Dynamical generation of fermion mass hierarchy in an extra dimension

Makoto Sakamoto
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References and collaborators

- Phase Structure of Gauge Theories on an Interval,
  Y. Fujimoto, T. Nagasawa, S. Ohya, M.S.,
  PTP 126 (2011) 841

- Quark mass hierarchy and mixing via geometry of extra dimension with point interactions,
  Y. Fujimoto, T. Nagasawa, K. Nishiwaki, M.S.,
  PTEP 023B07 (2013)

- CP phase from twisted Higgs vacuum expectation value in extra dimension,
  Y. Fujimoto, K. Nishiwaki, M.S.,
  Phys. Rev. D88 (2013)115007

- Realization of lepton masses and mixing angles from point interactions in an extra dimension,
  Y. Fujimoto, K. Nishiwaki, M.S., R. Takahashi,
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- Dynamical generation of fermion mass hierarchy in an extra dimension,
  Y. Fujimoto, T. Miura, M.S.,
  work in progress

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Purpose of my talk

We would like to show that

the quark & lepton flavor structure is naturally generated from extra dimensions.

# of generation

mass hierarchy

flavor mixing

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We would like to show that

the quark & lepton flavor structure is naturally generated from extra dimensions.

# of generation

closely related

mass hierarchy

flavor mixing

from an extra dimensional point of view

Plan of my talk

- ❑ Motivation to considering extra dimensions
- ❑ Mysteries of the Standard Model
- ❑ General features of extra dimensions
- ❑ Setup
- ❑ Point interactions
- ❑ Dynamical generation of fermion mass hierarchy
- ❑ Summary

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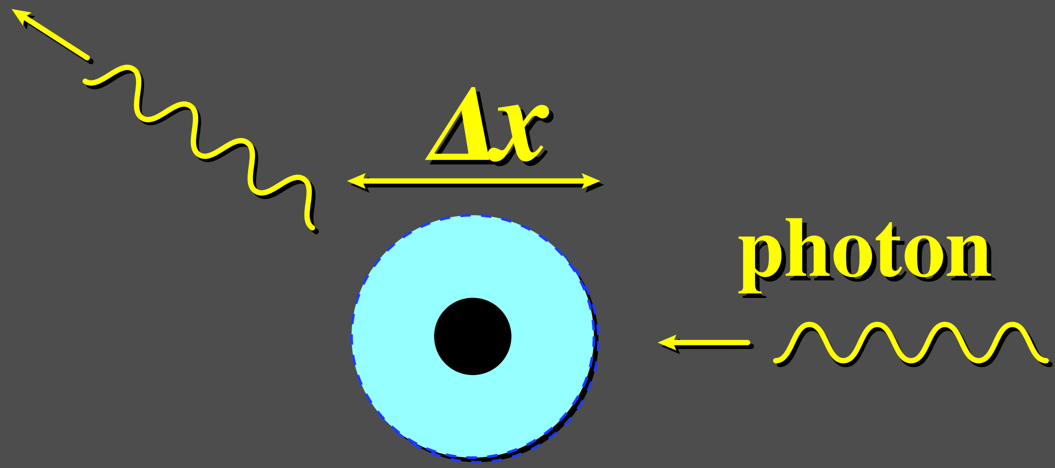
Minimum length



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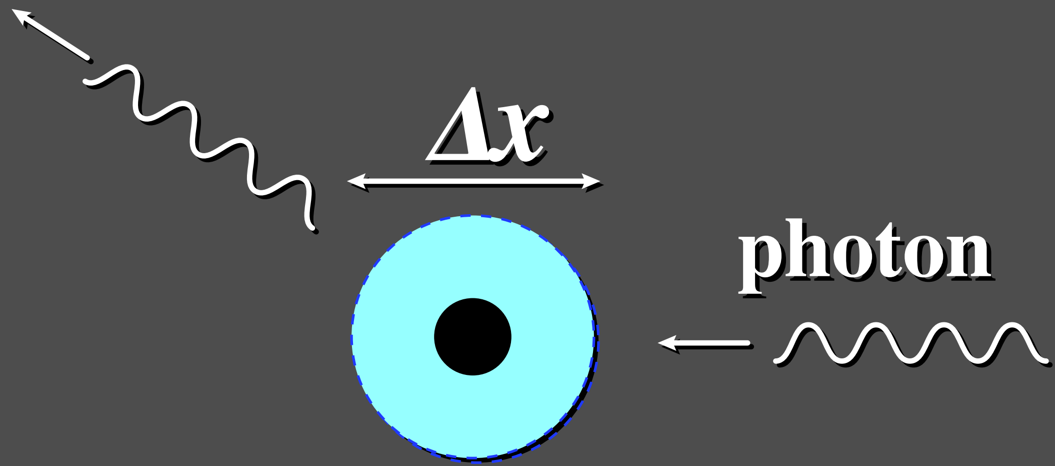
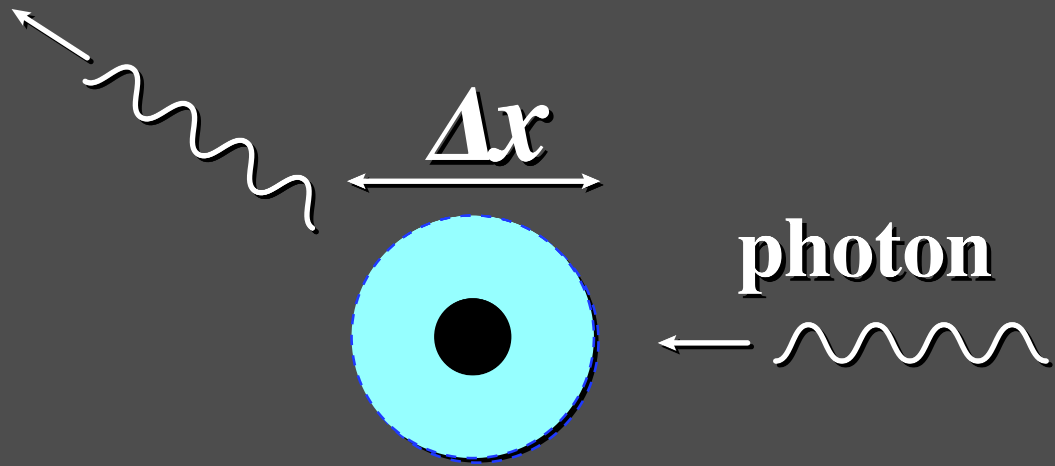


$$\Delta x \gtrsim \frac{\hbar}{\Delta p}$$

quamtum effect

Minimum length

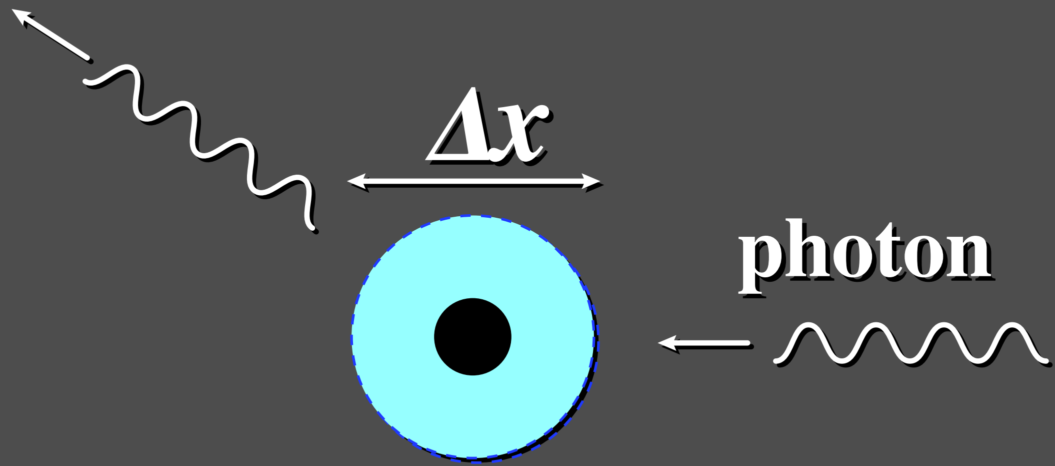


$$\Delta x \gtrsim \frac{\hbar}{\Delta p} \oplus \frac{G}{c^3}\Delta p$$

quamtum effect

gravity effect

Minimum length



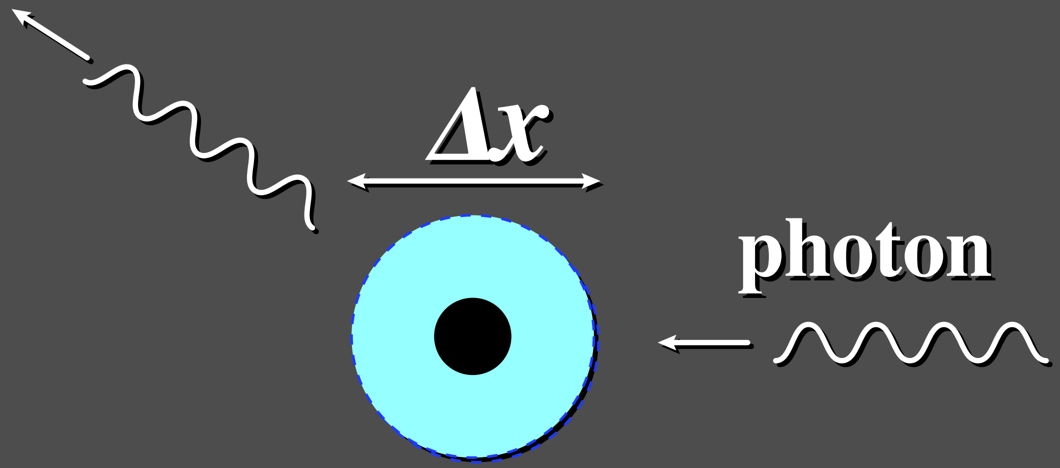
planck length

$$\Delta x \gtrsim \frac{\hbar}{\Delta p} \oplus \frac{G}{c^3}\Delta p \geq \sqrt{\frac{G\hbar}{c^3}} \equiv l_P$$

quamtum effect

gravity effect

Minimum length



planck length

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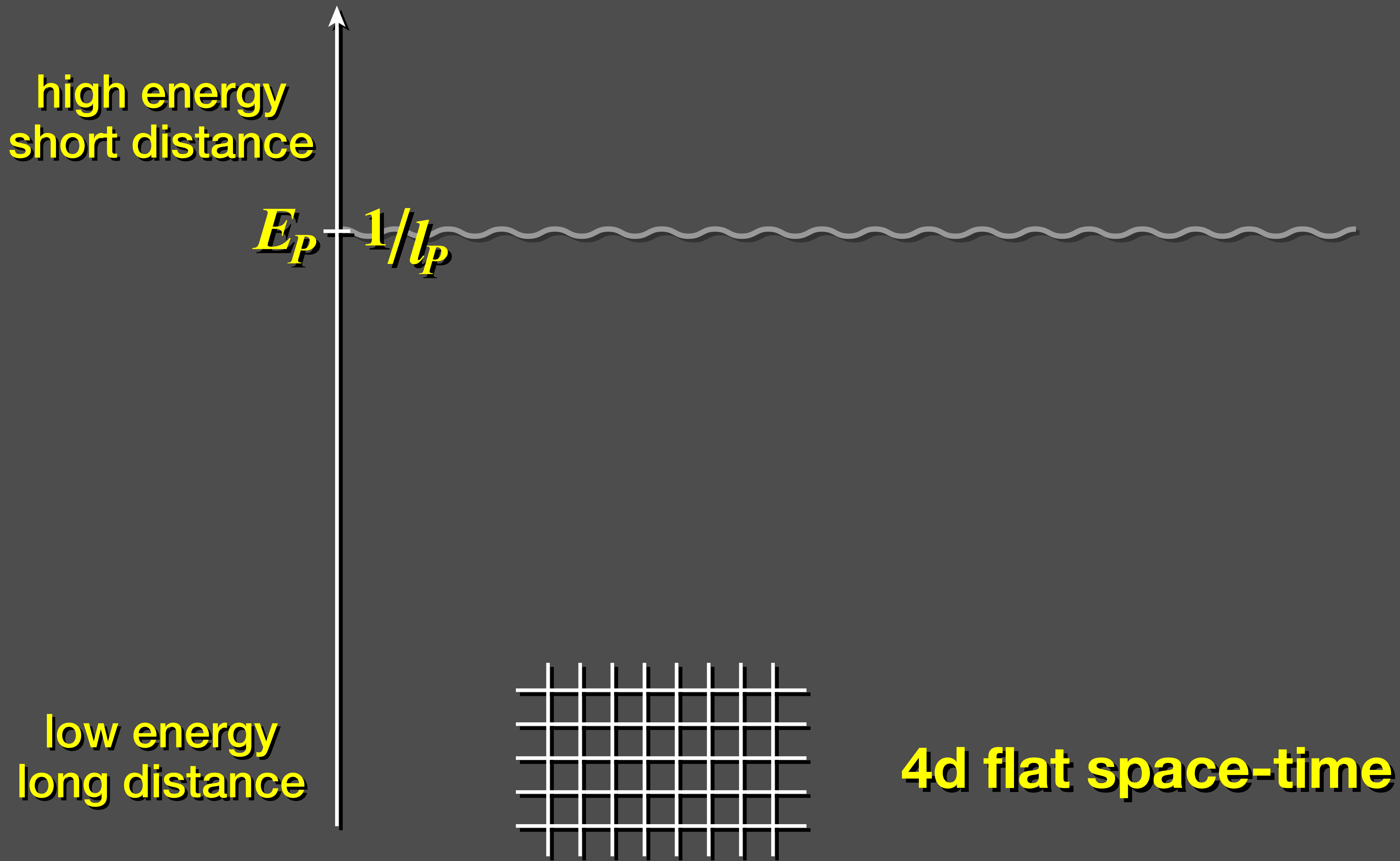
quamtum effect

gravity effect

quantum mechanics $\oplus$ gravity $\longrightarrow$ $\boxed{\text{No space-time less than } l_P}$

Disappearance of space-time



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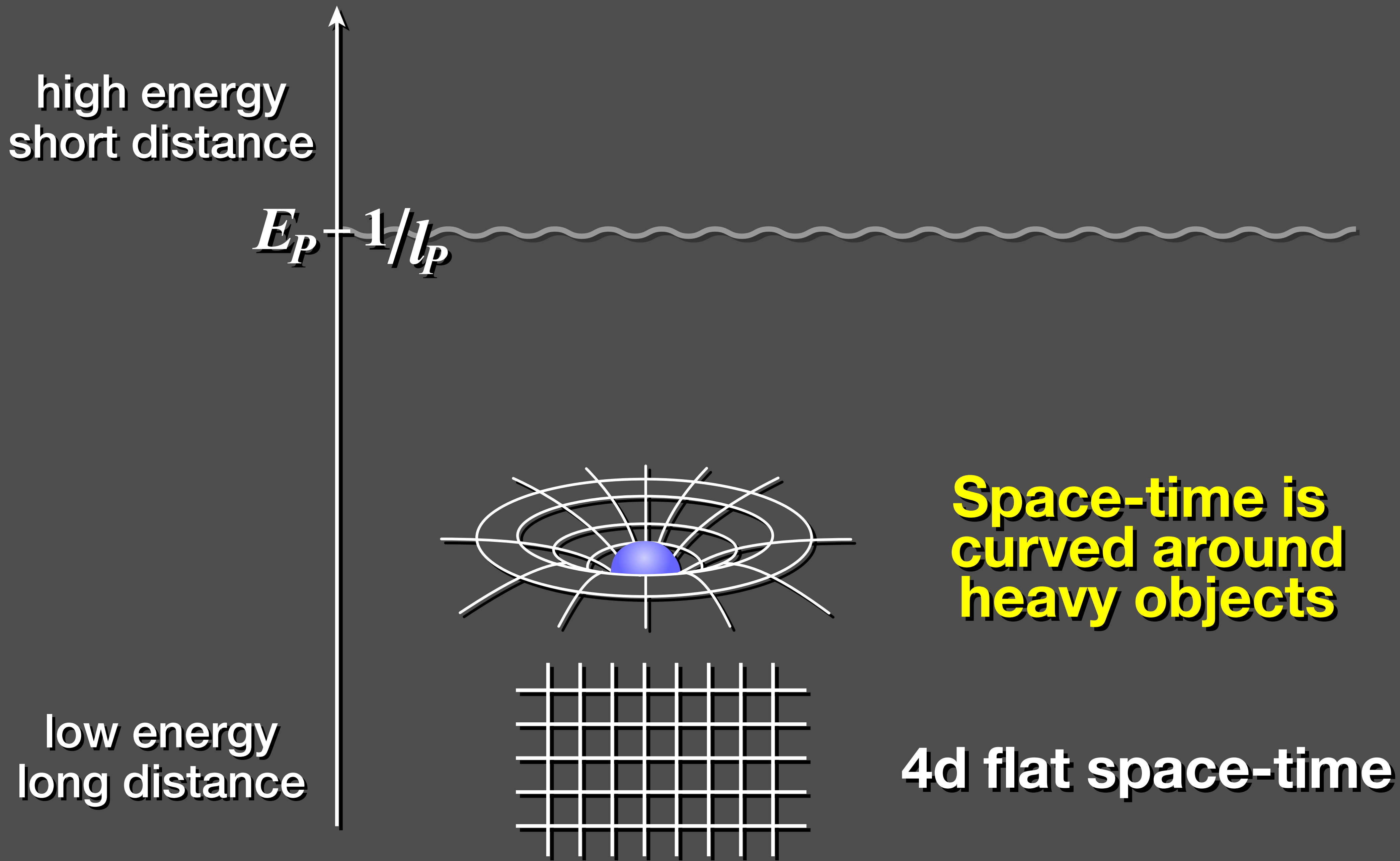

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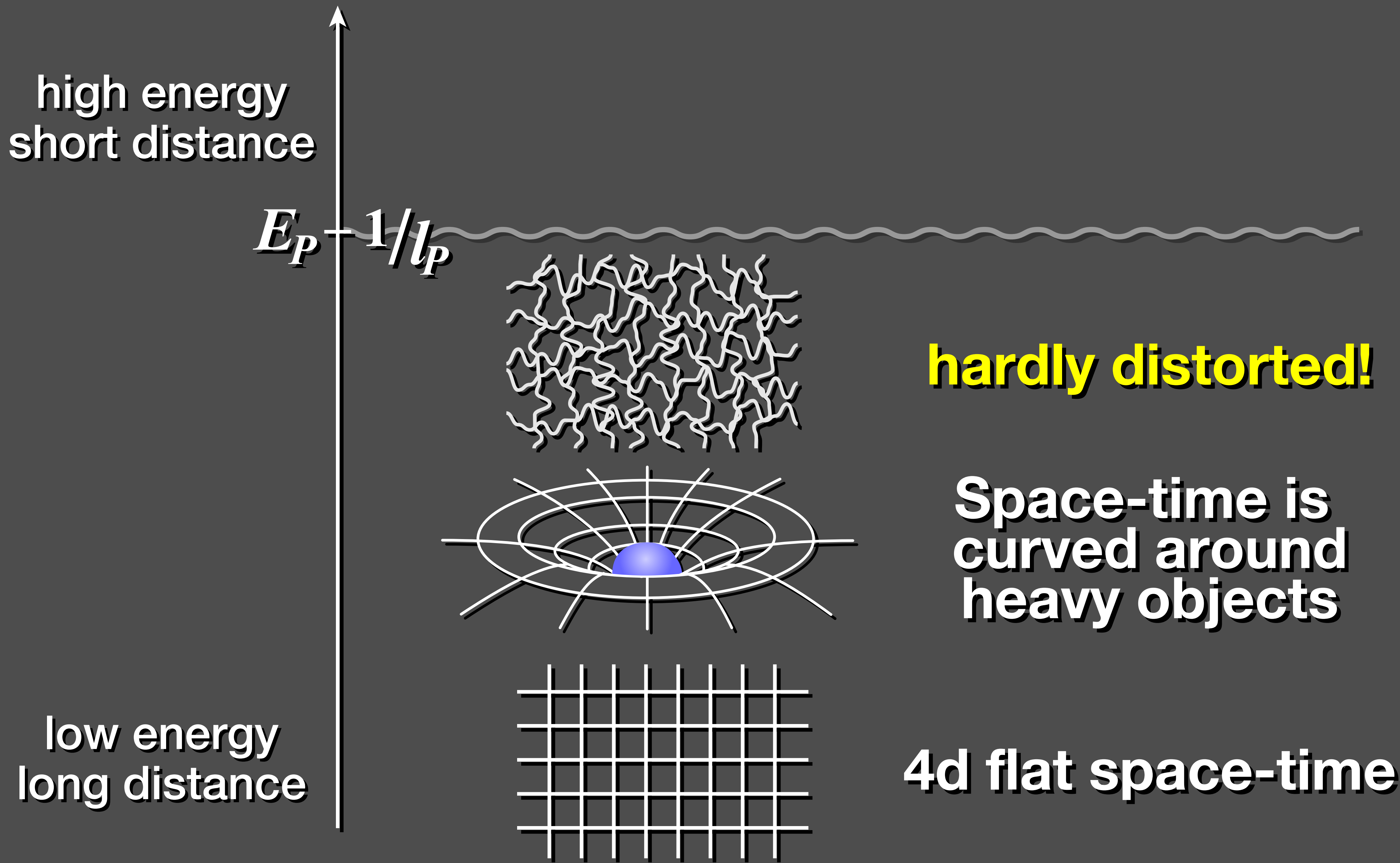
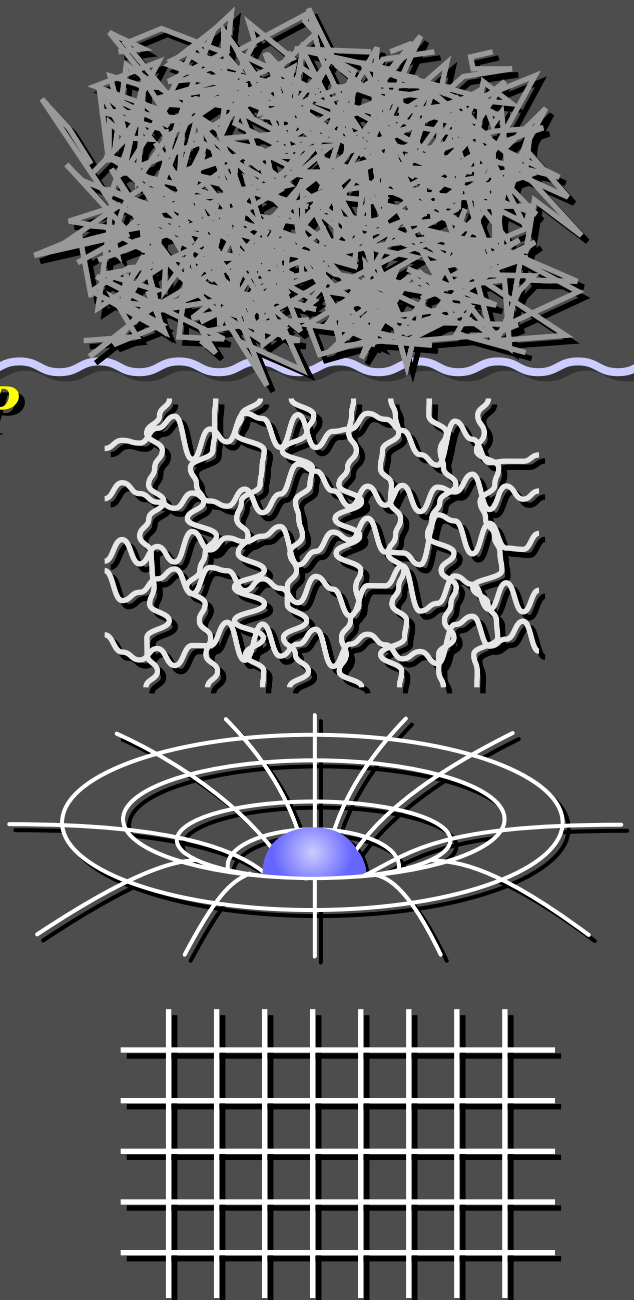


