Poisson generalized geometry and R-flux

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based on arXiv:1408.2649 [hep-th]

Why generalized geometry?

弦理論に特有の対称性

T-双対性:

時空の計量と NS-NS B場(Kalb-Ramond場)は同列

-奇妙な計量の出現(T-foldなど)

-奇妙なフラックスの出現(Non-geometric flux)

➢これらを“幾何学”として取り扱う枠組み:

(Poisson) Generalized geometry

What is generalized geometry?

Generalized tangent bundle: $TM \oplus T^*M$

-切断: $v+\xi = v^i\partial_i + \xi_i dx^i$

-括弧積: Courant括弧, Dorfman括弧

$$[v+\xi, w+\eta]_D$$
$$= \mathcal{L}_v w + \mathcal{L}_v \eta - \iota_w d\xi$$
$$=: \mathcal{L}_{v+\xi}(w+\eta)$$ ･･･一般化されたLie微分

-対称性: 座標変換+B場のゲージ変換

-*O(D,D)*不変な内積: $\langle u+\xi, v+\eta \rangle = \frac{1}{2}(u^i\eta_i + v^i\xi_i)$

O(D,D) 変換 $\supset$ T-双対変換

Outline

- Introduction (2)
- Generalized geometry

  -物理との関係 (9)

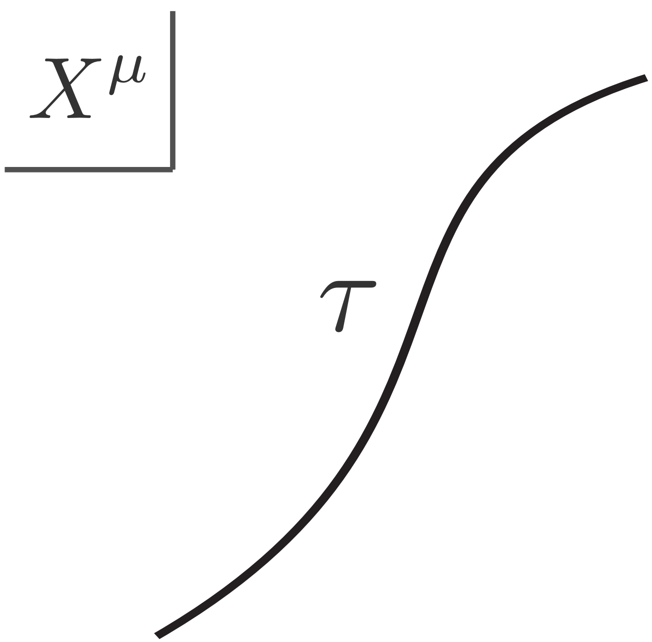
  -定義 & 性質 (2)

  -応用 (1)
- Non-geometricフラックス (4)
- Poisson generalized geometry (4)
- Conclusion and discussion (2)

点粒子の作用

世界線の長さ:

$$S_{PP} = -m \int d\tau \sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu}$$



