} = -p^a_{\rm tot} p_{{\rm tot}a}.$$

- Coordinate-independent and in principle observable

Formal divergence in collision energy

- The CM energy for near-horizon collision in the equatorial plane is formally given by

$$E_{\rm cm}^2 = \frac{m_1^2 r_H^2 + (L_1 - aE_1)^2}{r_H^2}\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2) + \cdots ,$$

  where both particle are assumed to be infalling.

- $E_{\rm cm}$ is apparently divergent if either of the particles is critical. i.e., $E - \Omega_H L = 0$.
- For massive particles, we define

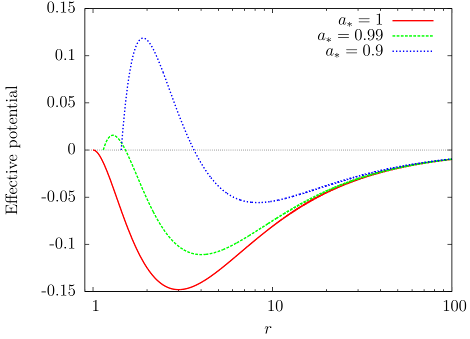
$$e = \frac{E}{m} \quad , l = \frac{L}{mM}, \quad l_c = \frac{E}{\Omega_H m M}.$$

Motion of the particles

- Massive particles which were at rest at infinity can reach the horizon if

$$-2(1+\sqrt{1+a_*}) = l_L < l < l_R = 2(1+\sqrt{1-a_*}).$$

- We can find $l_R \leq l_c$, where $l_R = l_c (= 2)$ holds only for $a = M$.
- The critical particle is bounced at $r \simeq r_H + \frac{2\sqrt{2}(E^2+m^2)M}{3E^2-m^2}\sqrt{1-a_*}$.



Unboundedly high CM energy

- Extremal Kerr, on-horizon collision, $l_1, l_2 < l_c(= 2)$:

$$\frac{E_{\rm cm}}{2m} = \sqrt{\frac{1}{2}\left(\frac{2-l_1}{2-l_2} + \frac{2-l_2}{2-l_1}\right)}.$$

  $E_{\rm cm} \to \infty$ as we fine-tune $l_1 \to 2$.
- Extremal Kerr, $l_1 = l_c(= 2)$, $l_2 < 2$:

$$\frac{E_{\rm cm}}{2m} \approx \sqrt{\frac{(2-\sqrt{2})(2-l_2)M}{2(r_{\rm col}-M)}}.$$

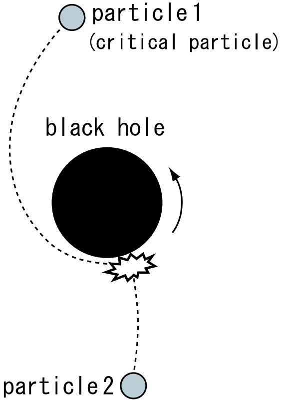
  $E_{\rm cm} \to \infty$ as $r_{\rm col} \to r_H = M$.
- Near-extremal Kerr, $l_1 = l_R(< l_c)$, $l_2 < l_R$:

$$\frac{E_{\rm cm}}{2m} \approx \frac{\sqrt{(2+\sqrt{2})(2-l_2)}}{2} \frac{1}{\sqrt[4]{1-a_*^2}}.$$

  $E_{\rm cm} \to \infty$ as $a_* \to 1$.

The orbit of the critical particle

The critical particle rotates infinitely many times around a maximally rotating BH and takes infinitely long proper time to reach the horizon.



CM energy in finite time

- The coordinate velocity is given by

$$\frac{dr}{dt}=\frac{(r-M)^2\sqrt{2M}}{(r^2+Mr+2M^2)\sqrt{r}},$$

  for the critical particle around the extremal Kerr.
- The Killing time T needed for particle 1 to reach $r=r_{\rm col}$ from $r=r_i$:

$$T=-\int_{r_i}^{r_{\rm col}}dr\frac{\sqrt{r}(r^2+Mr+2M^2)}{\sqrt{2M}(r-M)^2}\simeq\frac{2\sqrt{2}M^2}{r_{\rm col}-M}.$$

- We then obtain

$$\frac{E_{\rm cm}}{2m}\simeq\frac{1}{2}\sqrt{(\sqrt{2}-1)(2-l_2)\frac{T}{M}},$$

  or

$$E_{\rm cm}\simeq 2.5\times 10^{20}{\rm eV}\left(\frac{T}{10\text{ Gyr}}\right)^{1/2}\left(\frac{M}{M_\odot}\right)^{-1/2}\left(\frac{m}{1\text{ GeV}}\right).$$