Disappearance of space-time

high energy
short distance

E_P — $1/l_P$



No space-time

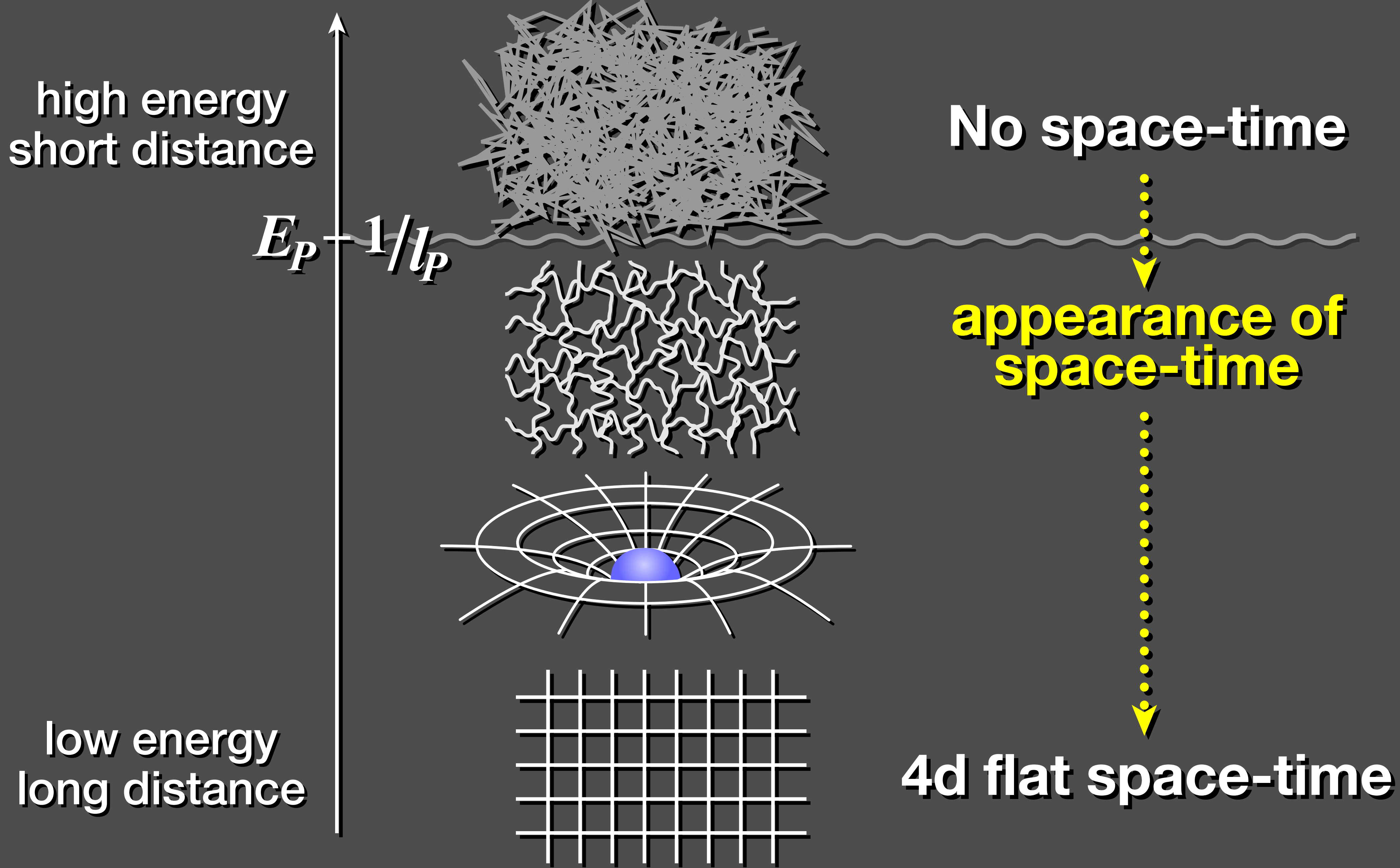
hardly distorted!

Space-time is curved around heavy objects

4d flat space-time

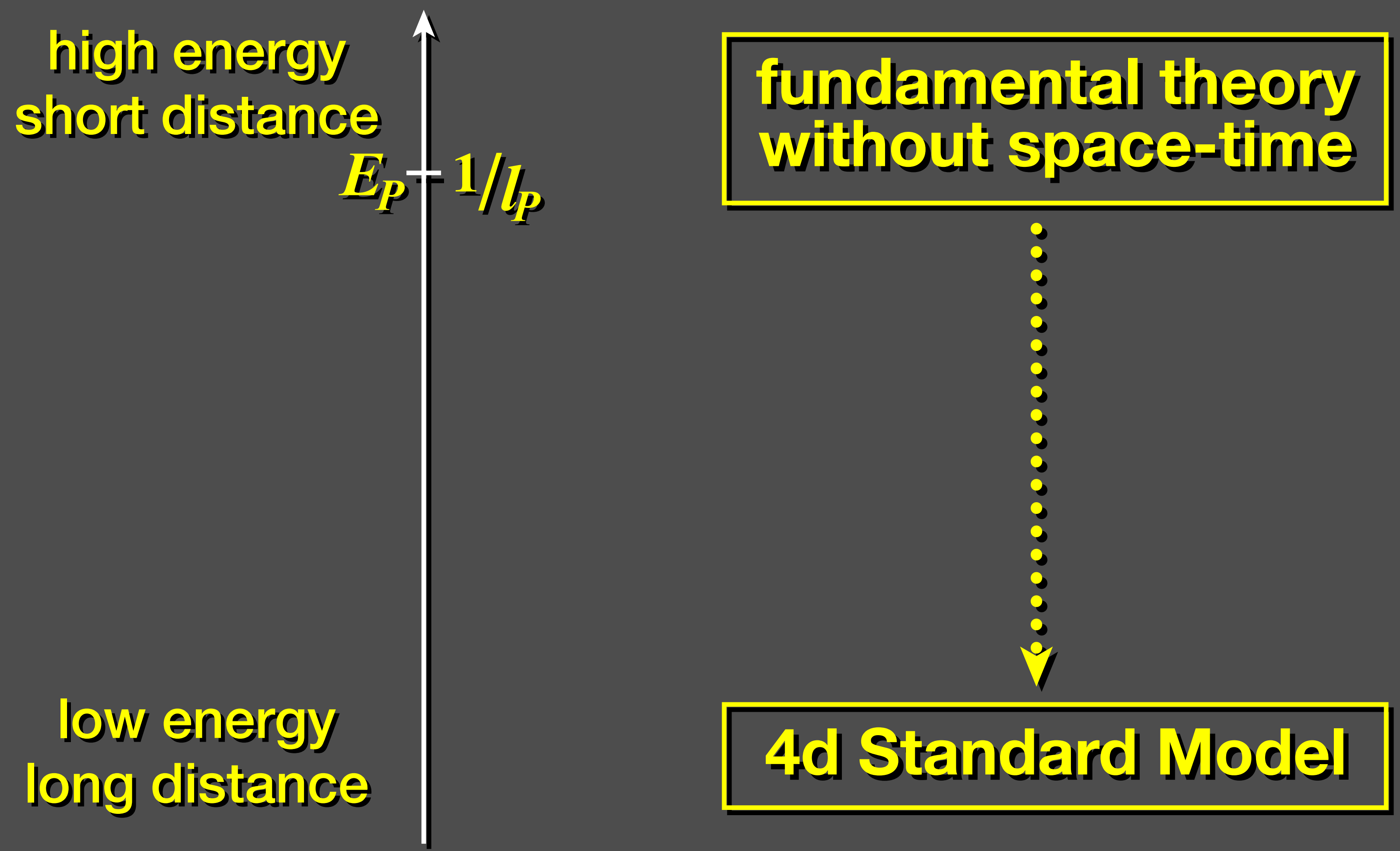
low energy
long distance

Disappearance of space-time



Implications of the minimum length

- The Standard Model should be regarded as a low energy effective theory.



Implications of the minimum length

- There is no distinction between scalar, spinor, vector and tensor at high energies because these cannot be defined without space-time.

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Supersymmetry

$$\Phi = (\,\phi\,,\,\psi\,)$$

$$W = (\,\lambda\,,\,A_\mu\,)$$

unifies fields whose spins differ by 1/2

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unifies fields whose spins differ by 1/2

Kaluza-Klein theory

$$A_M = (\, A_\mu \,,\, \phi \,)$$

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & A_\mu \\ A_\mu & \phi \end{pmatrix}$$

unifies fields whose spins differ by 1

Implications of the minimum length

- There is no distinction between space-time and matter at high energies because of vanishing space-tme.

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There is no reason to persist in 4-dimensions because extra-dimensions can be converted into matter, and vice versa.

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- There is no distinction between space-time and matter at high energies because of vanishing space-tme.

"degrees" of space-time $\xleftrightarrow{\text{convert}}$ "degrees" of matter

$\Downarrow$

There is no reason to persist in 4-dimensions because extra-dimensions can be converted into matter, and vice versa.

a 5d field on $M^4 \times S^1$

$$\Phi(x,y) = \sum_{n=-\infty}^{\infty} \phi_n(x) f_n(y) \qquad f_n(y) = \frac{1}{\sqrt{2\pi R}} e^{iy/R}$$

This Fourier expansion may tell us that *the degrees of S^1 can be converted into infinitely many 4d fields $\phi_n(x)$*.

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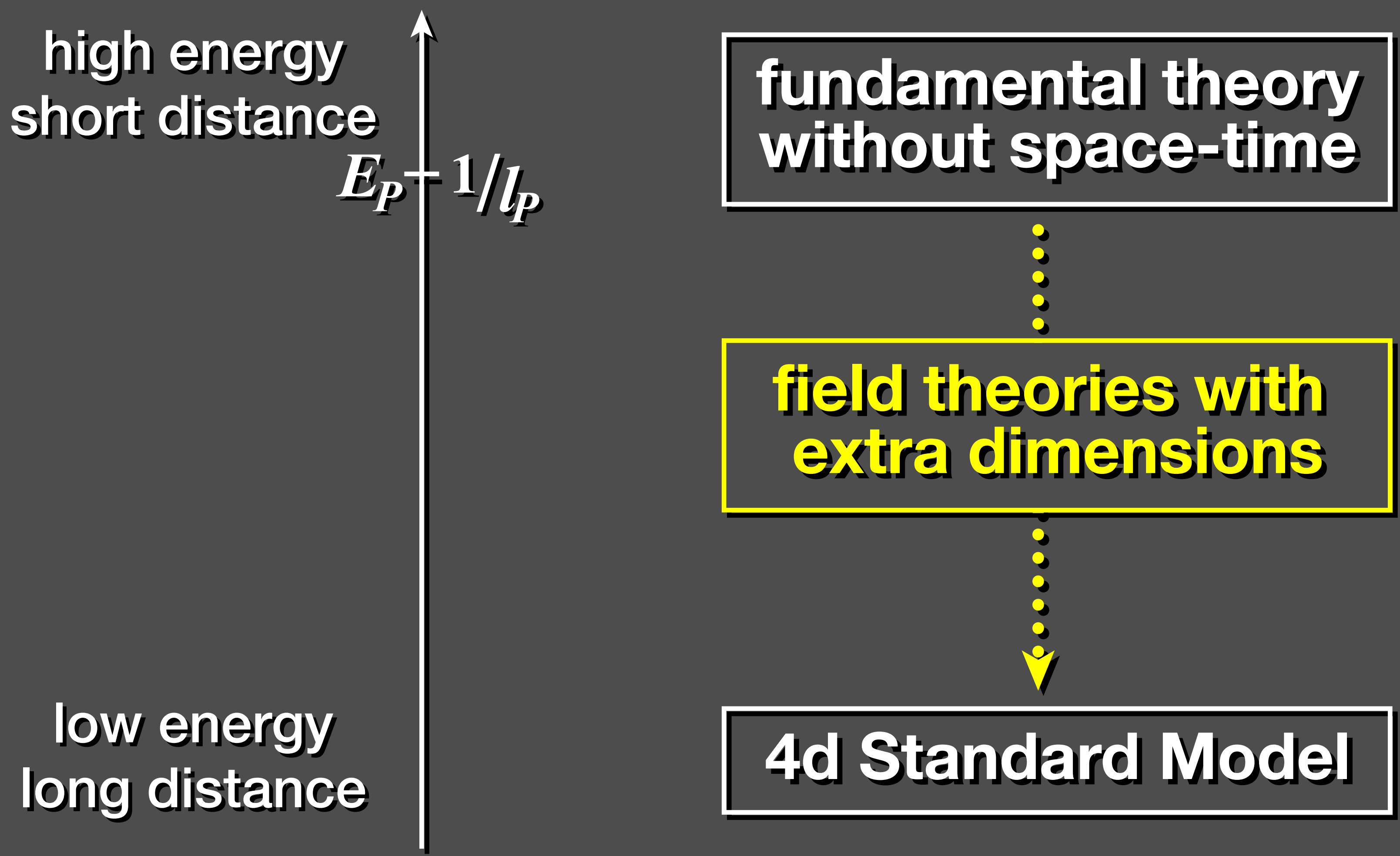

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Mystery of gauge group

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Why isn’t the SM gauge group $SU(1000000)$ but $SU(3)\times SU(2)\times U(1)$?

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I have the impression that small gauge groups are chozen !?

Mystery of matter

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Why are the matter representations chosen such that

	$SU(3)_C$ singlet	$SU(3)_C$ triplet
$SU(2)_W$ singlet	$e_R\,,\ (\nu_{eR})$	$(u^R\ \ u^G\ \ u^B)_R$ $(d^R\ \ d^G\ \ d^B)_R$
$SU(2)_W$ doublet	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$	$\begin{pmatrix}u^R & u^G & u^B\\ d^R & d^G & d^B\end{pmatrix}_L$

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Mystery of chiral structure of SM

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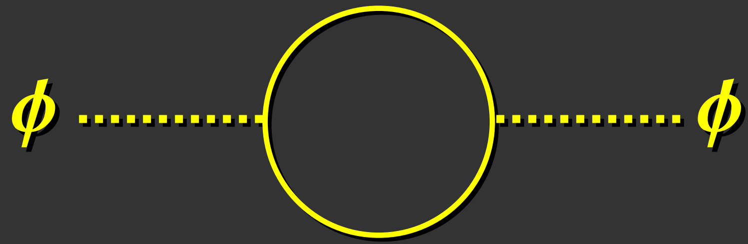
a left-right asymmetric gauge theory !

Mystery of scalar quadratic divergence

Mystery of scalar quadratic divergence

$\Delta m^2 =$ 

$\sim \Lambda^2$

It seems unnatural that fundamental scalars (Higgs?) appear in low energies !?

Mystery of fermion generations

Mystery of fermion generations

$\{\, e, \nu_e, u, d \,\}$

1st generation

$\{\, \mu, \nu_\mu, c, s \,\}$

2nd generation

$\{\, \tau, \nu_\tau, t, b \,\}$

3rd generation

Who ordered exactly the same three sets of quarks and leptons?

Mystery of mass hierarchy

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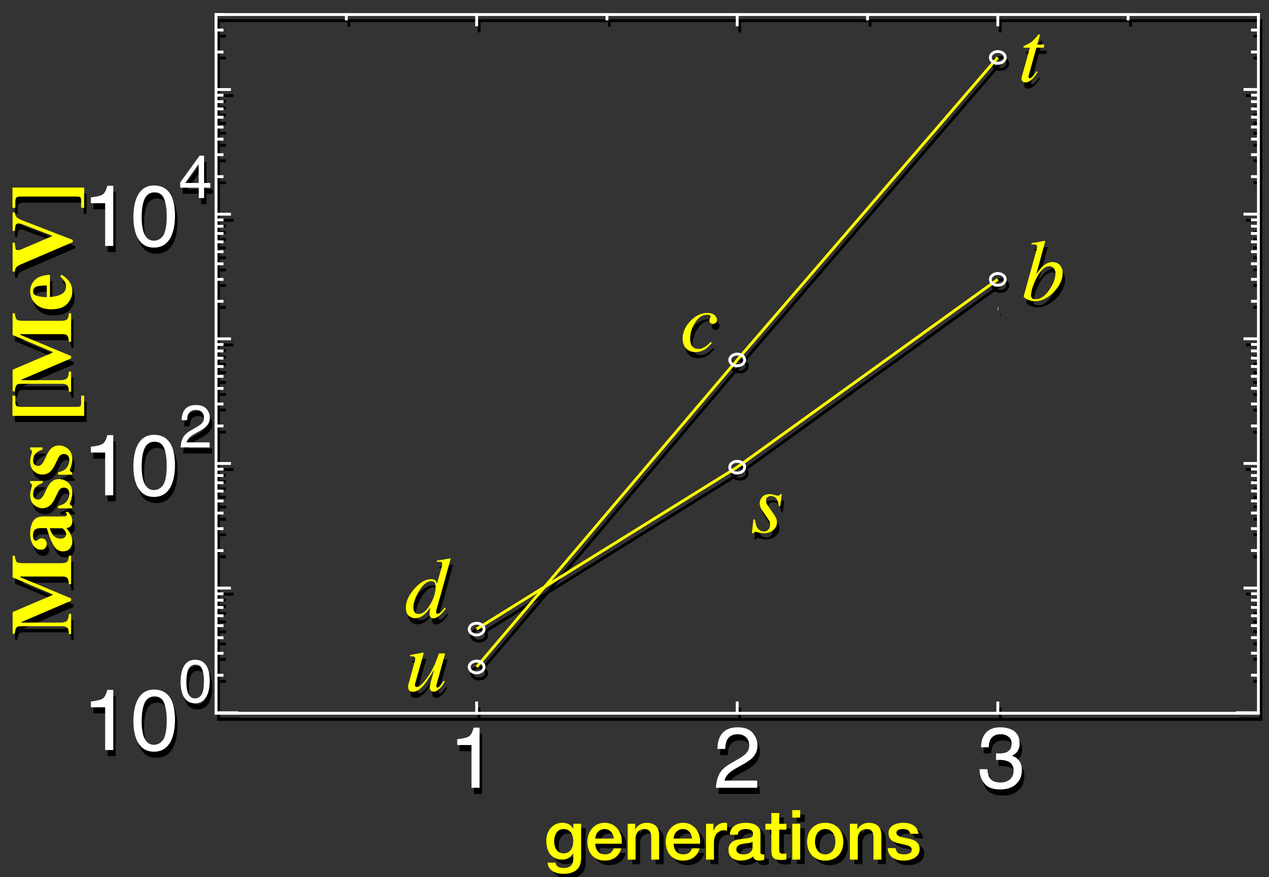
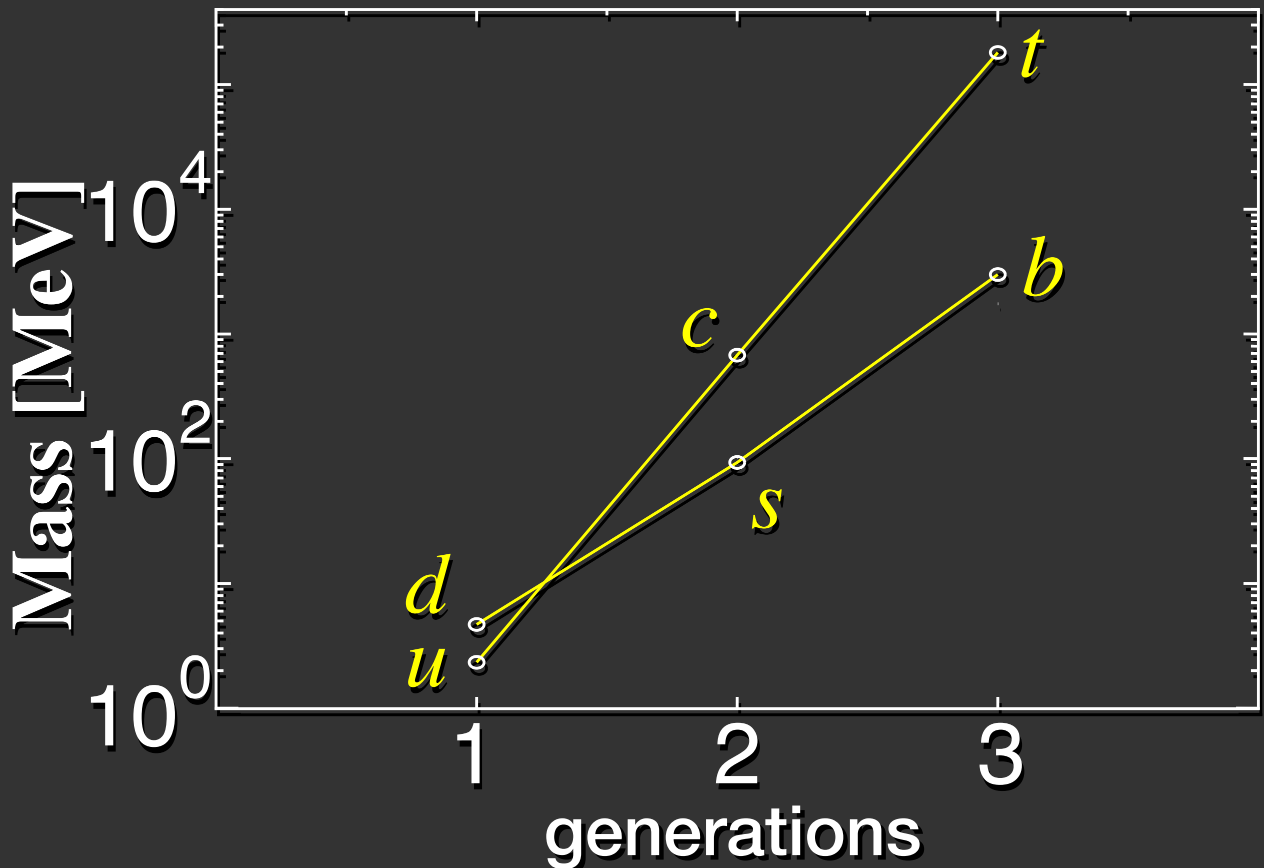