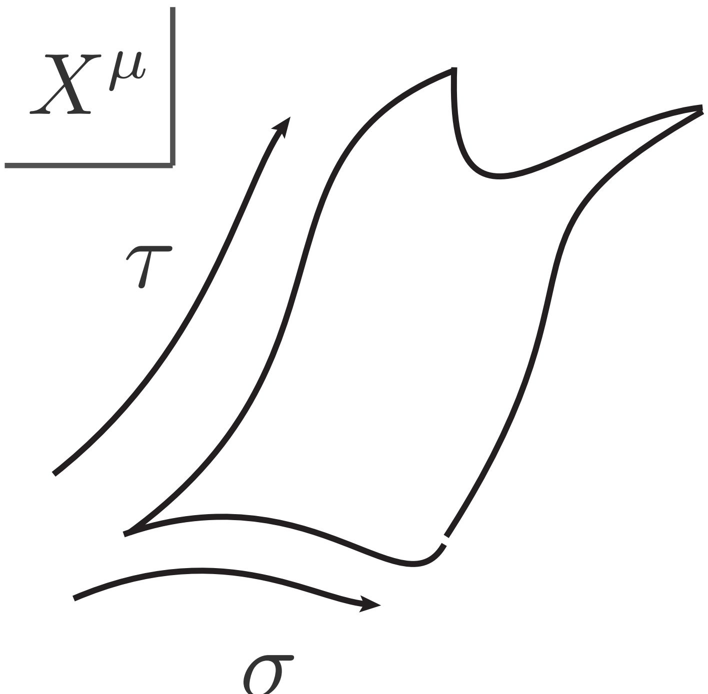
m 点粒子の質量

世界線:
時空 $g_{\mu\nu}$ の中を運動する
点粒子が描く軌跡

弦の作用 (南部・後藤作用)

世界面の広さ:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det(g_{\mu\nu}(X)\partial_i X^\mu \partial_j X^\nu)}$$



$\frac{1}{2\pi\alpha'}$ 弦の質量密度(張力)

世界面:
時空 $g_{\mu\nu}$ の中を運動する
弦が描く軌跡

弦の作用 (Polyakov作用)

世界面の広さ:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det(g_{\mu\nu}(X)\partial_i X^\mu \partial_j X^\nu)}$$

等価

⟺ $$S_P = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \sqrt{-h}h^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X)$$

解析しやすい：*e.g.* Minkowski時空のとき量子化可能

⟹ 弦の物理的状態スペクトル

質量ゼロモード（閉弦）

$g_{\mu\nu}$ 重力場　　$B_{\mu\nu}$ NS-NS *B*場　　ϕ ディラトン場

もっと一般的な弦の作用（非線形シグマ模型）

重力場 $g_{\mu\nu}$ + B場 $B_{\mu\nu}$ 中の弦

$$S_P = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma [\sqrt{-h}h^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X) + \\ + \underline{\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X)]}$$

c.f. 重力場 $g_{\mu\nu}$ + 電磁場 A_μ 中の荷電粒子

$$S_{PP} = -m\int d\tau \sqrt{-g_{\mu\nu}(X)\dot{X}^\mu \dot{X}^\nu} + e\int \underline{d\tau \dot{X}^\mu A_\mu(X)}$$

作用の対称性

一般座標変換

$$X^\mu \to X'^\mu = X'^\mu(X)$$

$$g_{\mu\nu} \to g'_{\mu\nu} = \frac{\partial X^\alpha}{\partial X'^\mu}\frac{\partial X^\beta}{\partial X'^\nu} g_{\alpha\beta}; \;\; B_{\mu\nu} \to B'_{\mu\nu} = \frac{\partial X^\alpha}{\partial X'^\mu}\frac{\partial X^\beta}{\partial X'^\nu} B_{\alpha\beta}$$

B場ゲージ変換

$$B_{\mu\nu} \to B'_{\mu\nu} = B_{\mu\nu} + \underline{(\partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu)} \qquad B \to B' = B + d\Lambda$$

c.f. $F = dA = d(A + d\lambda) = dA'$

$$\delta S_P = \frac{1}{2\pi\alpha'} \int d\sigma^2 \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu \partial_\nu \Lambda_\mu = \int d\sigma^2 \partial_b [\epsilon^{ab} \partial_a X^\mu \Lambda_\mu]$$

Current代数とDorfman括弧 [Alekseev, Strobl]

対称性に付随するカレント

$$\mathcal{J}_{(\xi,\Lambda)}(\sigma) = \xi^\mu(X)P_\mu(\sigma) + \Lambda_\mu(X)\partial X^\mu(\sigma)$$

Poisson括弧 $\{X^\mu(\sigma), P_\nu(\sigma')\}_{PB} = \delta^\mu_\nu \delta(\sigma - \sigma')$