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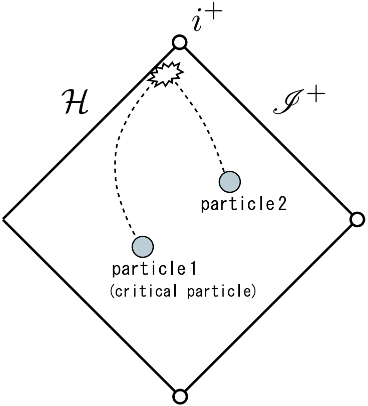
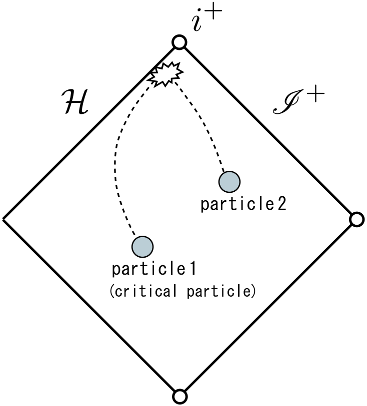
The critical particle is accelerated unboundedly.



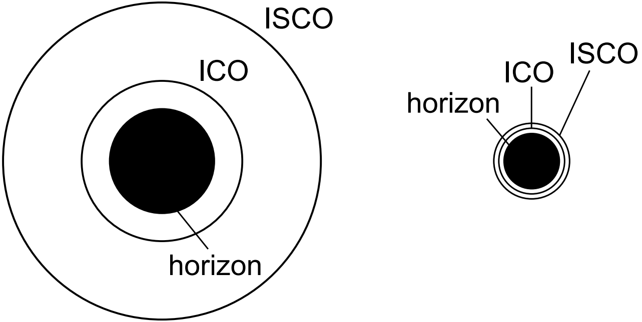
The critical particle asymptotically approaches the event horizon which is a null hypersurface. This implies that the critical particle is accelerated infinitely to the speed of light in infinite time.

The infalling particle is accelerated unboundedly.



The infalling particle is accelerated to the light speed with respect to the static observer. If the observer can stay at a constant radius near the horizon, he or she will see the particle falls with the almost speed of light.

The structure of circular orbits around a BH



The observer can stay at a constant radius near the horizon only for a near-extremal Kerr BH, where both the Innermost Stable Circular Orbit (ISCO) and Innermost Circular Orbit (ICO) are close to the horizon.

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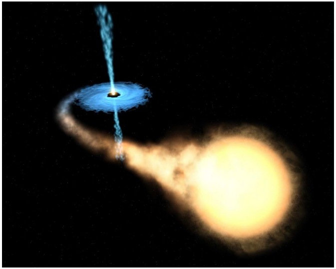
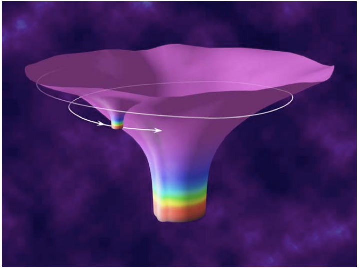
Astrophysical significance of the ISCO





- ISCO = Innermost Stable Circular Orbit
- The inner edge of the standard accretion disk is given by the ISCO. (Shakura and Sunyaev 1973, Novikov and Thorne 1973)
- A compact object inspirals adiabatically around a supermassive BH and begins to plunge into the horizon at the ISCO.

Natural fine-tuning for the ISCO particle

- The ISCO for a Kerr BH is analytically given by Bardeen, Press and Teukolsky (1972).
- As $a_* \to 1$, we find

$$r_{\rm ISCO} \to r_H, \; E_{\rm ISCO} \to m_0/\sqrt{3}, \; L_{\rm ISCO} \to 2m_0 M/\sqrt{3}, \; \Omega_H \to 1/(2M).$$

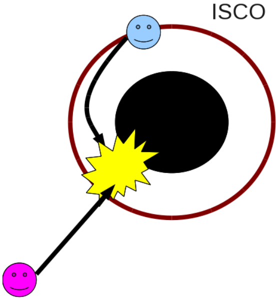
- Thus,

$$E_{\rm ISCO} - \Omega_H L_{\rm ISCO} \to 0.$$

Therefore, the ISCO particle satisfies the critical condition in the maximal rotation limit.