$$\boxed{\frac{m_t}{m_u} \sim 10^5}$$

Mystery of mass hierarchy



USS Enterprise aircraft

$\sim 10^5$ t

$$\frac{m_t}{m_u} \sim 10^5$$

~ 1 t

Why is there the hierarchical mass difference between different generations of quarks and leptons?

Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?

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"degrees" of space-time $\xleftrightarrow{\text{convert}}$ "degrees" of gauge group

KK theory on $M^4\times S^1$:

$$g_{MN} = \begin{pmatrix} \underbrace{g_{\mu\nu}}_{\text{4d metirc}} & \underbrace{A_\mu}_{\text{4d } U(1) \text{ vector}} \\ A_\mu & \phi \end{pmatrix}$$

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⇨ the rank of gauge groups $\leq$ # of extra dimensions

⇨ E_6, $SO(10)$, $SU(5)$, $SU(3)\times SU(2)\times U(1)$, ••• for 6 dim.

I will not discuss this subject in my talk.

Mystery of matter

Why are small representations for the quarks & leptons are chosen ?

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$SU(2)_{\rm W}$ singlet	$e_{\rm R}$, $(\nu_{e{\rm R}})$	$(u^{\rm R}\ u^{\rm G}\ u^{\rm B})_{\rm R}$ $(d^{\rm R}\ d^{\rm G}\ d^{\rm B})_{\rm R}$
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- 5d field on $M^4 \times S^2$

$$\Psi(x;\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_{l,m}(x)\, Y_{l,m}(\theta,\phi)$$

spherical harmonics on S^2

4d fields belonging to spin l representation of SU(2)

Answer from our point of view

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- mass^2 of $\psi_{l,m}$

$$m_l^2 = \frac{l(l+1)}{R^2}$$

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- 5d field on $M^4 \times S^2$

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4d fields belonging to spin l representation of SU(2)

- mass² of $\psi_{l,m}$

$$m_l^2 = \frac{l(l+1)}{R^2} \longrightarrow$$

At low energies, only 4d fields belonging to ***small*** dimensional representations can appear!

I will not discuss this subject in my talk.

Mystery of chiral structure of SM

Why is the SM a chiral gauge theory?

	$SU(3)_c$ singlet	$SU(3)_c$ triplet
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The SM should be regarded as a low energy effective theory, which will be described by ***massless*** particles.

massless spinors

massless vectors

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massless spinors $\longrightarrow$ chiral fermions

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massless vectors $\longrightarrow$ gauge fields

$\downarrow$

chiral gauge theory!

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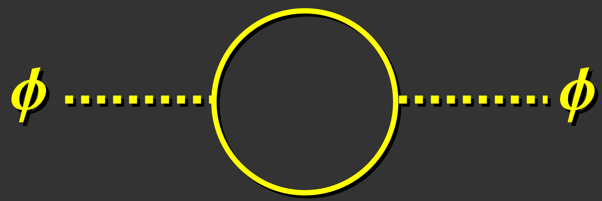
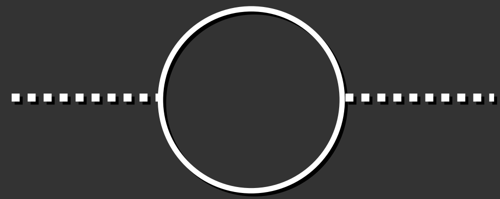
$\Delta m^2 =$ 

$\sim \Lambda^2$

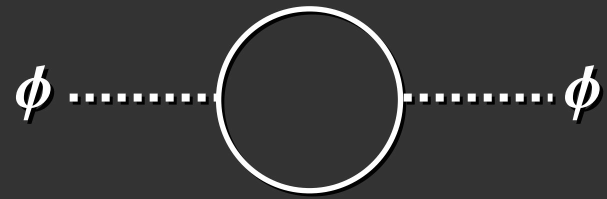
Mystery of scalar quadratic divergence

$\Delta m^2 = \phi$  $\phi \sim \Lambda^2$

Answer from our point of view

There is no distinction between scalars, spinors, vectors at high energies.

Mystery of scalar quadratic divergence

$\Delta m^2 =$ 

$\sim \Lambda^2$

Answer from our point of view

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$\downarrow$

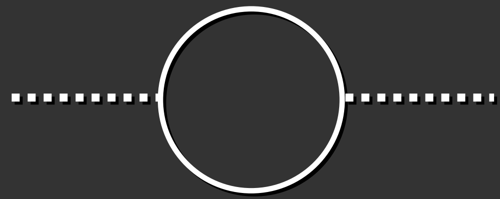
Scalars should belong to some multiplets with spinors and/or vectors at high energies.

e.g.

$\Phi = (\, \phi \,,\, \psi \,)$ ············➤ supersymmetry

$A_M = (\, A_\mu \,,\, \phi \,)$ ············➤ extra dimensions

Mystery of scalar quadratic divergence

$\Delta m^2 = \phi$  $\phi \quad \sim \Lambda^2$

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$\Phi = (\phi, \psi)$ ⟶ supersymmetry

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No quadratic divergences!

Mystery of fermion generations

What is the origin of generations?

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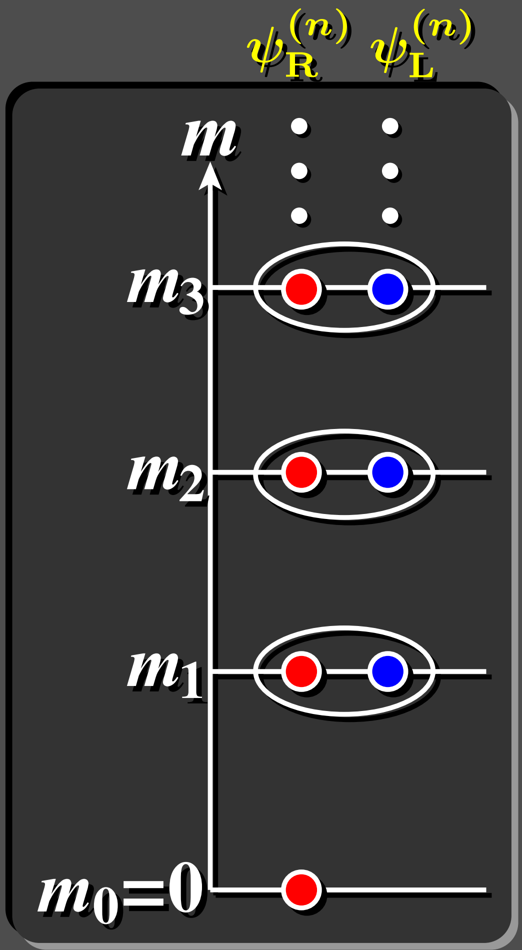
Answer from our point of view



higher dim. spinor

4d left-handed spinors

$$\Psi(x,y) = \sum_n \left\{ \psi_R^{(n)}(x) f_n(y) + \psi_L^{(n)}(x) g_n(y) \right\}$$

4d right-handed spinors

wavefunctions on extra dim.

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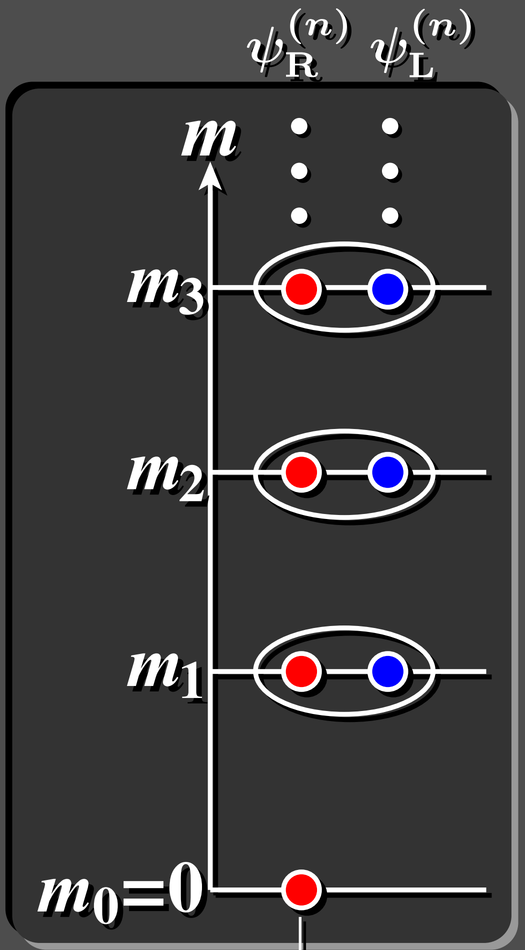

chiral zero modes $\psi_R^{(0)}$

higher dim. spinor

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chiral zero modes

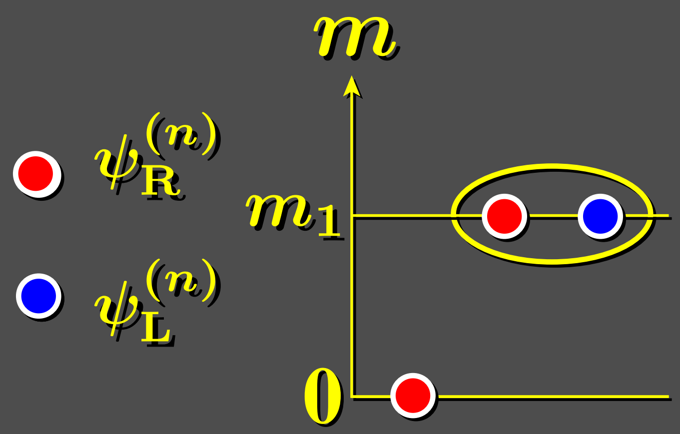
$$\text{\# of generations} \equiv \left| \text{\# of } \psi_R^{(0)} - \text{\# of } \psi_L^{(0)} \right|$$
$$= \text{a topological \# of extra dimensions}$$

Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

Change the parameters $(m, g, L, \hbar, \cdots)$ of the theory.



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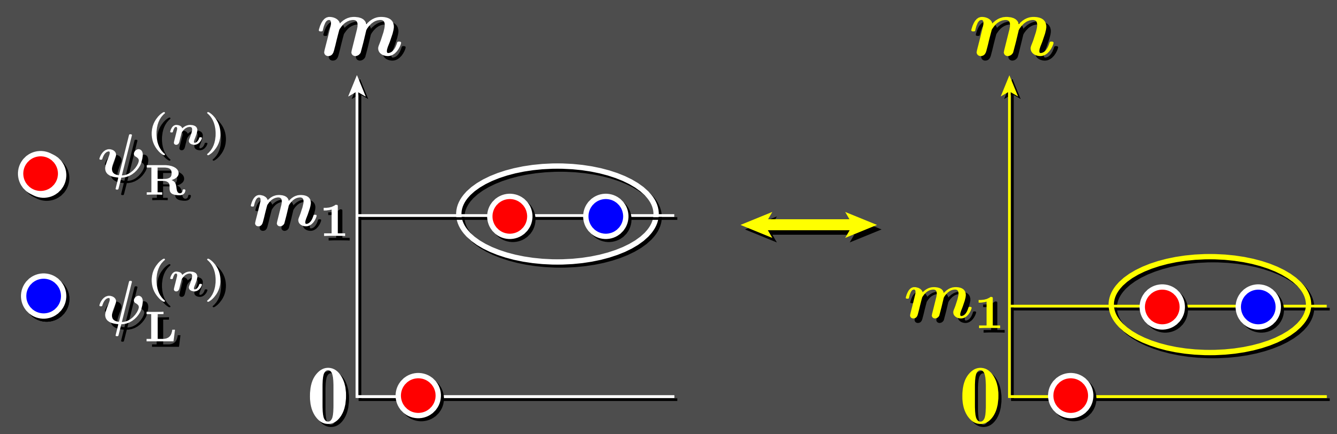

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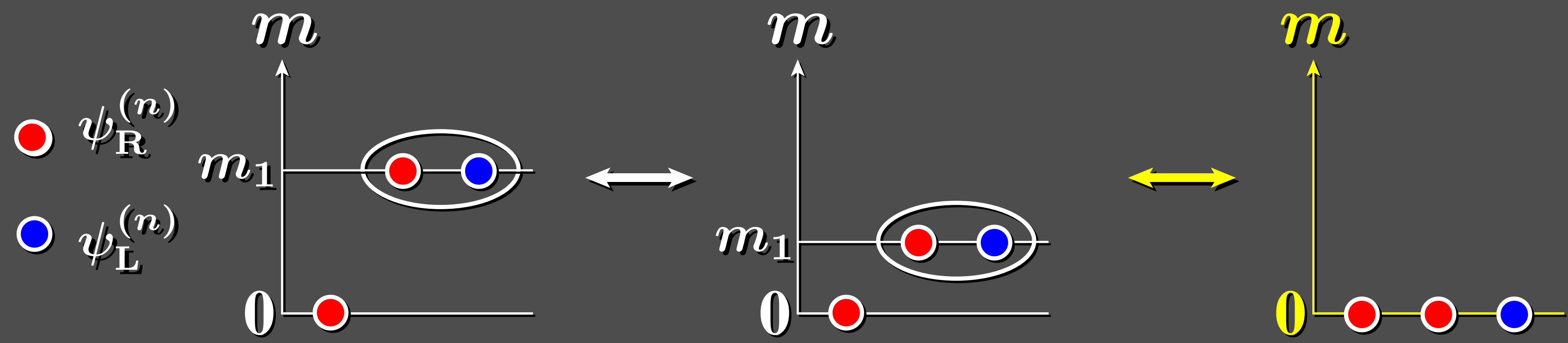

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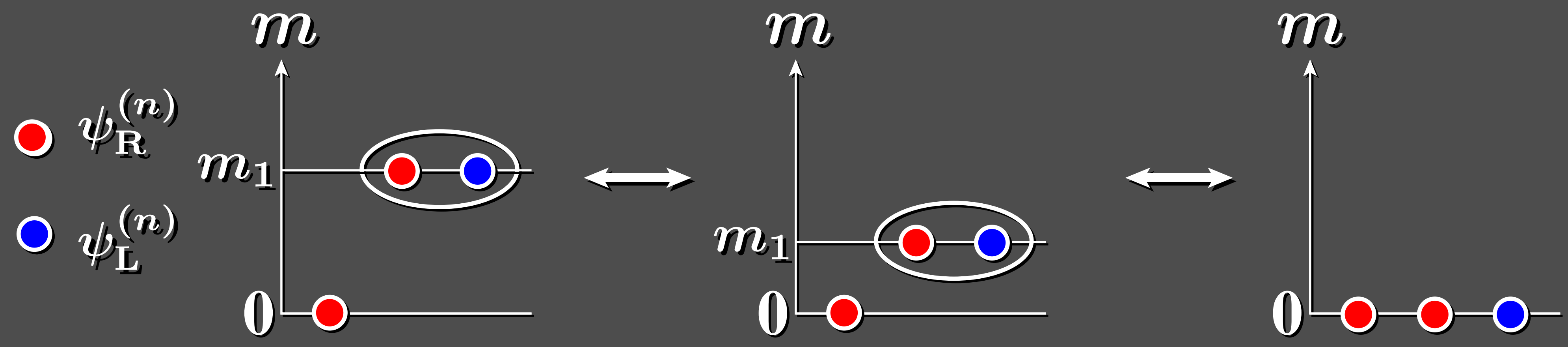

Each number of $\psi_{\mathrm{R}}^{(0)}$ and $\psi_{\mathrm{L}}^{(0)}$ can change but ***NOT*** their difference!

Mystery of fermion generations

What is the origin of generations?

Answer from our point of view

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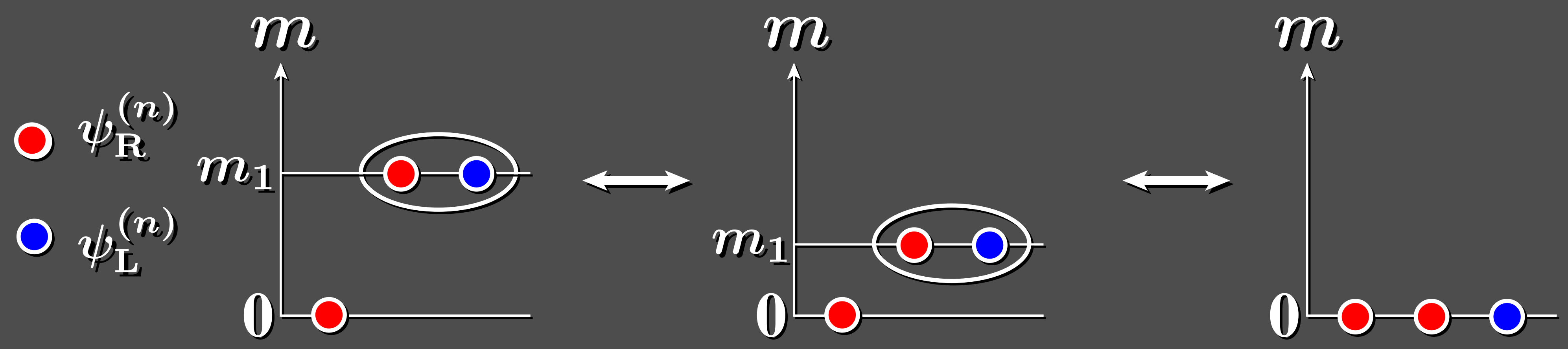


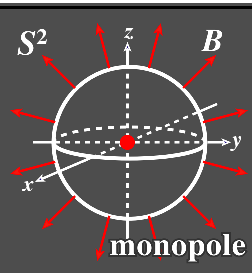
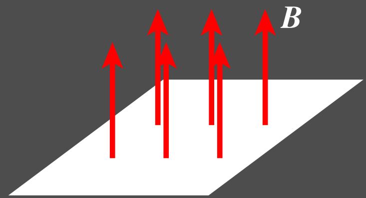
Each number of $\psi_R^{(0)}$ and $\psi_L^{(0)}$ can change
but *NOT* their difference!

⇨ # of generators is a *topological number!*

Mystery of fermion generations

What is the origin of generations?