カレントのなす代数

$$\{\mathcal{J}_{(u,\alpha)}(\sigma), \mathcal{J}_{(v,\beta)}(\tau)\}$$

$$= -\mathcal{J}_{[(u,\alpha),(v,\beta)]}(\sigma) + (i_u\beta + i_v\alpha)(\tau)\partial_\sigma\delta(\sigma - \tau)$$

ただし $[(u,\alpha),(v,\beta)] = ([u,v], \mathcal{L}_u\beta - \mathcal{L}_v\alpha + i_v d\alpha)$

・・・Dorfman括弧

T-双対性

$g_{\mu\nu}\ B_{\mu\nu}$ が $X^0 \sim X^0 + 2\pi R$（周期的, コンパクト）によらないとき

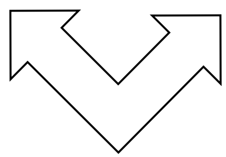
KK運動量

$$\frac{K}{R} = \frac{1}{2\pi}\int d\sigma P_0(\sigma)$$

巻き付き数

$$WR = \frac{1}{2\pi}\int d\sigma \partial X^0(\sigma)$$

$$\partial_\tau$$



$$\partial_\sigma$$

$$R \to \frac{\alpha'}{R}$$

入れ替えても物理は同じ:
T-双対性

Buscher則とT双対性

[Buscher]

$g_{\mu\nu}$ $B_{\mu\nu}$ が X^0 によらないとき

$$S' = \frac{1}{4\pi\alpha'}\int d^2\sigma[\sqrt{-h}h^{ab}g_{00}V_aV_b + 2\sqrt{-h}h^{ab}g_{0i}V_a\partial_bX^i + \sqrt{-h}h^{ab}g_{ij}\partial_aX^i\partial_bX^j$$
$$+ 2\epsilon^{ab}B_{0i}V_a\partial_bX^i + \epsilon^{ab}B_{ij}\partial_aX^i\partial_bX^j + 2\epsilon^{ab}\hat{X}^0\partial_aV_b]$$

⇓ 補助場 $\hat{X}^0$ の拘束を解く

⇓ 場 V_a について変分をとる

非線形シグマ模型

g_{00}, g_{0i}, g_{ij},
B_{0i}, B_{ij}

⇔ T-双対性

非線形シグマ模型

$$\tilde{g}_{00} = \frac{1}{g_{00}}, \quad \tilde{g}_{0i} = \frac{B_{0i}}{g_{00}}, \quad \tilde{g}_{ij} = g_{ij} - \frac{g_{0i}g_{0j} - B_{0i}B_{0j}}{g_{00}},$$
$$\tilde{B}_{0i} = \frac{g_{0i}}{g_{00}}, \quad \tilde{B}_{ij} = B_{ij} - \frac{g_{0i}B_{0j} - g_{0j}B_{0i}}{g_{00}}$$

Buscher則(もう少し特殊な場合)とO(D,D)変換対称性

[Duff]

$g_{\mu\nu}$ $B_{\mu\nu}$ が X^μ によらないとき

$$S'' = \frac{1}{4\pi\alpha'}\int d^2\sigma[\sqrt{-h}h^{ab}g_{\mu\nu}V_a^\mu V_b^\nu + \epsilon^{ab}B_{\mu\nu}V_a^\mu V_b^\nu + 2\epsilon^{ab}\partial_a\hat{X}_\mu V_b^\mu]$$

⇩ 補助場 $\hat{X}^\mu$ の拘束を解く

非線形シグマ模型 X^μ

T-双対性 ⇔

⇩ 場 V_a について変分をとる

非線形シグマ模型 $\hat{X}^\mu$

$$Z^M = \begin{pmatrix} X^\mu \\ \hat{X}_\mu \end{pmatrix}$$

⇨ 場同士の関係 ～ $\partial_\tau \leftrightarrow \partial_\sigma$

$$\epsilon^{ab}\Omega_{MN}\partial_b Z^N = \sqrt{-h}h^{ab}G_{MN}\partial_b Z^N$$

$$\Omega_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad G_{MN} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix}$$