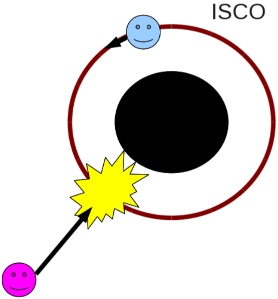
Collision of the ISCO particle

Near-horizon collision



$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/2}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[4]{1-a_*^2}}$$

On-ISCO collision



$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/6}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[6]{1-a_*^2}}$$

Both diverge as $a_* \to 1$ but in different manners. (Harada and Kimura 2011a)

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Non-equatorial motion

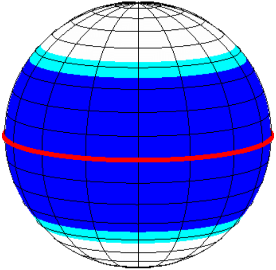
- 4 conserved quantities: m, E, L and the Carter constant Q.
- Forward-in-time condition near the horizon requires $L \le L_c \equiv E/\Omega_H$.
- In the near-horizon limit, the CM energy is given by (Harada and Kimura 2011b)

$$E_{\rm cm}^2 = \frac{m_1^2 r_H^2 + Q_1 + (L_1 - aE_1)^2}{r_H^2 + a^2 \cos^2\theta} \frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2) + \cdots ,$$

  where Q is the Carter constant.
- If we fine-tune $L_1 \to L_{1c} = E_1/\Omega_H$, i.e. for critical particles, $E_{\rm cm}$ formally diverges.

High-energy collision belt

- We require that the critical particles should be allowed to reach the horizon from the outside.
- Then, non-equatorial high-energy collisions are possible near a maximally rotating Kerr BH and only between latitudes $\pm 42.94°$.



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Effects of gravitational radiation reaction

- One might expect that gravitational radiation and its backreaction constrain the collision energy. (Berti et al. 2009)
- The radiation power of the ISCO particle is, however, strongly suppressed in the near-extremal Kerr. Numerical results can be fit by

$$\dot{E}_{\rm GW} \propto (1 - a_*)^{\lambda}$$

  where $\lambda \simeq 0.317$ (Hughes, Kesden 2011).
- A detailed analysis shows that radiative effects are subdominant if the mass ratio m/M is sufficiently small. (Harada and Kimura 2011c).

Effects of self-gravity

- If the CM energy is as large as the energy of the central BH, we must take into account the self-gravity of the particles.
- We can take the analogous system of electrically charged dynamical spherical shells around a static spherically symmetric electrically charged BH.
- We can find the upper bound on the

$$E_{\rm cm} \lesssim 2^{1/4} M^{1/4} \mu^{3/4},$$

where μ is the proper mass of each shell. (Kimura, Nakao and Tagoshi 2011)

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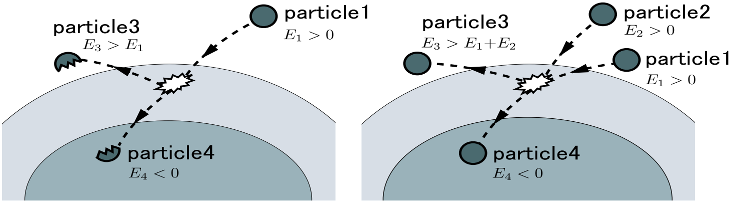
Collisional Penrose process



- The high energy collision may produce superheavy and/or superenergetic particles.
- The upper limit on the energy gain efficiency is 109.3 % for pair annihilation. Hence, the ejecta can be slightly more energetic at infinity than the incident particles. (Bejger et al. 2012, Harada, Nemoto and Miyamoto 2012)

Too low flux to be observed by a distant observer

- The escape fraction is so diminished that the Fermi satelite cannot detect the ejecta directly from the BSW collision (McWilliams 2013).

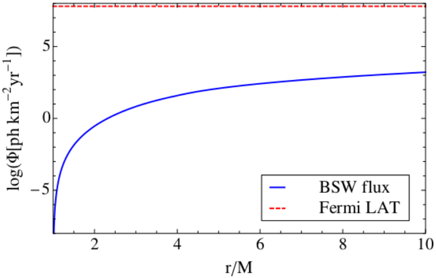


FIG. 1 (color online). Integrated flux Φ reaching an observer at $D_L = 10$ kpc from inside radius r (solid line), compared to the flux sensitivity of the Fermi LAT for a one year exposure (dashed line).

- The possible indirectly observable effects are discussed. (Gariel, Santos and Silk 2014, ...)

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Effects of astrophysical magnetic field

- Magnetic fields around astrophysical BHs
  - $B \sim 10^8$ Gauss for stellar mass BHs
  - $B \sim 10^4$ Gauss for supermassive BHs
- Negligible to the geometry

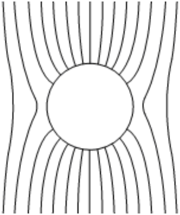
$$B \ll B_{\max} = \frac{c^4}{G^{3/2}M} \sim 10^{19} \left(\frac{M}{M_\odot}\right)^{-1} \text{Gauss}$$

- Crucial to the orbits of charged particles

$$b = \frac{qBGM}{mc^4} \sim 10^{11} \left(\frac{q}{e}\right) \left(\frac{m}{m_e}\right)^{-1} \left(\frac{B}{10^8 \text{Gauss}}\right) \left(\frac{M}{10 M_\odot}\right)$$

Magnetised BHs as particle accelerators

- We assume a BH immersed in a uniform magnetic field.