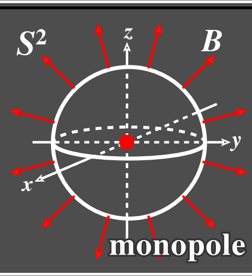
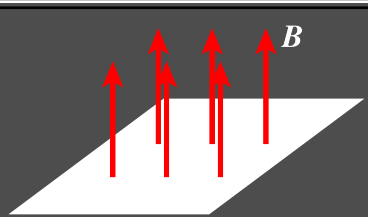
Answer from our point of view

background fields		generation numbers
kink		**1**
monopole on S^2		**M (monopole charge)**
magnetic flux on T^2		**M (magnetic flux charge)**
point interactions		**M (# of point interactions -1)**

Mystery of fermion generations

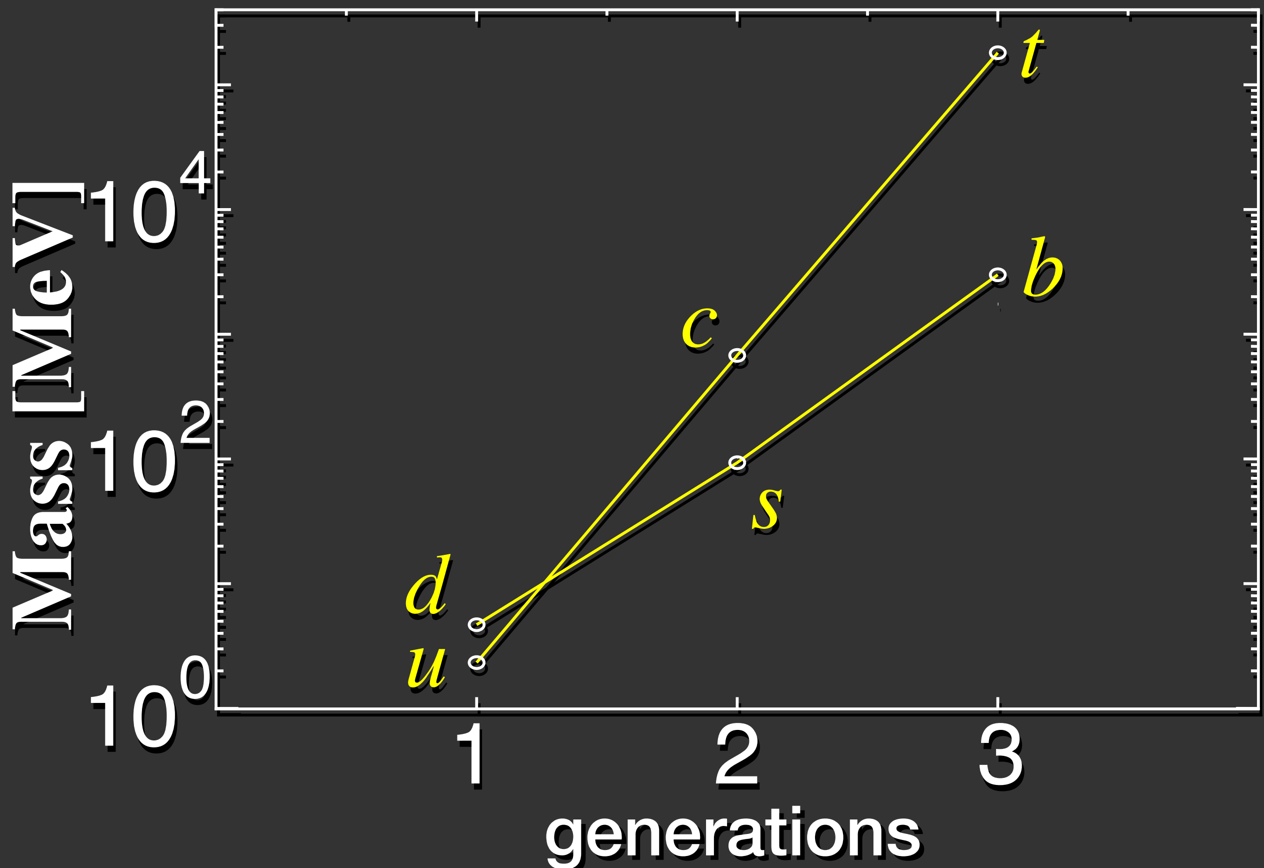
What is the origin of generations?

Answer from our point of view

background fields		generation numbers
kink		1
monopole on S^2		M (monopole charge)
magnetic flux on T^2		M (magnetic flux charge)
point interactions		M (# of point interactions -1)

Mystery of mass hierarchy

What is the origin of the hierarchical masses?



USS Enterprise aircraft

$\sim 10^5$ t

$$\boxed{\frac{m_t}{m_u} \sim 10^5}$$

~ 1 t

Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

Yukawa interactions

4 dim.

$$\int d^4x \, \lambda \, \bar{\psi}_{\mathrm{L}}(x) \phi(x) \psi_{\mathrm{R}}(x)$$

$$\Downarrow$$

$$\int d^4x \, m \, \bar{\psi}_{\mathrm{L}}(x) \psi_{\mathrm{R}}(x)$$

$$m = \lambda \, \langle \phi \rangle$$

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$$m = \lambda \langle \phi \rangle$$

4 dim. + extra dim.

$$\int d^4x \int dy \, \lambda \, \bar{\Psi}(x,y)\Phi(x,y)\psi,(x,y)$$

$$\Downarrow$$

$$\int d^4x \, m \, \bar{\psi}^{(0)}_{\mathrm{L}}(x)\psi^{(0)}_{\mathrm{R}}(x)$$

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4 dim. + extra dim.

$$\int d^4x \int dy \; \lambda \, \bar{\Psi}(x,y)\Phi(x,y)\psi'(x,y)$$

$$\Downarrow$$

$$\int d^4x \; m \, \bar{\psi}^{(0)}_{\mathrm{L}}(x)\psi^{(0)}_{\mathrm{R}}(x)$$

$$m = \lambda \int dy \; (g_0(y))^* \langle \Phi(y) \rangle f_0(y)$$

$$\begin{cases} \Psi(x,y) = \psi^{(0)}_{\mathrm{L}}(x) g_0(y) + (\text{massive modes }) \\ \Psi'(x,y) = \psi^{(0)}_{\mathrm{R}}(x) f_0(y) + (\text{massive modes }) \\ \Phi(x,y) = \langle \Phi(y) \rangle + (\text{massive modes }) \end{cases}$$

Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

$$\boxed{m = \lambda \int dy \; (g_0(y))^* \langle \Phi(y) \rangle f_0(y)}$$

Two ways to produce mass hierarchy:

Mystery of mass hierarchy

What is the origin of the hierarchical masses?

Answer from our point of view

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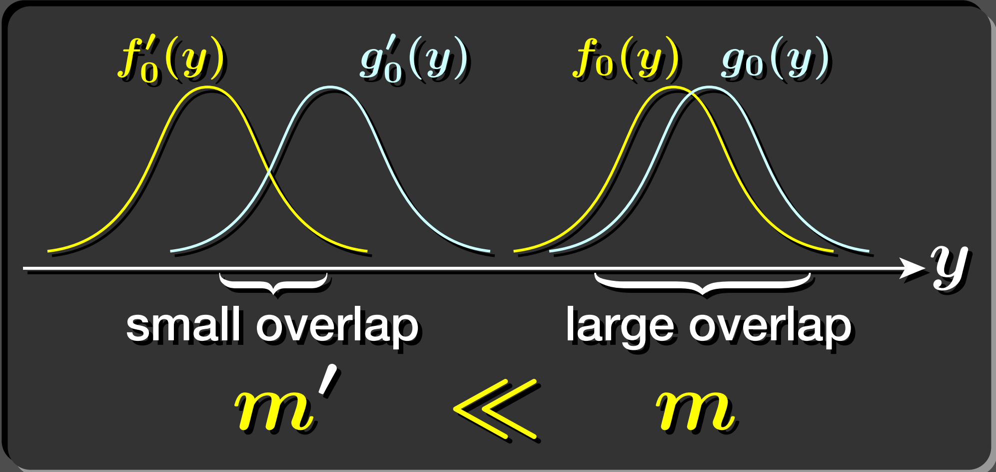
Two ways to produce mass hierarchy:



Localization naturally leads to mass hierarchy!

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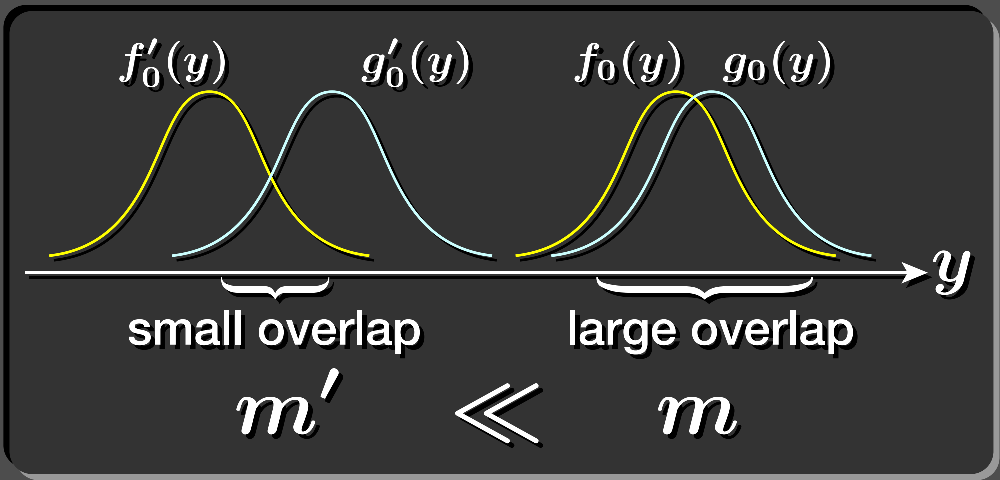
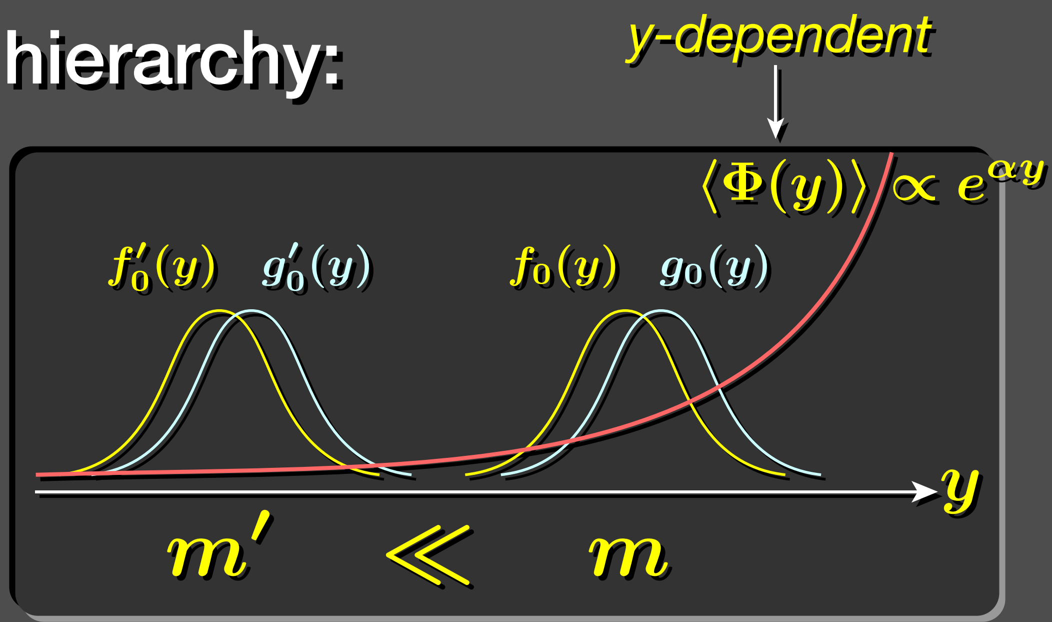




Localization naturally leads to mass hierarchy!

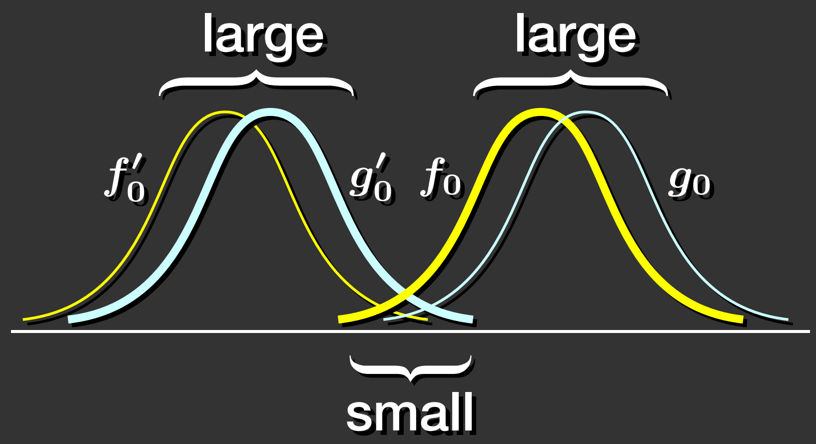
The y-dependent vacuum expectation value can happen in extra dimensions!

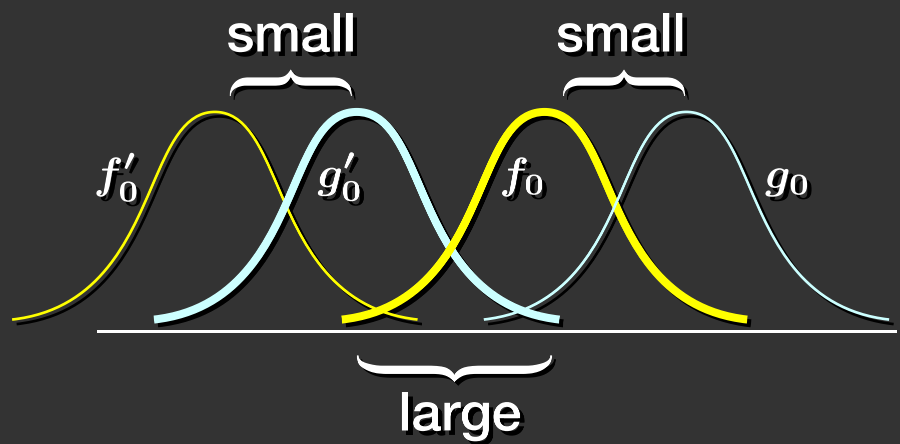
Mystery of flavor mixing

What is the origin of the fermion flavor mixings?

Why are the quark flavor mixings small but the lepton flavor mixings large?

Answer from our point of view





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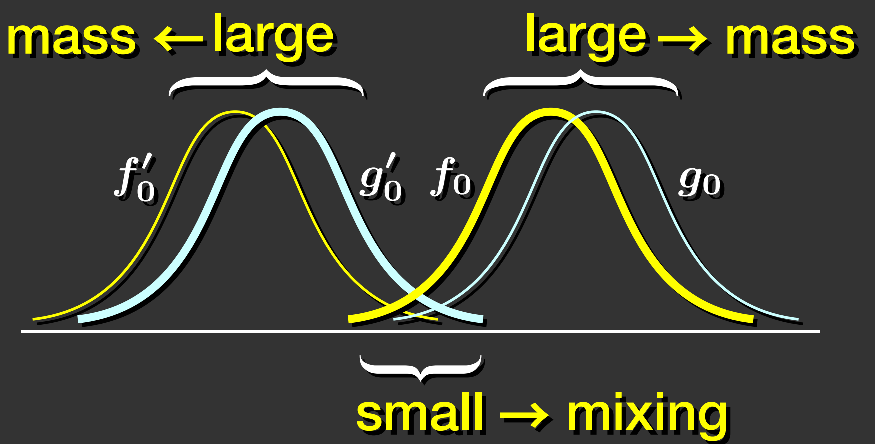

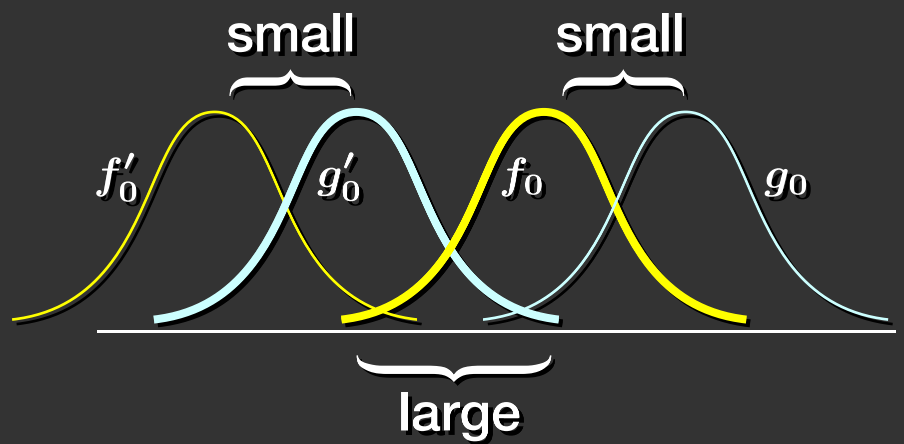

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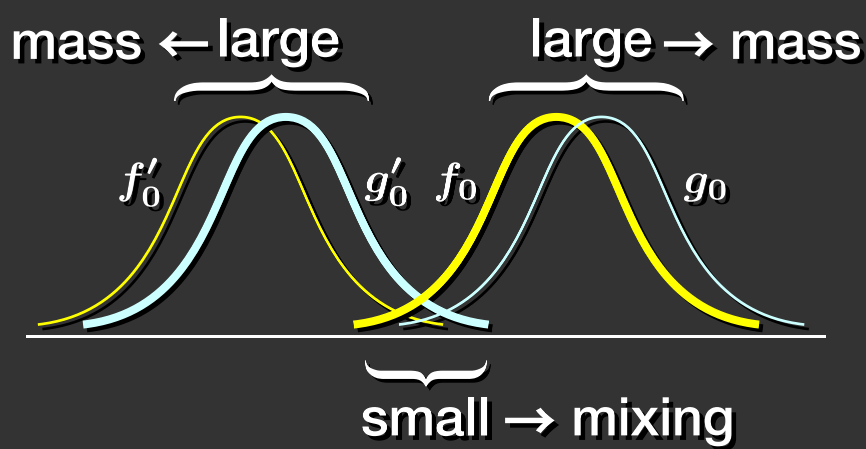
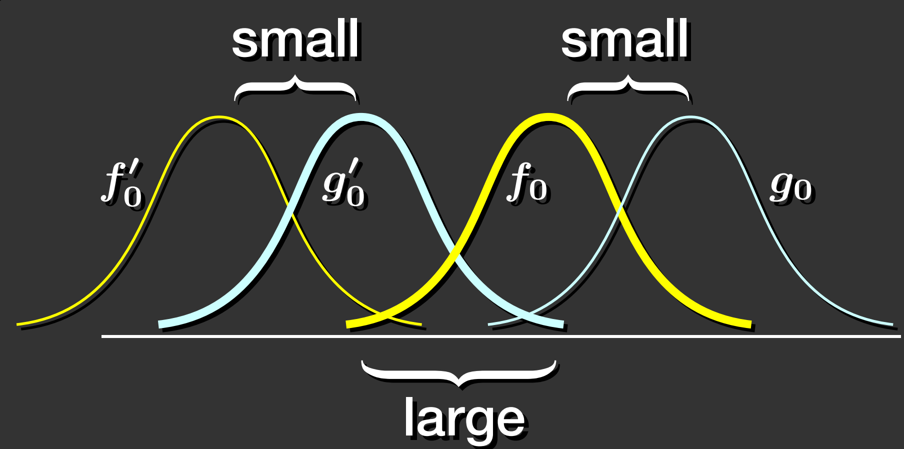


⇓

large mass → small mixing



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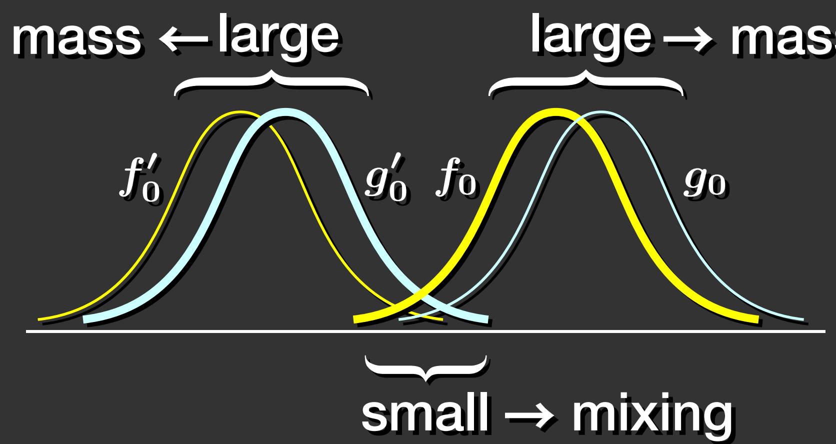
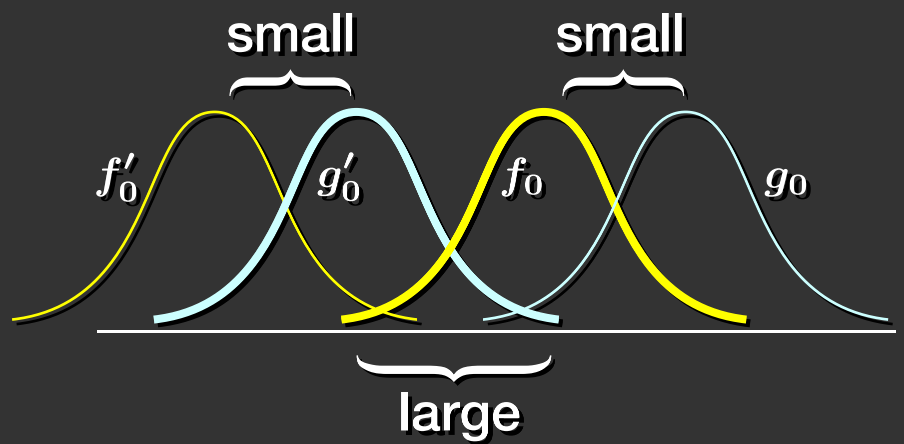