O(D,D)変換

$$S^T\Omega S = \Omega$$

⟶ $Z' = S^{-1}Z$ ⇨ $G' = S^T G S$

Ordinary Geometry

- Tangent Bundle
  TM
- Section: vector field
  $v = v^i \partial_i$
- Lie括弧(反対称○,ヤコビ○)
  $[v, w] = \mathcal{L}_v w$
- Symmetry:
  -Diffeomorphism

Generalized Geometry

['03 Hitchin]

- Generalized Tangent Bundle
  $E = TM \oplus T^*M$
- Section: vector field + 1-form
  $v + \xi = v^i \partial_i + \xi_i dx^i$
- Dorfman括弧(反対称✕,ヤコビ○)
  $[v + \xi, w + \eta]_D$
  $= \mathcal{L}_v w + \mathcal{L}_v \eta - \iota_w d\xi$
  $=: \mathcal{L}_{v+\xi}(w + \eta)$
- Symmetry:
  -Diffeomorphism
  -Gauge transf. of B-field

- Courant括弧(反対称⭕,ヤコビ❌)

$$[u+\xi, v+\eta]_C = [u,v] + \mathcal{L}_u\eta - \mathcal{L}_v\xi - \frac{1}{2}d(i_u\eta - i_v\xi)$$

- $O(D,D)$ invariant canonical inner Product

$$\langle u+\xi, v+\eta\rangle = \frac{1}{2}(i_u\eta + i_v\xi) = \frac{1}{2}\begin{pmatrix} u \\ \xi \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} v \\ \eta \end{pmatrix}$$

O(D,D)変換

$$S^T\Omega S = \Omega$$

無限小変換

$$S = 1 + X = 1 + \begin{pmatrix} A & \beta \\ B & \alpha \end{pmatrix}$$

$$\Rightarrow \alpha = -A^T,\ \beta^T = -\beta,\ B^T = -B$$

座標変換

$$\begin{pmatrix} e^A & 0 \\ 0 & e^{-A^T} \end{pmatrix}$$

B変換

$$e^B := \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix}$$

β変換

$$e^\beta := \begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix}$$

- B-transformation and H-flux $H = dB$

$$[e^B(u+\xi), e^B(v+\eta)]_C = [u+\xi+B(u), v+\eta+B(v)]_C$$
$$= e^B([u+\xi, v+\eta]_C) + i_v i_u dB$$

$dB = 0$ => Courant括弧について準同型

$dB \neq 0$ => Courant括弧の再定義(Twisted br.)で吸収可能

- β -transformation

$$[e^\beta(u+\xi), e^\beta(v+\eta)]_C = [u+\xi+\beta(\xi), v+\eta+\beta(\eta)]_C$$
$$\neq e^\beta([u+\xi, v+\eta]_C)$$

β の性質だけでは括弧を閉じさせられない

Generalized Riemannian geometry [Baraglia, etc.]

O(D,D) => Positive definite ⨁Negative definite

$$C_\pm = \{X(\pm g + B)(X) \mid X \in TM\}$$

- C_+ 上に接続が定義できる: $\nabla_X u = \pi_+([X^-, u]_C)$

ただし $X^- = X + (-g+B)(X) \in C_-$, $u \in C_+$, $\pi_+ : TM \oplus T^*M \to C_+$

i.e. Leibniz則 $\nabla_{fX}(gu) = fg\nabla_X u + f(\mathcal{L}_X g)u$

-曲率: $R(X,Y)u := (\nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]_C})u$

i.e. Tensor性 $R(fX, gY)hu = fghR(X,Y)u$

-Ricci scalar: $R - \frac{1}{4}H^{ijk}H_{ijk}$ ・・・TorsionとしてH-fluxが入る

T-dualities and non-geometric fluxes [Kaloper,...]