- For a nonrotating BH, the ISCO for a charged particle can be inside $3r_g$ and $E_{\rm cm} \simeq 1.74 b^{1/4} m_0$ (Frolov 2011).
- For a rotating BH, $E_{\rm cm}(b) \simeq \sqrt{b}/(3^{1/4}) E_{\rm cm}(b=0)$ (Igata, Harada and Kimura 2012).
- In short, astrophyical magnetic field substantially enhances the capability of the particle accelerators.

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High energy particle collision with bounded physical quantities

- It is interesting that the CM energy can be arbitrarily high even though the energy and angular momentum of each particle are finite in the original BSW scenario.
- This can happen in the non-extremal Kerr BHs.
  - Head-on collision near the BH horizon (Piran and Shaham 1977)
  - Near-critical particles inside the potential barrier (Grib and Pavlov 2011)
- No physical processes are known to give particles such special initial conditions.

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High energy particle collision in non-Kerr BHs

- Neutral particle accelerators
  - Kerr BHs (Bañados, Silk and West 2009, ...)
  - Kerr-Newmann family (Wei et al. 2010, Liu, Chen and Jing 2011)
  - Accelerating and rotating BHs (Yao et al. 2011)
  - Dirty BHs (Zaslavskii 2010, 2012)
  - Sen BHs (Wei et al. 2010)
- Charged particle accelerator
  - Reissner-Nordström BHs (Zaslavskii 2010)
  - General stationary charged BHs (Zhu et al. 2011)
- Higher-dimensions
  - Myers-Perry BHs (Abdujabbarov et al. 2013, Tsukamoto, Kimura and Harada 2014): fine-tuning of the angular momenta is still needed.
- Link with field instability on the extremal horizon (Aretakis 2011-2013, Murata 2013, Murata, Reall and Tanahashi 2013)

High energy particle collision in Myers-Perry BHs (1)

- Motivation
  - A test particle is a good probe into the geometry.
  - A link between the BSW process and the Aretakis's field instability
  - Test particles are the limiting objects of waves and other extended objects.
- Myers-Perry BH
  - Higher-dimensional counterpart of the Kerr BH
  - Spin parameter: $\{a_i\}_{i=1,\ldots,n}$, where $n = [D/2]$. If D is even, $a_n = 0$.
  - The geodesic equations in the MPBH are completely integrable.

High energy particle collision in Myers-Perry BHs (2)

- Geodesic particles in the MPBH
  - We assume $a_i = a$ $(i = 1, \ldots, q)$, $a_{j+q} = b$ $(j = 1, \ldots, p)$ $(p + q = n)$.
  - Geodesic particles: energy E, angular momenta $\{\Phi_i\}_{i=1,\ldots,q}$, $\{\Psi_j\}_{j=1,\ldots,p}$
  - Forward-in-time condition near the horizon imply

$$E - \frac{a}{r_+^2 + a^2}\sum_{i=1}^{q}\Phi_i - \frac{b}{r_+^2 + b^2}\sum_{j=1}^{p}\Psi_j \geq 0.$$

  - Critical particles are defined by the equality of the above.
- The CM energy for the two colliding particles:
  - For $r_+ > 0$, $E_{\rm cm}$ is formally divergent if either of the particles is critical so that the fine-tuning of the angular momenta is needed.
  - The critical particle can reach the horizon only if the BH is extremal.
  - Critical particles can reach the horizon for the case of equal spins.

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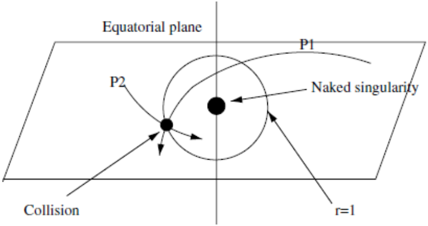
High energy particle collision in non-back hole spacetimes

- High energy particle collision also occurs in the deep gravitational potential well. Head-on collision is physically motivated if there is no horizon.
- Examples
  - Overspinning Kerr/Superspinar (Patil and Joshi 2011, Stuchlík and Schee 2012, 2013)
  - Bardeen magnetic monopoles (Patil and Joshi 2012)
  - Janis-Newman-Winicour singularities (Patil and Joshi 2012)
  - Overcharged Reissner-Nordström (Patil et al. 2012)



(Patil and Joshi 2011)

Summary

- The gravitational particle acceleration by nearly rotating BHs is founded on the basic properties of near-extremal Kerr BHs.
- The maximum achievable energy is subjected to several physical effects, such as finite acceleration time.
- The CM energy is in principle physically observable, although the ejecta from high energy collision will not be directly observed.
- The gravitational particle acceleration without a horizon is advantageous to observation, if there is an extremely deep potential well.