$\Downarrow$

large mass → small mixing

$\Downarrow$

quarks → small mixing



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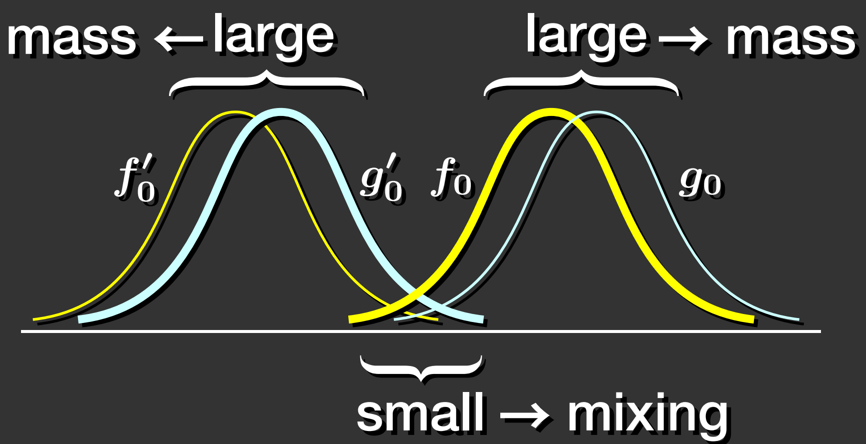

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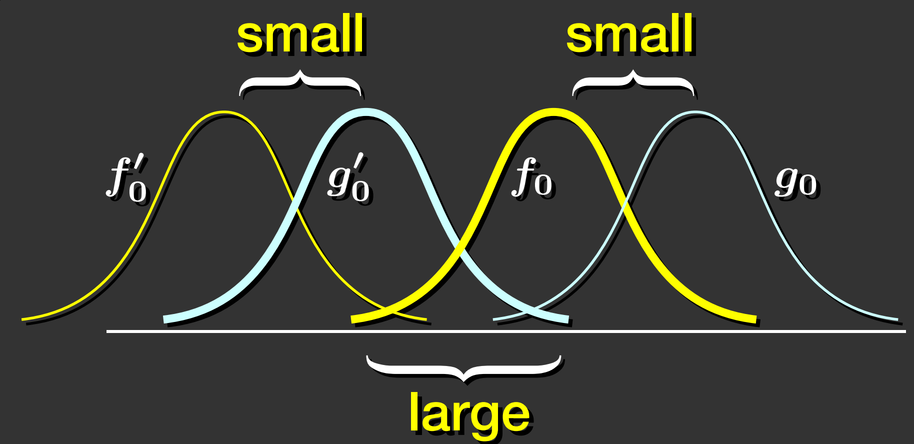

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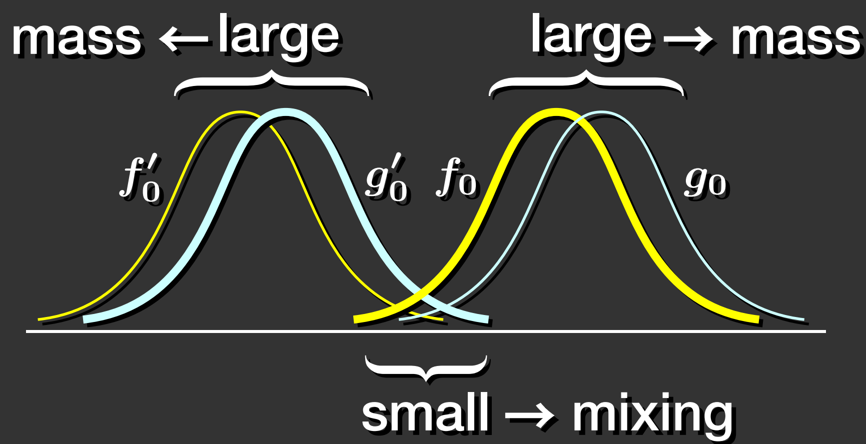

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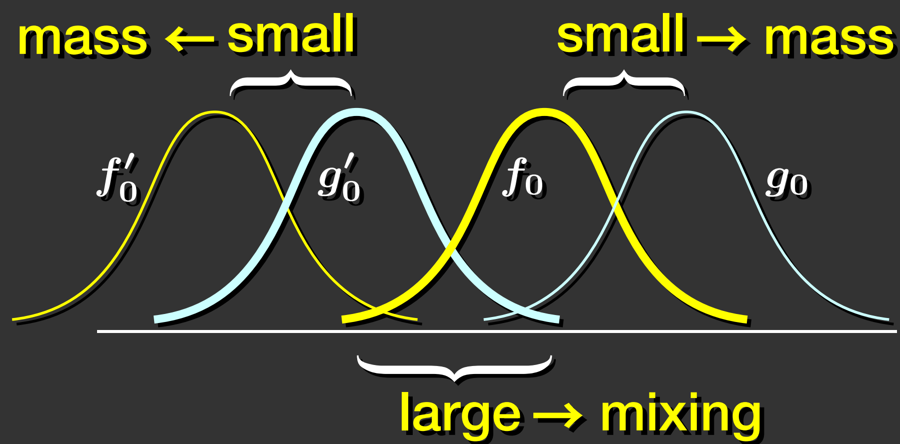

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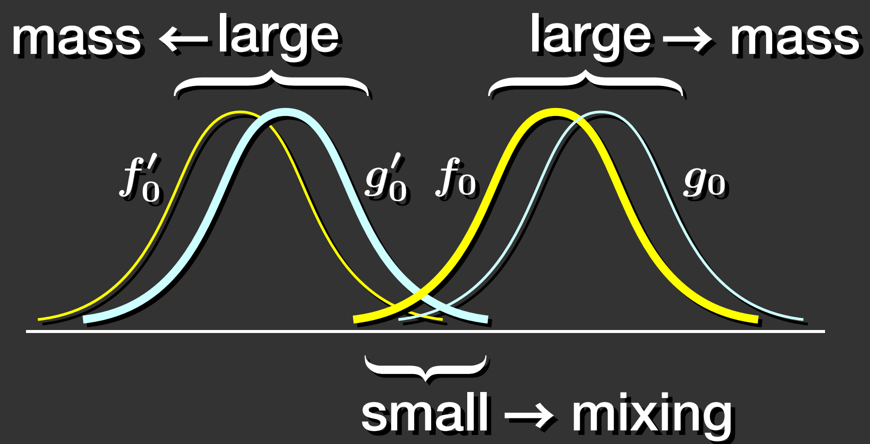
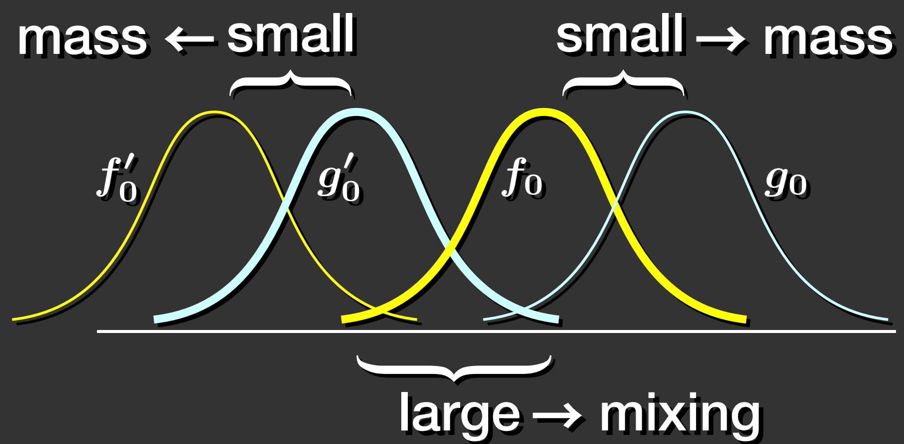


⇓

large mass → small mixing

⇓

quarks → small mixing



⇓

small mass → large mixing

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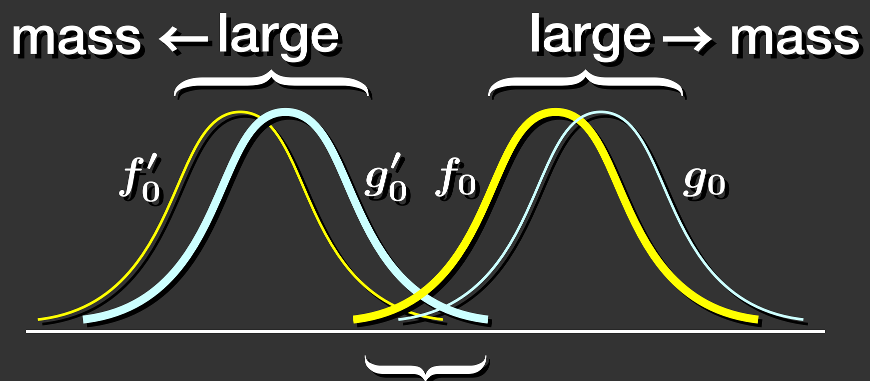


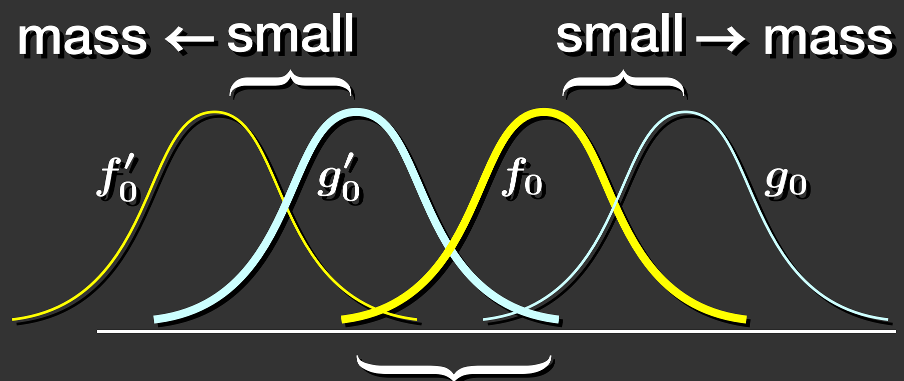
small → mixing

⇓

large mass → small mixing

⇓

quarks → small mixing



large → mixing

⇓

small mass → large mixing

⇓

leptons → large mixing
(neutrinos)

- ❑ Motivation to considering extra dimensions
- ❑ Mysteries of the Standard Model
- ❑ General features of extra dimensions
- ❑ Setup
- ❑ Point interactions
- ❑ Dynamical generation of fermion mass hierarchy
- ❑ Summary

Setup

We want to find an extra-dimensional model which realizes the ideas discussed so far!

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We propose an extra-dimensional model such that

5-dimensional gauge theory on an interval with ***point interactions***.

$$\begin{pmatrix} U(x,y) \\ D(x,y) \end{pmatrix}$$

$U'(x,y)$ $D'(x,y)$

quarks

$$\begin{pmatrix} N(x,y) \\ E(x,y) \end{pmatrix}$$

$N'(x,y)$ $E'(x,y)$

leptons

$\Phi(x,y)$

Higgs

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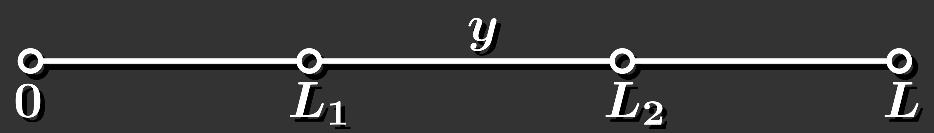


$$\begin{pmatrix} U(x,y) \\ D(x,y) \end{pmatrix} \quad U'(x,y) \quad D'(x,y)$$

quarks

$$\begin{pmatrix} N(x,y) \\ E(x,y) \end{pmatrix} \quad N'(x,y) \quad E'(x,y)$$

leptons

$$\Phi(x,y)$$

Higgs

We prepare only one generation of quarks & leptons!

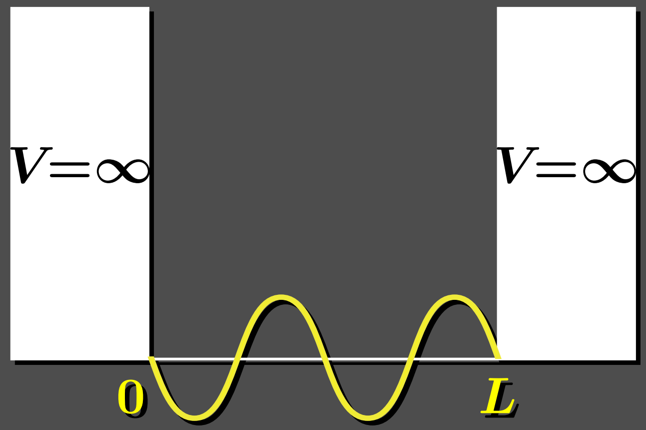
Point interactions

- delta-function potential

Point interactions

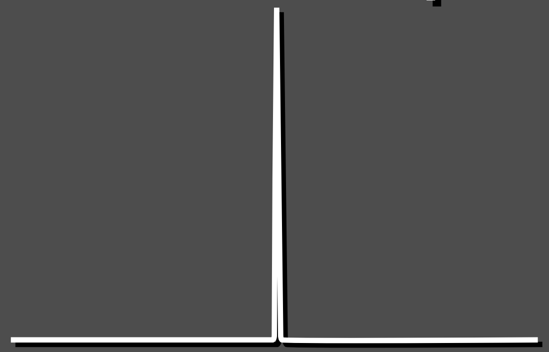
- ❑ delta-function potential

- ❑ infinite square well



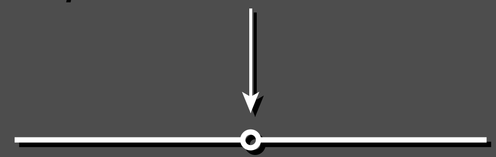
Point interactions

❑delta-function potential

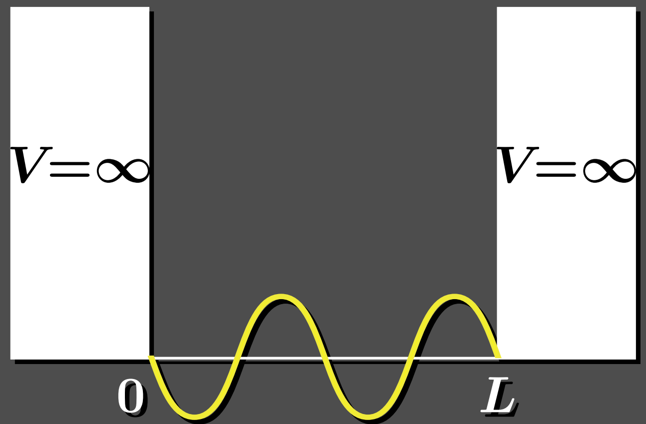


=

can be ragarded as
a point interaction



❑infinite square well



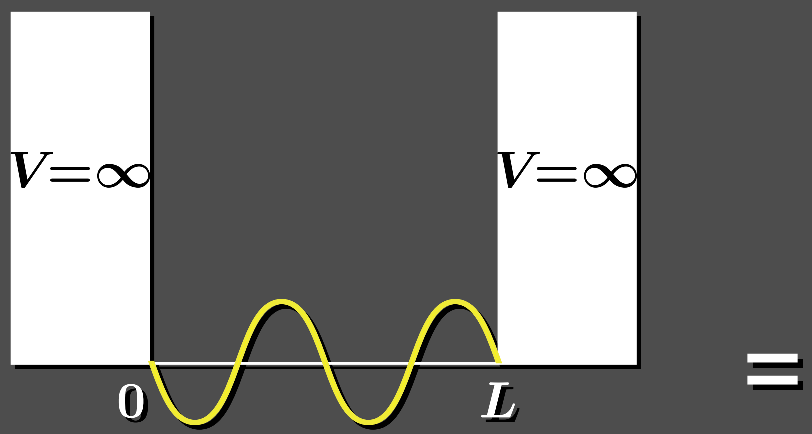
=

Dirichlet b.c. $\psi(0) = \psi(L) = 0$

Point interactions

- delta-function potential

- infinite square well



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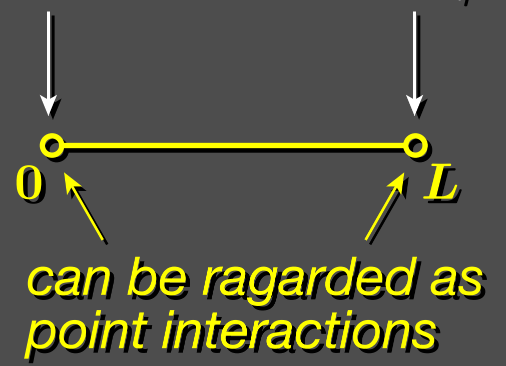
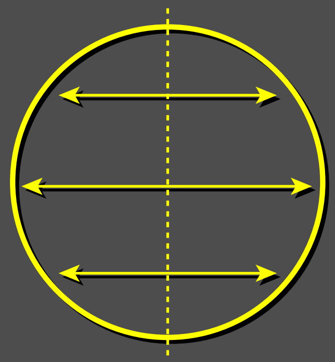


Point interactions

- fixed points on orbifolds

S^1/Z_2



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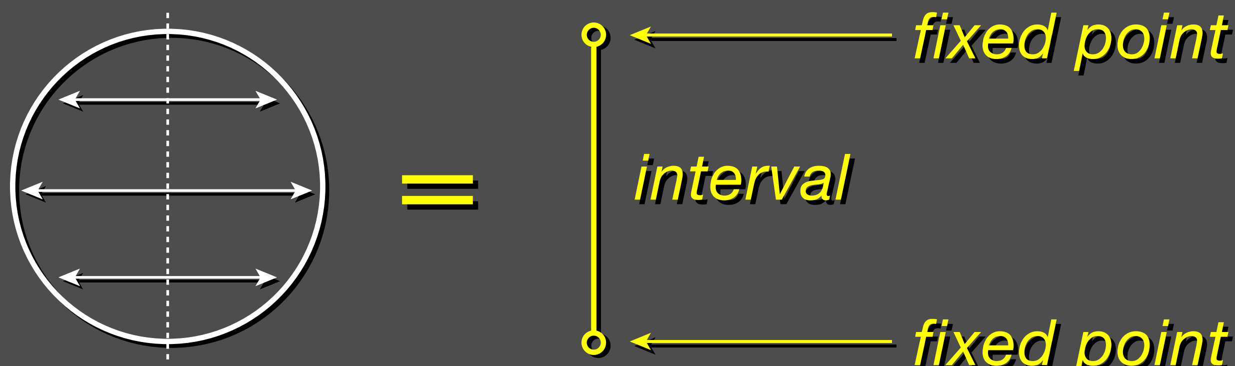

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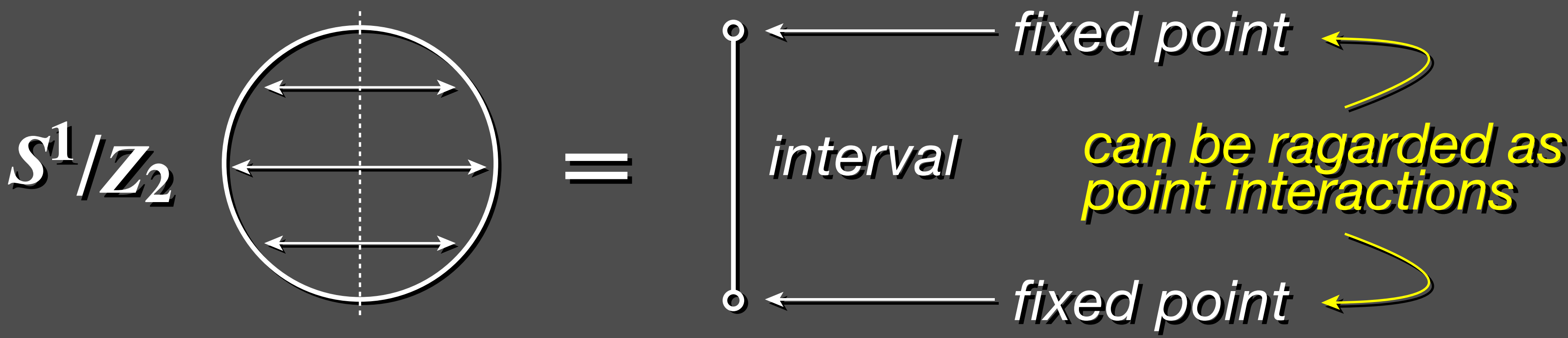