T-双対変換のフラックスへの作用:

$$H_{abc} \longrightarrow f^c_{ab} \longrightarrow Q^{bc}_a \longrightarrow R^{abc}$$

ここで各フラックスはKaloper-Myers代数

$$[e_a, e_b] = f^c_{ab} e_c + H_{abc} e^c,$$
$$[e_a, e^b] = Q^{bc}_a e_c + f^b_{ac} e^c,$$
$$[e^a, e^b] = R^{abc} e_c + Q^{ab}_c e^c$$

e_a : Vector field

e^a : 1-form

に現れる係数, *e.g.*

Lie代数 $[X_a, X_b] = f^c_{ab} X_c$

Example

一様な *H*-flux 背景下のトーラス T^3 $x \sim x + 1$ *etc.*

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad B = kx dy \wedge dz$$

y-方向とz-方向に計量と*B*場は依存してない

y-方向に沿って
T-双対変換

$\Longrightarrow$

Buscher則

$$ds^2 = dx^2 + dz^2 + \underline{(dy + kx dz)^2} \quad B = 0$$

周期性がツイストされる：Twisted torus

Geometric interpretation of f-flux

$$ds^2 = dx^2 + dz^2 + \underline{(dy + kxdz)^2} \qquad B = 0$$

周期性がツイストされる: Twisted torus

$$\eta^1 = dx \quad \eta^2 = dy + kxdz \quad \eta^3 = dz$$

$$\eta_1 = \partial_x \quad \eta_2 = \partial_y \quad \eta_3 = \partial_z - kx\partial_y$$

$$\eta^i(\eta_j) = \delta^i_j$$

f-flux ⇒

$$[\eta_1, \eta_3] = [\partial_x, \partial_z - kx\partial_y] = -k\partial_y = \underline{f^2_{13}\eta_2}$$

$$[\eta^2, \eta_1] = [dy + kxdz, \partial_x] = -kdz = \underline{f^2_{13}\eta^3}$$

$$[\eta^2, \eta_3] = \underline{-f^2_{13}\eta^1}$$

Non-geometric flux

[Hull, etc.]

$$ds^2 = dx^2 + dz^2 + (dy + kxdz)^2 \qquad B = 0$$

z-方向に計量とB場は依存してない

z-方向に沿って
T-双対変換
⇒
Buscher則

$$ds^2 = dx^2 + \frac{1}{1+k^2x^2}(dy^2 + dz^2) \qquad B = \frac{-kxdy \wedge dz}{1+k^2x^2}$$

$x \to x + 1$ の貼り合わせに 計量とB場の混合が必要

manifold: 座標変換で貼り合う ⇒ T-fold

付随するフラックス　Q-flux　$Q_1^{23} \sim k$

R-flux?

T-双対変換 ~ Vector ⇔ 1-form

$$H_{123} \xrightarrow{\mathrm{T}_y} f^{2}_{13} \xrightarrow{\mathrm{T}_z} Q^{23}_{1} \xrightarrow[?]{\mathrm{T}_x} R^{123}$$

Tensor Type	3-form (0,3)	(1,2)	(2,1)	Tri-vector? (3,0)

Geometrical meaning is UNCLEAR!

⇨ Analogue of Gen. Geom. but Vector ⇔ 1-form can describe R ?

Poisson geometry

Lie algebroid $(T^*M,\, \theta,\, [\cdot,\cdot]_\theta)$

-section: 1-from $\xi = \xi_i dx^i$

-anchor map: $\theta : T^*M \to TM,\ \xi \mapsto \theta(\xi) = \xi_i \theta^{ij} \partial_j$

-Lie bracket: $[\xi, \eta]_\theta = \mathcal{L}_{\theta(\xi)} \eta - i_{\theta(\eta)} d\xi$:Koszul bracket

Poisson bi-vector $\theta = \frac{1}{2} \theta^{ij} \partial_i \wedge \partial_j$

-Poisson bracket $\{f, g\}_{PB} = \theta(df, dg) = \theta^{ij} \partial_i f \partial_j g$