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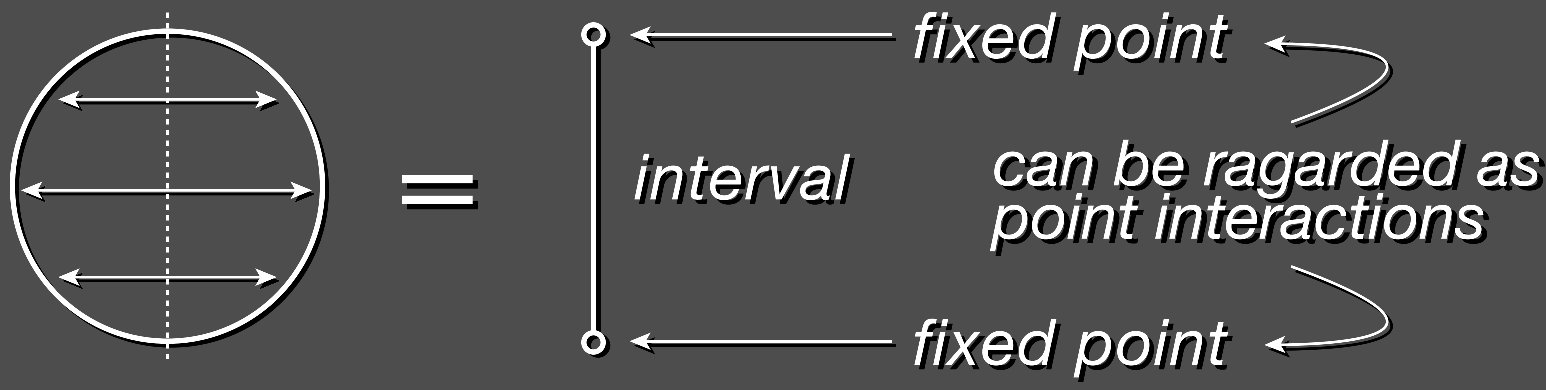
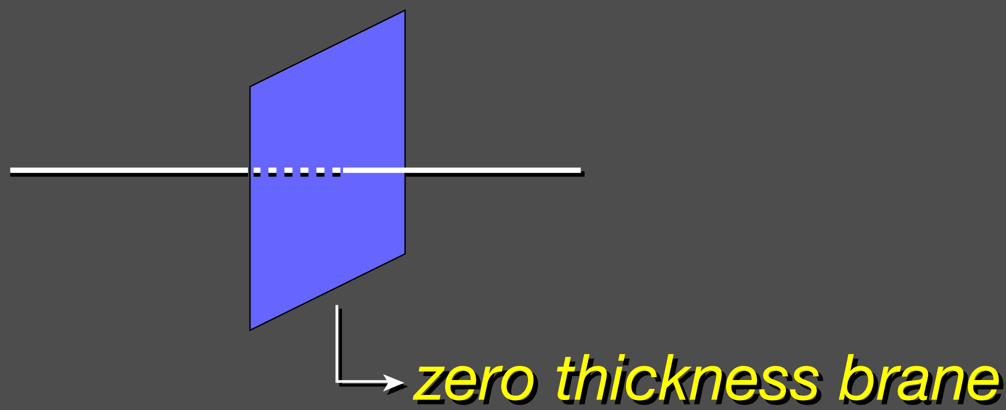


- zero thickness brane



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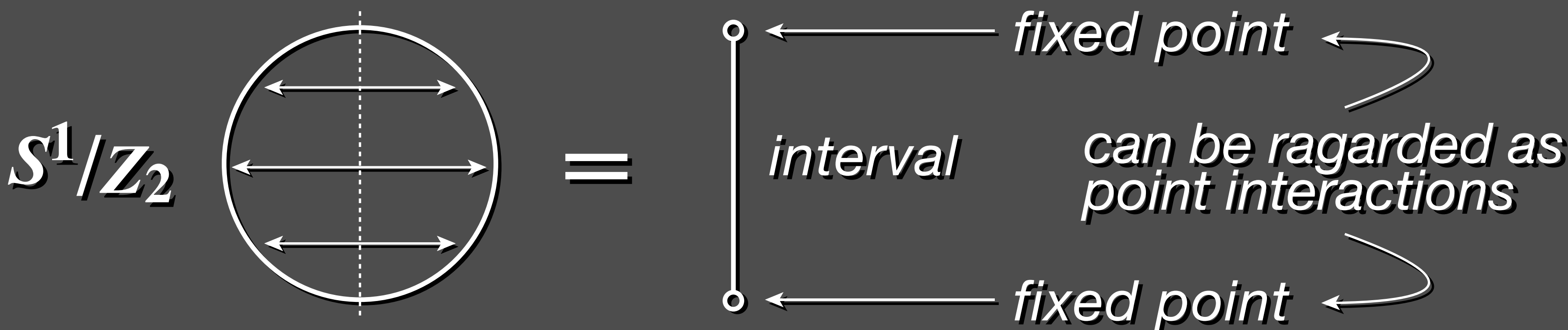

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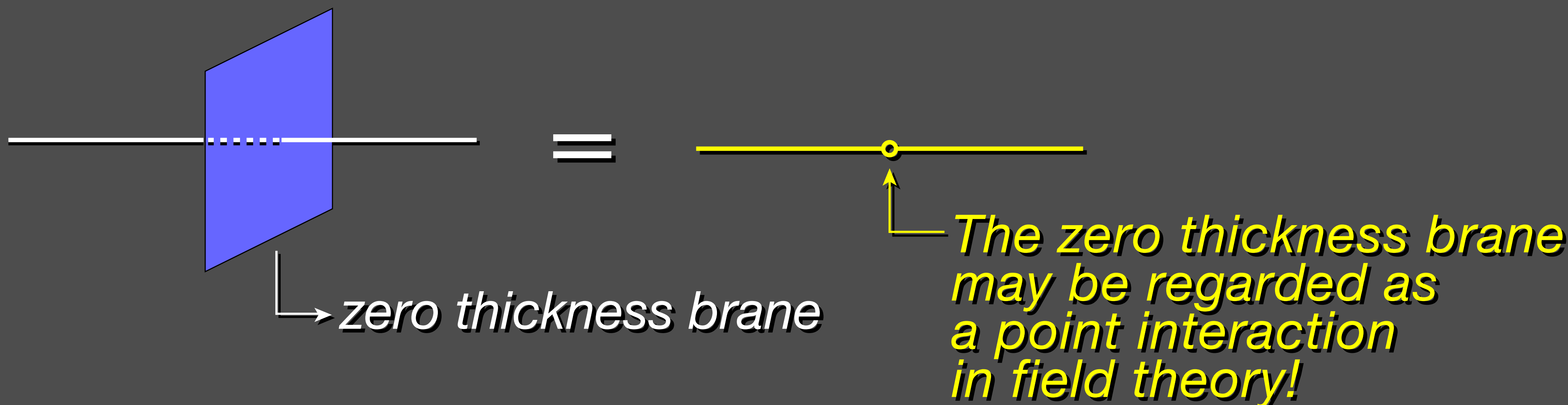
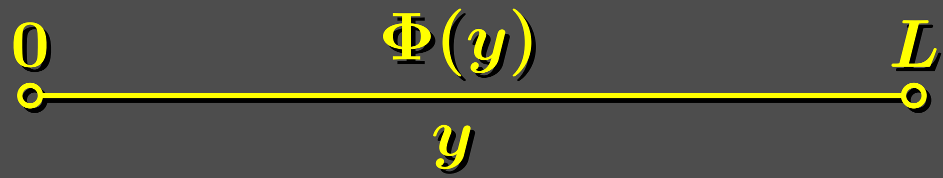


5d scalar on an interval



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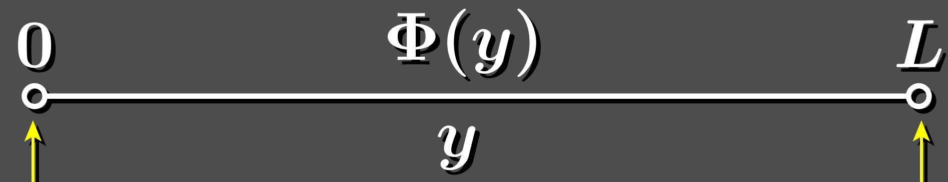
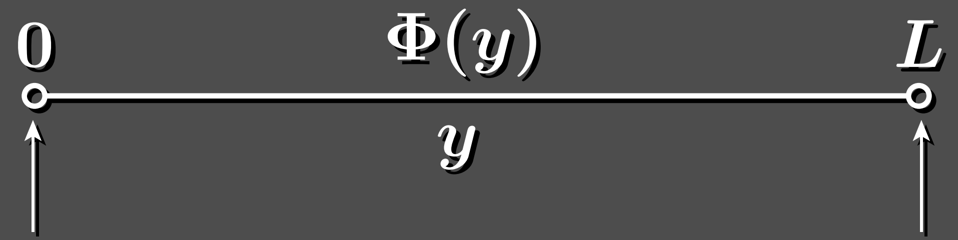


$$\Phi'(0) + L_+ \Phi(0) = 0 \qquad \Phi'(L) - L_- \Phi(L) = 0$$

most general b.c. (Robin b.c.)

5d scalar on an interval



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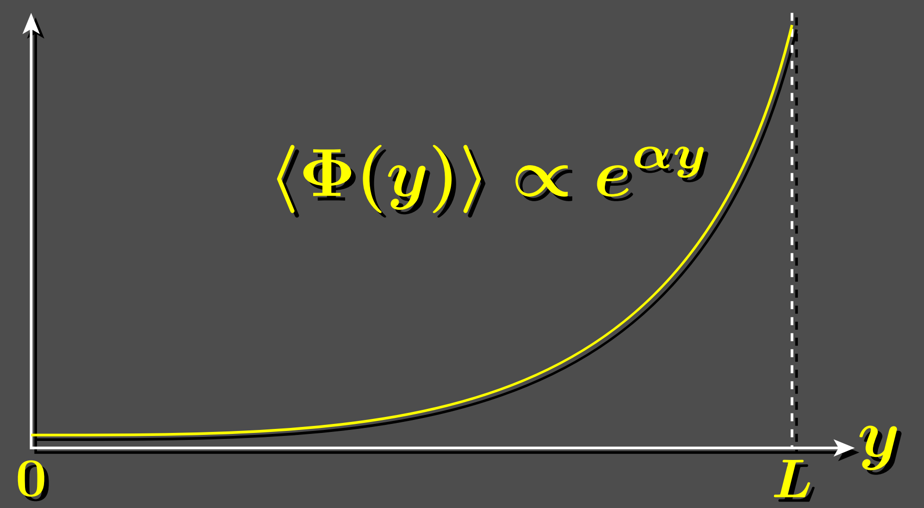
Interesting observations are that

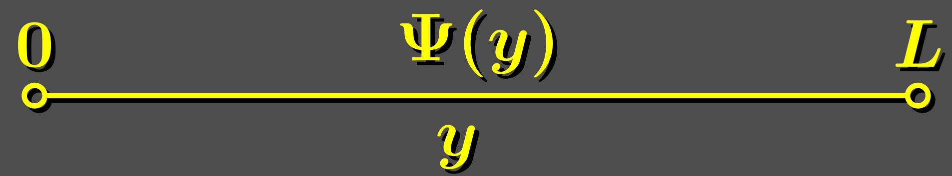
★non-vanishing vacuum expectation value $\langle \Phi(y) \rangle$ can occur even for $M^2>0$!

★ $\langle \Phi(y) \rangle$ can depend on y!



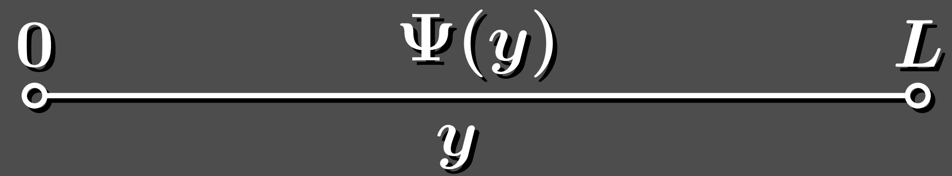
Y. Fujimoto, T. Nagasawa, S. Ohya, M.S., PTP 126 (2011) 841

5d spinor on an interval



$$S = \int d^4x \int_0^L dy \; \bar{\Psi}(y)\Big(i\Gamma^\mu\partial_\mu + i\Gamma^y\partial_y - M\Big)\Psi(y)$$

5d spinor on an interval



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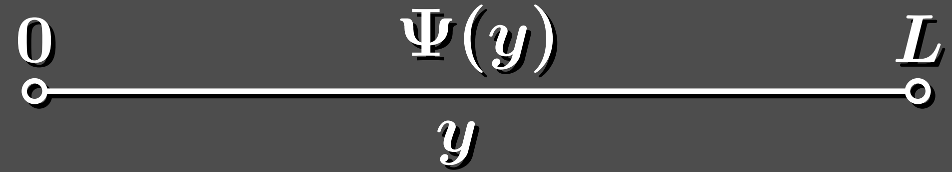
The action principle $\delta S = 0$ gives

$\rightarrow \begin{cases} \text{eq. of motion} \\ \text{b.c. } \bar{\Psi}_{\mathrm{R}}(y)\Psi_{\mathrm{L}}(y) = 0 \quad \text{at } y = 0, L \end{cases}$

5d spinor on an interval



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boundary conditions

$$\Psi_{\mathrm{R}}(y)=0 \ \text{ or } \ \Psi_{\mathrm{L}}(y)=0 \quad \text{at } y=0,L$$

5d spinor on an interval

boundary conditions

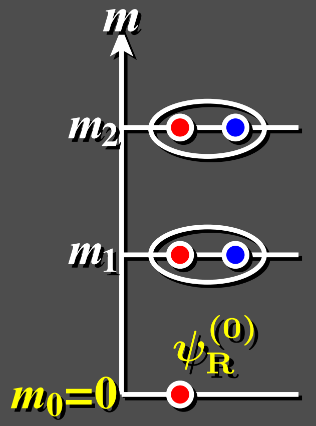
$$\Psi_{\mathrm{R}}(y) = 0 \quad \text{or} \quad \Psi_{\mathrm{L}}(y) = 0 \quad \text{at } y = 0, L$$

5d spinor on an interval

boundary conditions

$$\Psi_{\mathrm{R}}(y) = 0 \quad \text{or} \quad \Psi_{\mathrm{L}}(y) = 0 \quad \text{at } y = 0, L$$

$$\Psi_{\mathrm{L}}(0) = \Psi_{\mathrm{L}}(L) = 0$$



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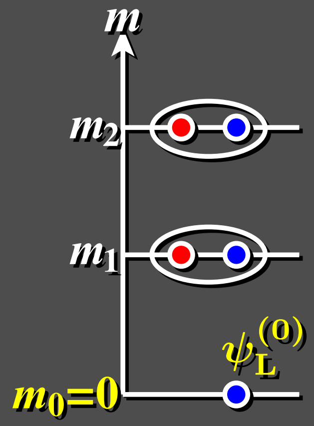

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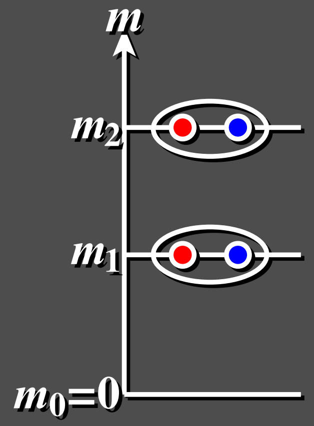

5d spinor on an interval

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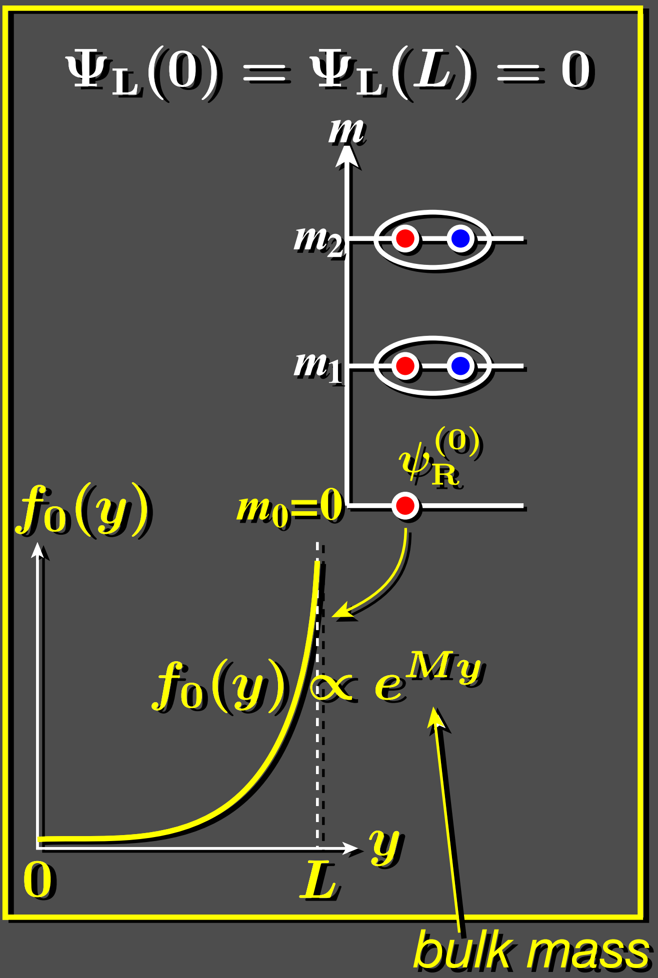

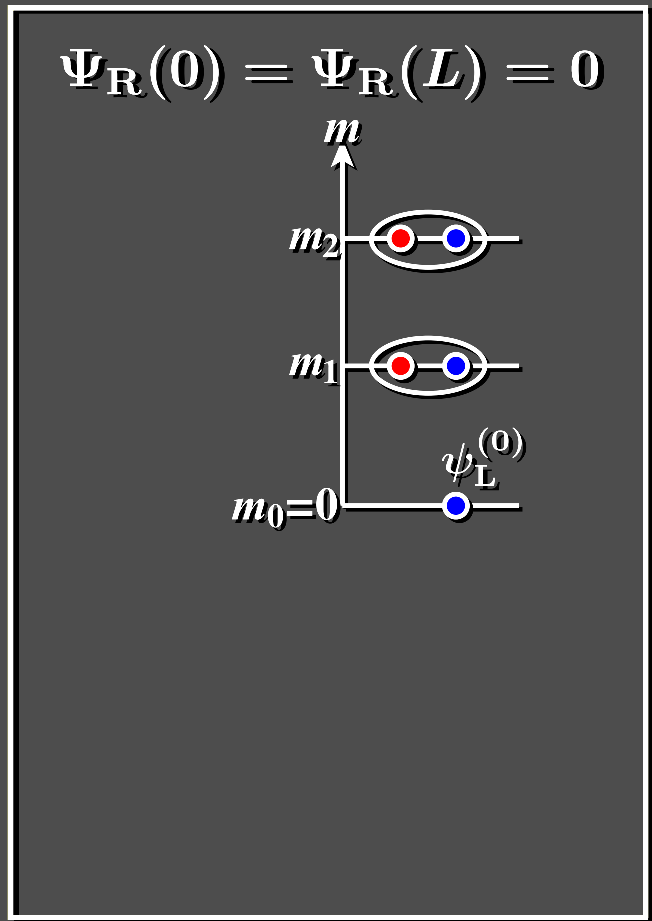

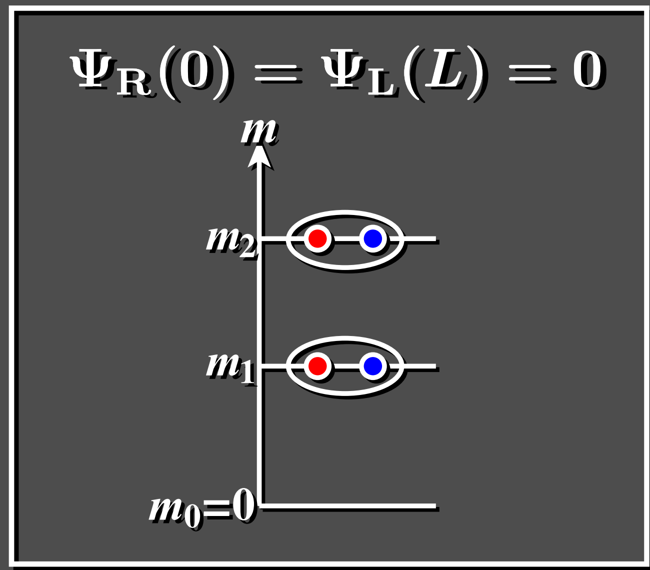

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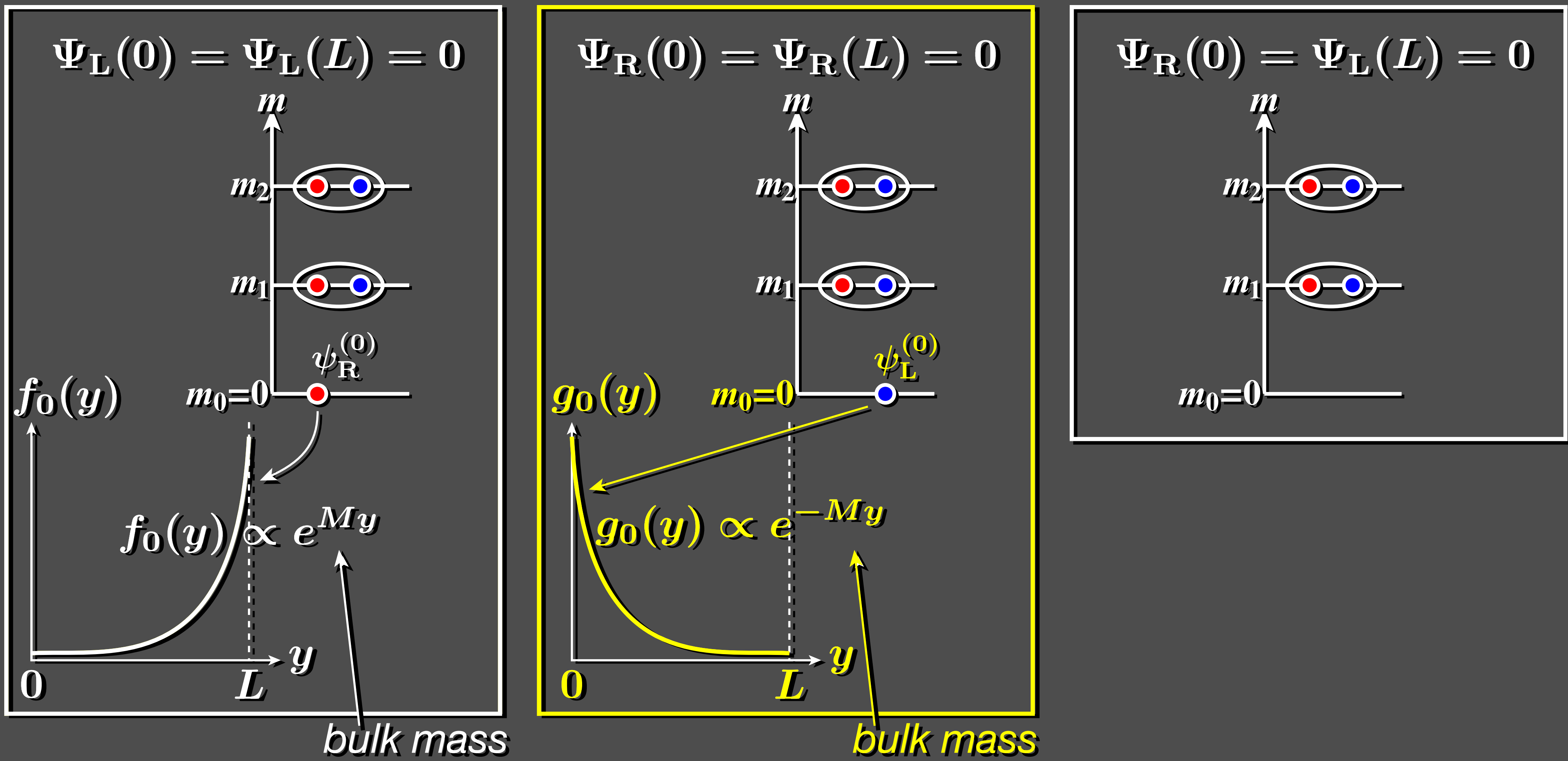