-Jacobi identity $\{\{f,g\},h\} + \{\{g,h\},f\} + \{\{h,f\},g\} = 0$

$\Longleftrightarrow [\theta, \theta]_S = 0$

Schouten bracket: *e.g.* $[X \wedge Y, Z]_S = [X, Z] \wedge Y - [Y, Z] \wedge X$

Extension of Lie br. to multi-vector

Cartan algebra in Poisson geometry

``Exterior derivative” $d_\theta = [\theta, \cdot]_S$ Schouten bracket: Extension of Lie br. to multi-vector

-Nilpotency $d_\theta^2 = 0 \Longleftarrow [\theta, \theta]_S = 0$

``Lie derivative”

$$\mathcal{L}_\zeta f := i_\zeta d_\theta f$$

$$\mathcal{L}_\zeta \xi := [\zeta, \xi]_\theta$$

$$\mathcal{L}_\zeta X := (d_\theta i_\zeta + i_\zeta d_\theta) X$$

``Cartan algebra”

$$\{i_\xi, i_\eta\} = 0, \quad \{d_\theta, i_\xi\} = \mathcal{L}_\xi, \quad [\mathcal{L}_\xi, i_\eta] = i_{[\xi,\eta]_\theta},$$

$$[\mathcal{L}_\xi, \mathcal{L}_\eta] = \mathcal{L}_{[\xi,\eta]_\theta}, \quad [d_\theta, \mathcal{L}_\xi] = 0.$$

Poisson generalized geometry [ASMW]

Analogue of generalized geometry based on Poisson geom.
$(T^*M,\, \theta,\, [\cdot,\cdot]_\theta)$

-Same as a vector bundle $T^*M \oplus TM$

-New Courant bracket

$$[X+\xi, Y+\eta] = [\xi,\eta]_\theta + \mathcal{L}_\xi Y - \mathcal{L}_\eta X + \frac{1}{2} d_\theta (i_X \eta - i_Y \xi)$$

c.f. $[u+\xi, v+\eta]_C = [u,v] + \mathcal{L}_u \eta - \mathcal{L}_v \xi - \frac{1}{2} d(i_u \eta - i_v \xi)$

Vector field $\Longleftrightarrow$ 1-form

-Same *O(D,D)*-inv. inner product

Symmetry: β-diffeo. + β-transf. [Andriot, etc.]

β-diffeo. $e^{\mathcal{L}_\xi}$: generated by 1-form ξ

$$[\mathcal{L}_\xi(X+\xi), Y+\eta] + [(X+\xi), \mathcal{L}_\xi Y+\eta] = \mathcal{L}_\xi[(X+\xi), Y+\eta]$$

: Leibniz rule

β-transf. e^{β}

$$[e^\beta(X+\xi), e^\beta(Y+\eta)] = e^\beta([X+\xi, Y+\eta]) + [\theta,\beta]_S(\xi,\eta)$$

R-flux $\boxed{R = d_\theta\beta = [\theta,\beta]_S}$

c.f. *H*-flux $H = dB$

$$[e^B(u+\xi), e^B(v+\eta)]_C = e^B([u+\xi, v+\eta]_C) + i_v i_u dB$$

Conclusion

新たなCourant括弧を見出し，性質を調べた

$$[X+\xi, Y+\eta] = [\xi,\eta]_\theta + \mathcal{L}_\xi Y - \mathcal{L}_\eta X + \frac{1}{2} d_\theta (i_X \eta - i_Y \xi)$$

対称性: β-diffeo. + β-transf. ⇨ $R = d_\theta \beta$

R-フラックスの大域的にwell-definedな定義を与えた

Discussions

- ✓ 他のフラックスとの関係, T-双対？
- ✓ 新しいCourant括弧の起源
- ✓ Gen. geom. との関係？
- ✓ WZW-like modelや超重力理論での実現?
  - -*R*-flux chargeの量子化?

We can do almost the same things done in ge. ge.!

じぇじぇ!?

ありがとうございました

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