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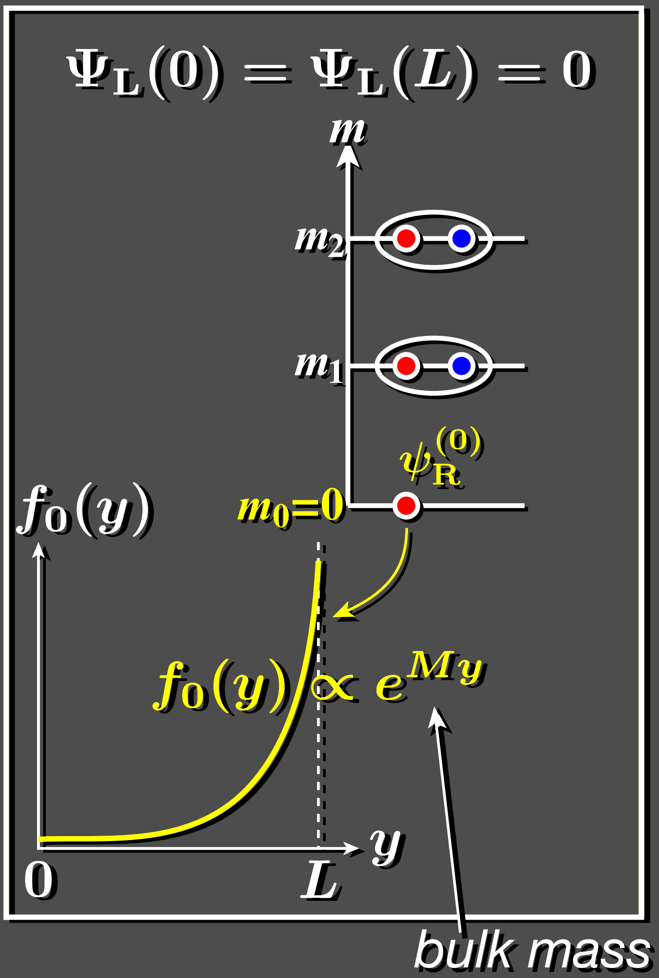

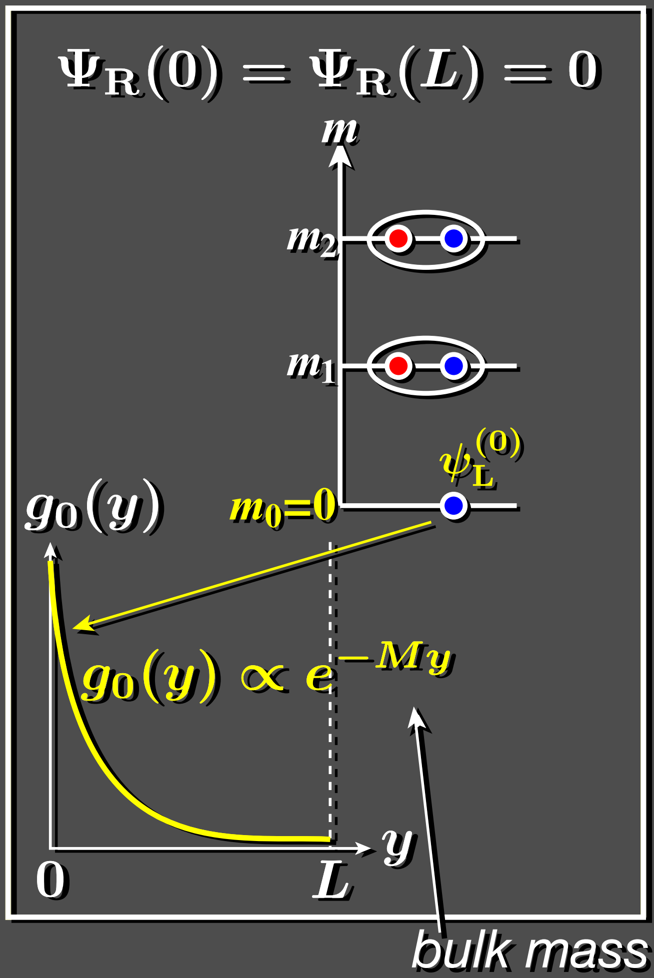

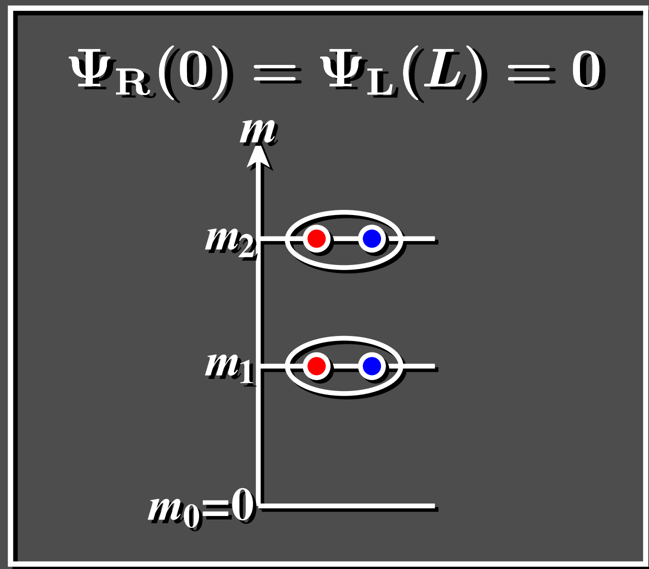

good news

- *a chiral fermion*
- *localization*

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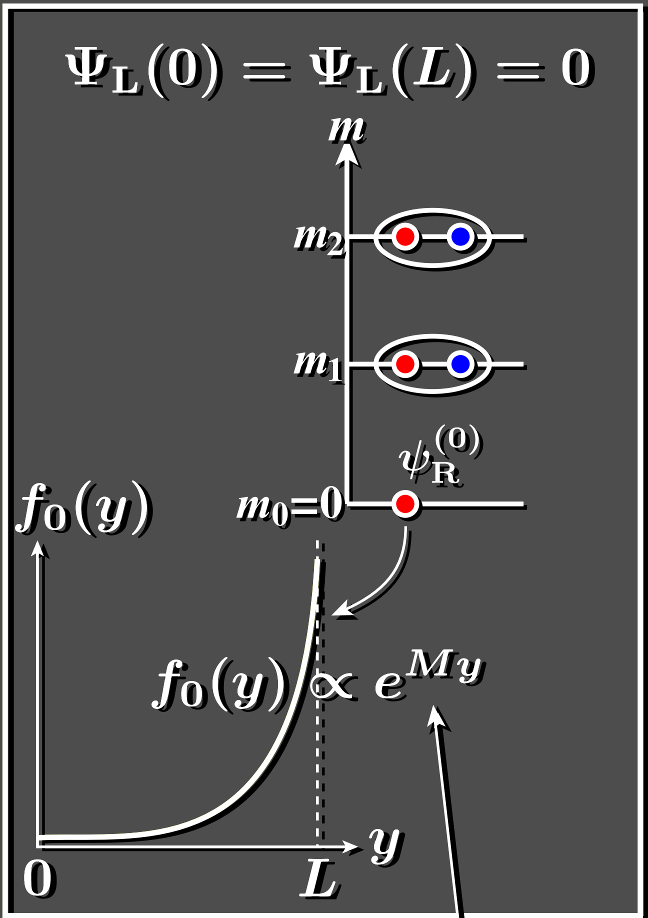
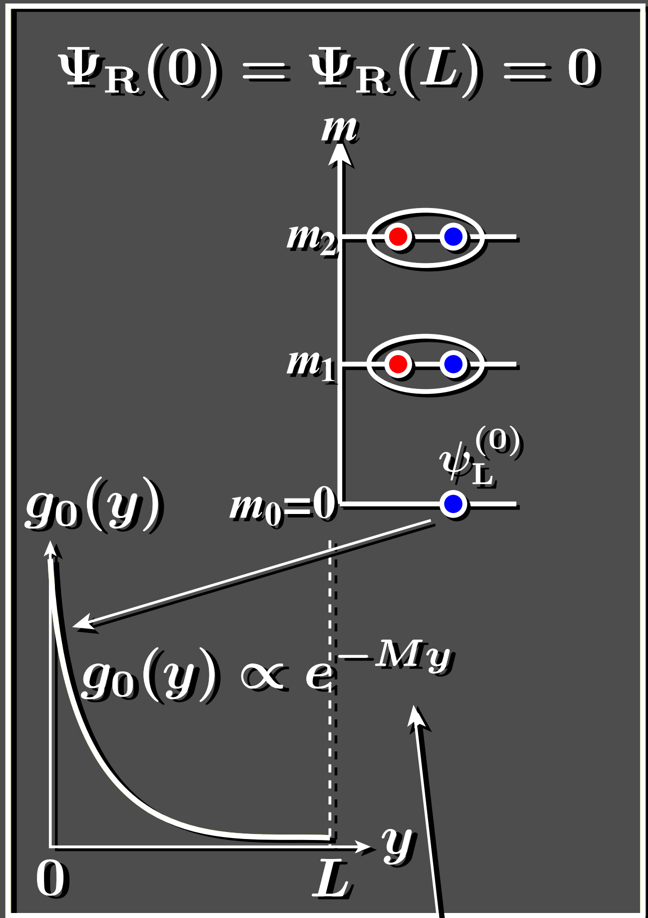


bulk mass



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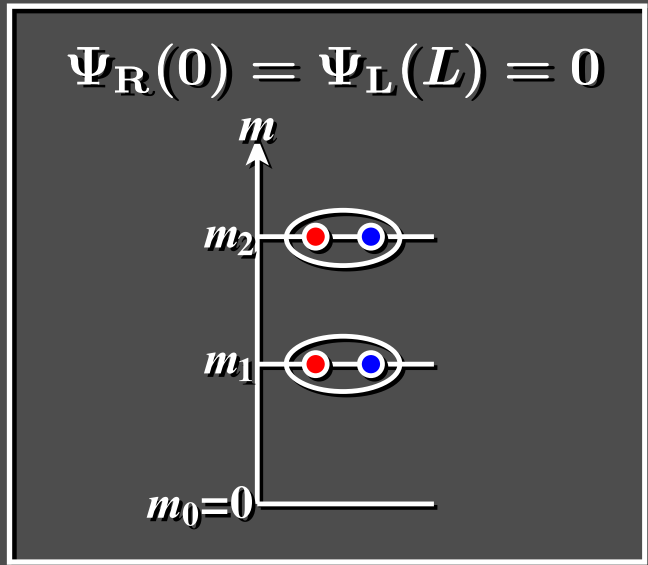

good news

- *a chiral fermion*
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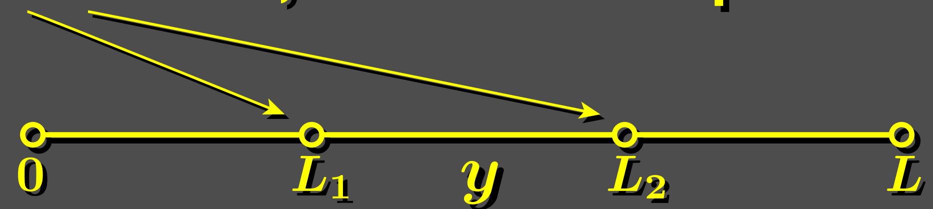
bad news

one generation

5d spinor on an interval with point interactions

To realize ***three generations***, we introduce point interactions, which are specified by BC's.



$\Psi_L(0) = 0$ $\quad$ $\Psi_L(L_1) = 0$ $\quad$ $\Psi_L(L_2) = 0$ $\quad$ $\Psi_L(L) = 0$

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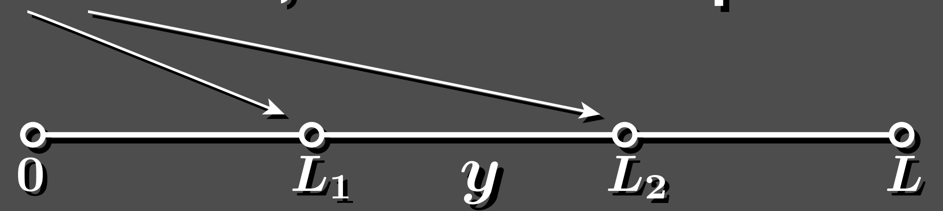

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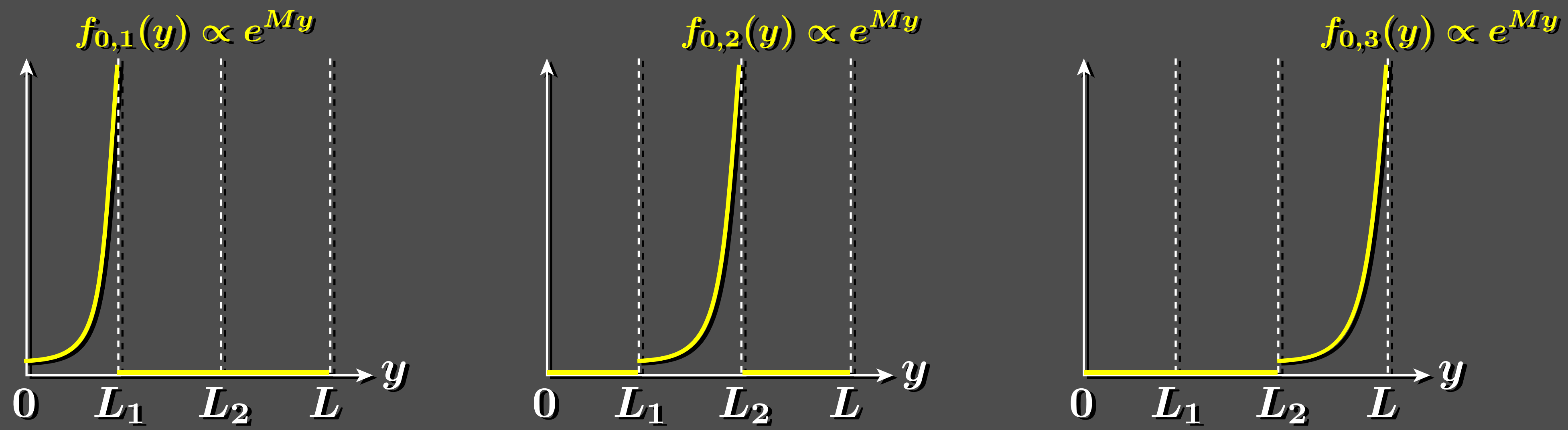

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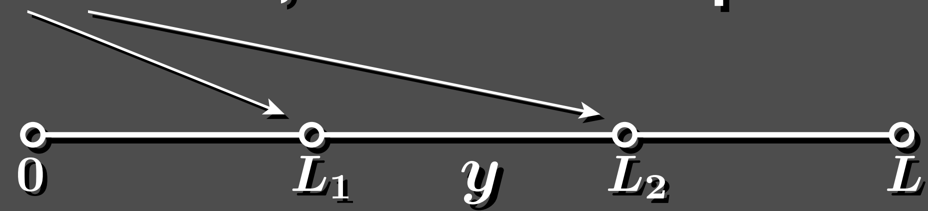

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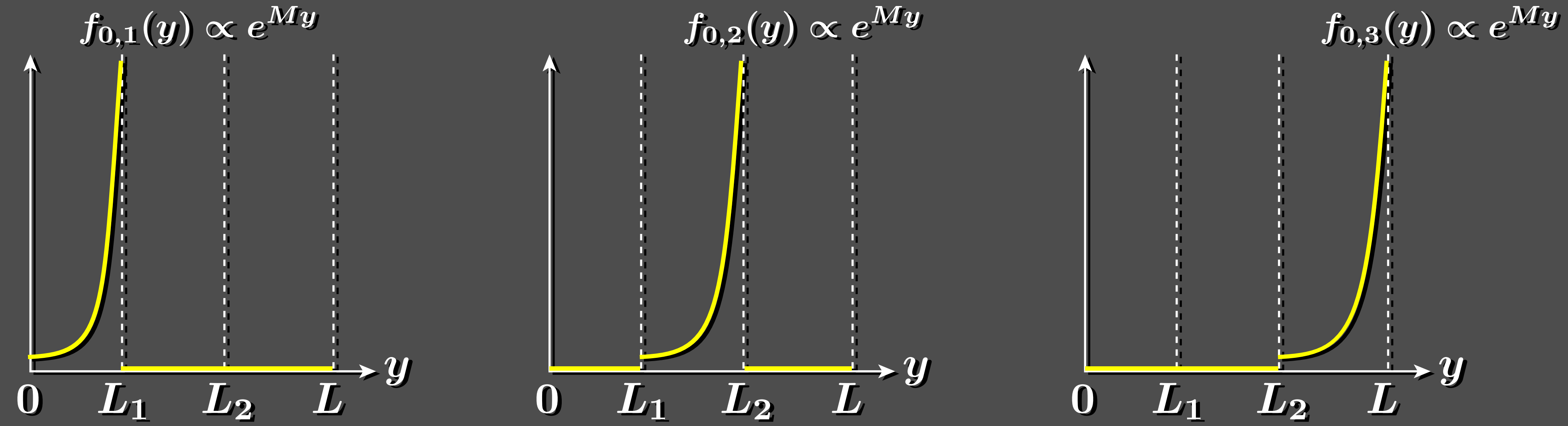
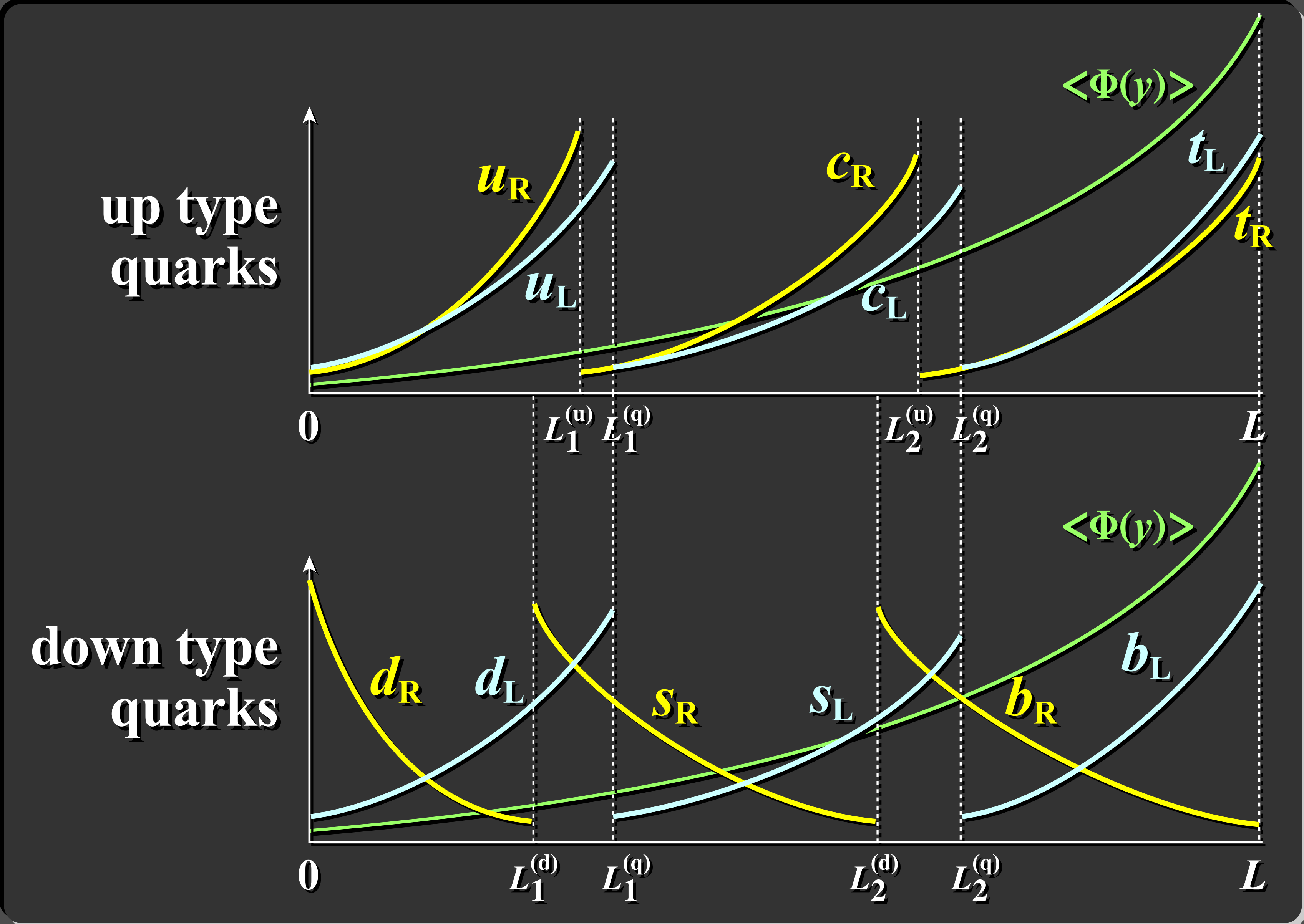
localized at point interactions ($f_{0,i}(y)$)

three generations ($\psi^{(0)}_{\mathrm{R},i}$)



Quark flavor structure in our model



$m_u \ll m_c \ll m_t$

$m_d \ll m_s \ll m_b$

$m_b \ll m_t$

Quark flavor mixing is small!

Y.Fujimoto, T.Nagasawa, K.Nishiwaki, M.S.
PTEP 2013(2013)023B07

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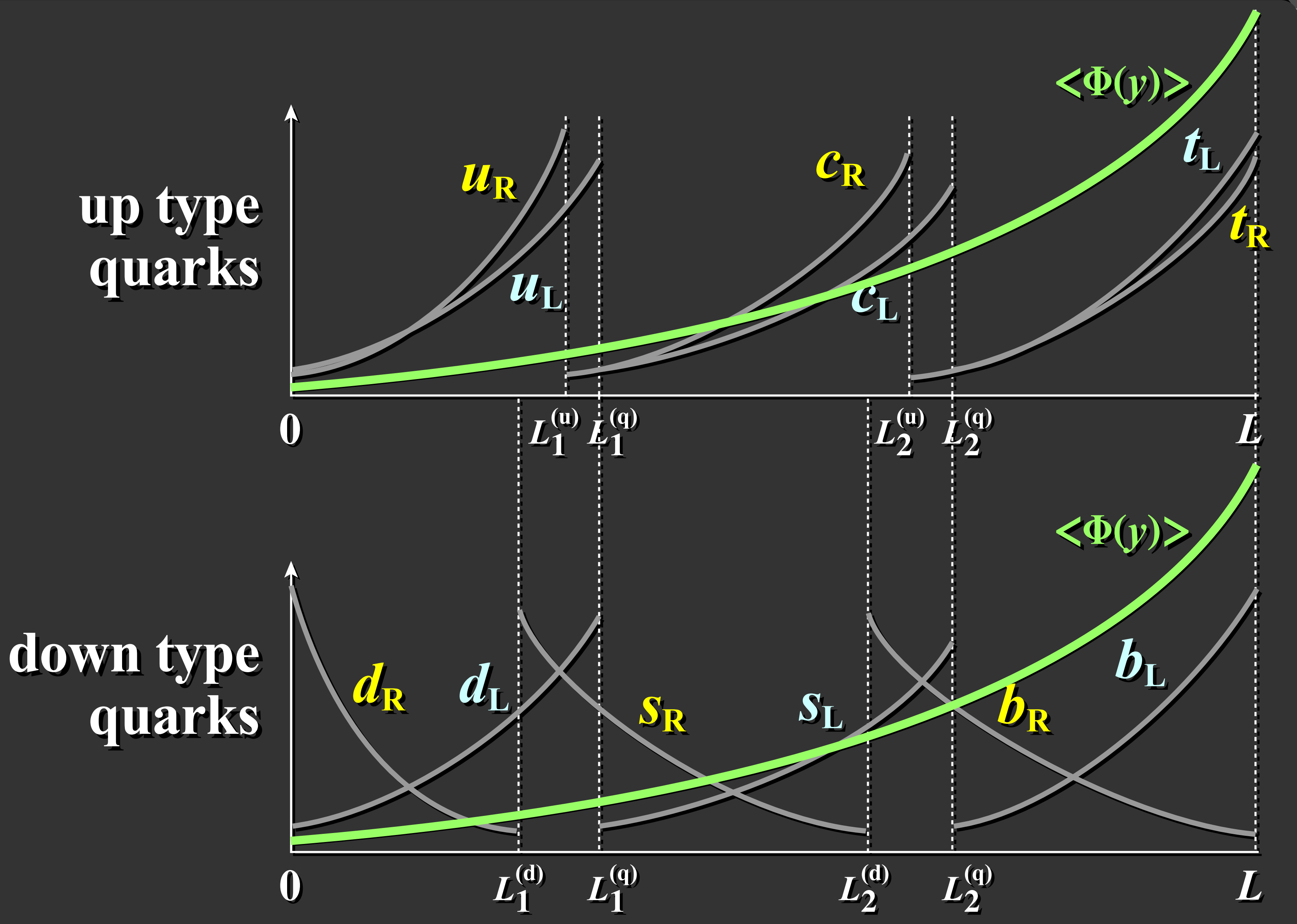

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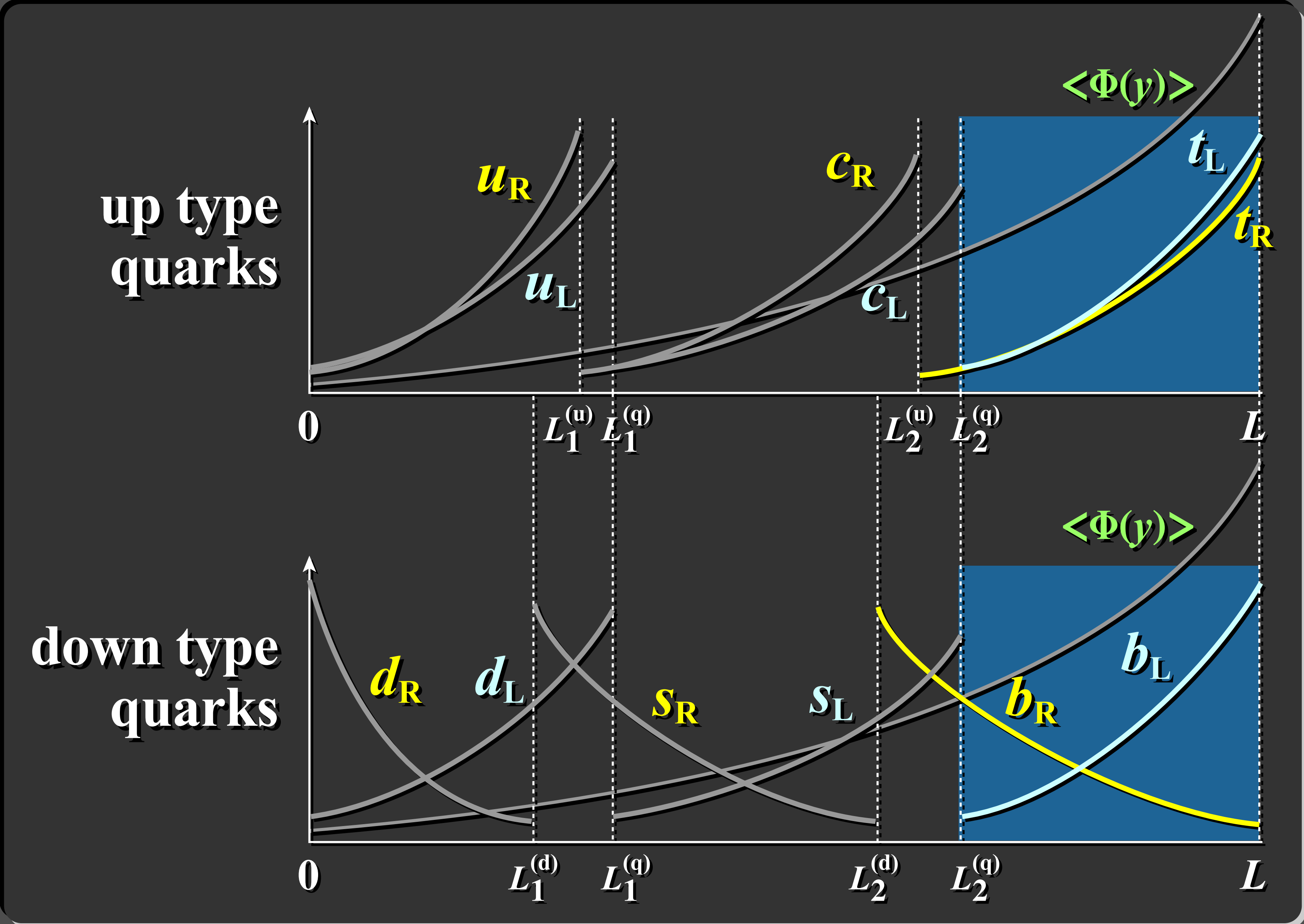

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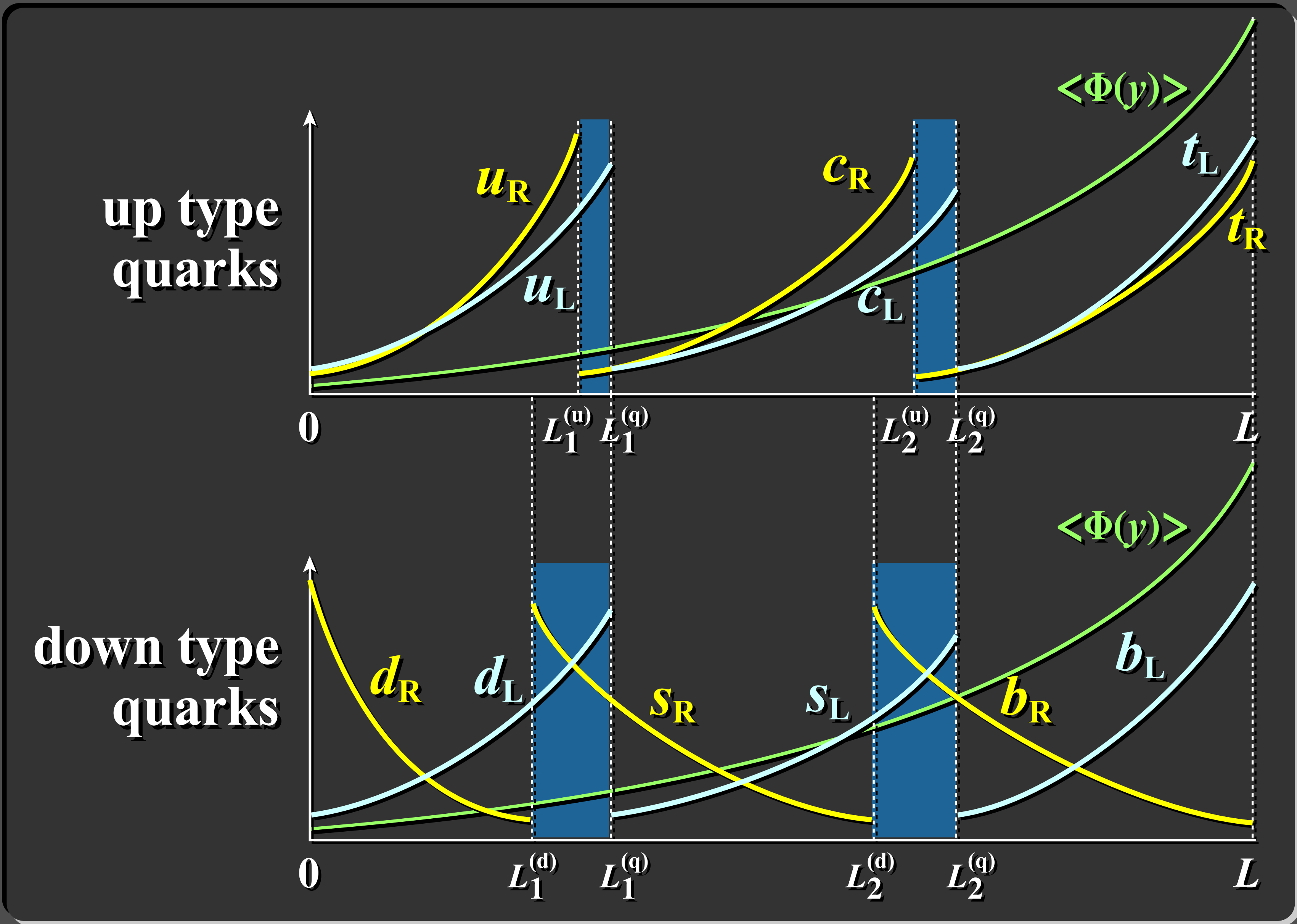

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Characteristic features of our model

★ The mass hierarchy is naturally realized.

★ Our model naturally produces small quark flavor mixings and large lepton flavor ones.

★ The mass matrices are severely restricted from the geometry of the extra dimension.

$$M_{ij} = \begin{pmatrix} m_{11} & m_{12} & 0 \\ 0 & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{pmatrix}$$

★ It is impossible to obtain observed quark masses without quark flavor mixing in our model.

★ The observed values of quark & lepton masses and mixings can be realized within 10% errors.

- ❑ Motivation to considering extra dimensions
- ❑ Mysteries of the Standard Model
- ❑ General features of extra dimensions
- ❑ Setup
- ❑ Point interactions
- ❑ Dynamical generation of fermion mass hierarchy
- ❑ Summary

Parameters of our models

- Quark sector

quark bulk mass:	3
# of point interactions:	6 (+3)
size of interval:	1
Higgs parameters:	2
CP phase:	1

Parameters of our models

❑ Quark sector

quark bulk mass:	3
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CP phase:	1

The number of the physical parameters (=10) is less than our input parameters, although it does not mean, due to the gemetrical restriction, that our model could reproduce any values of physical observables.

Dynamical generation of mass hierarchy

We have determined the positions of the point interactions to reproduce the observed values.

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Can the positions of the point interactions be determined dynamically?

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Yes!

Dynamical generation of mass hierarchy

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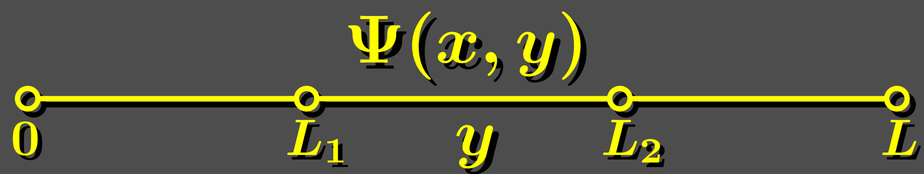
Can the positions of the point interactions be determined dynamically?

Yes!

They can be determined by minimizing the vacuum energy (= ***Casimir energy***), which is a function of the positions of the point interactions!

A preliminary result

A 5d fermion on an interval with 2 point interactions



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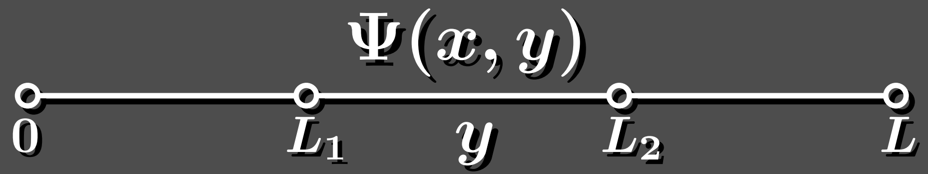

Minimizing the Casimir energy $E_0(L_1, L_2)$

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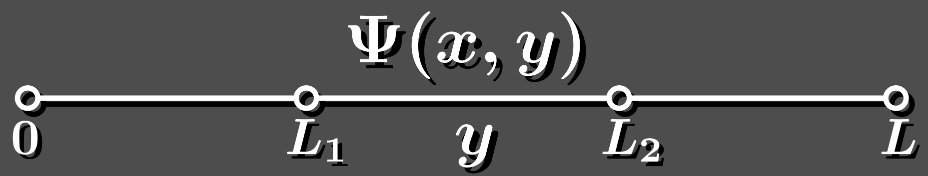

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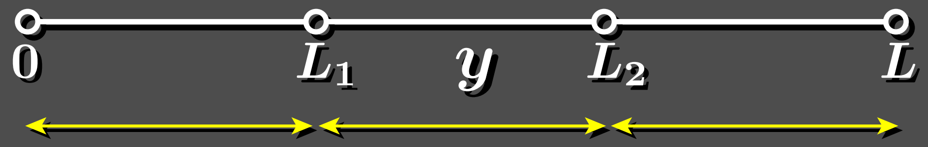

The ***regular intervals*** are a vacuum configuration!

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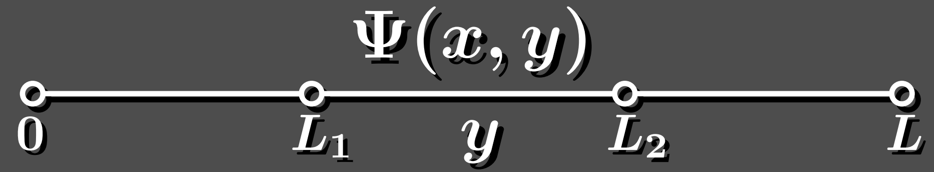

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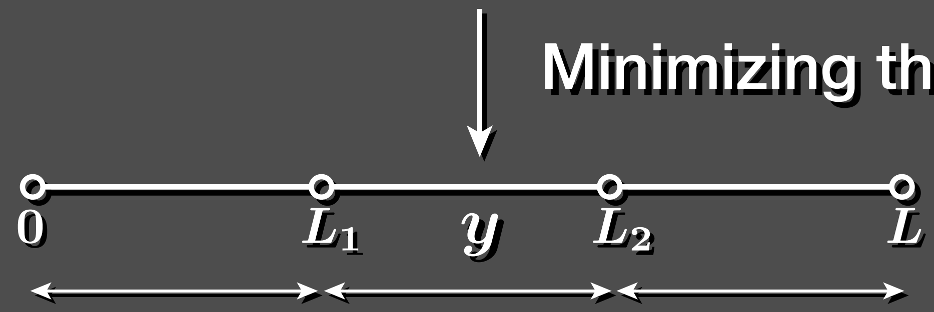

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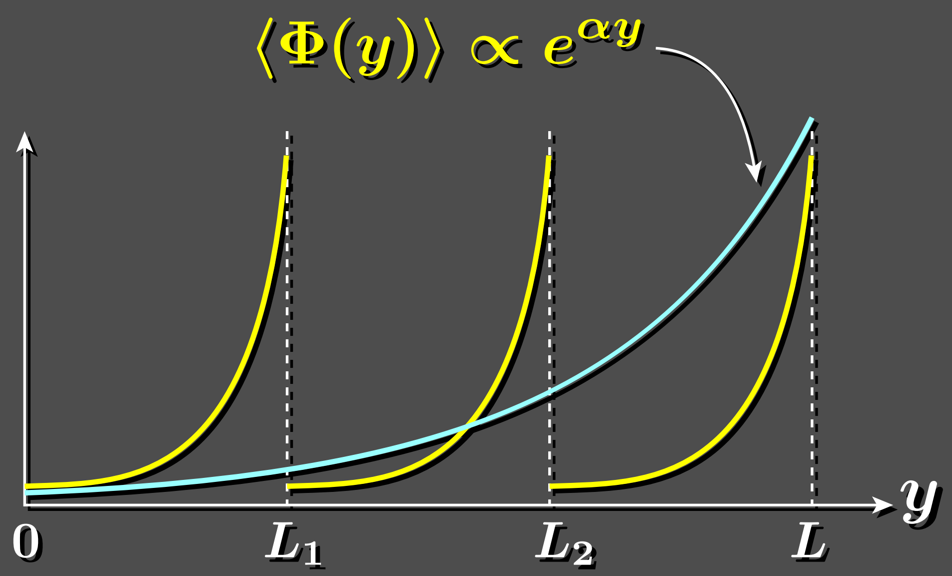

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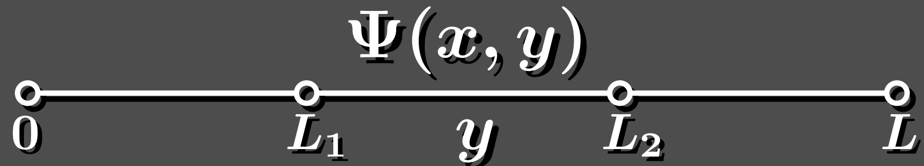

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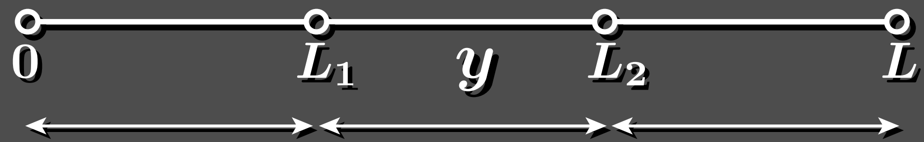
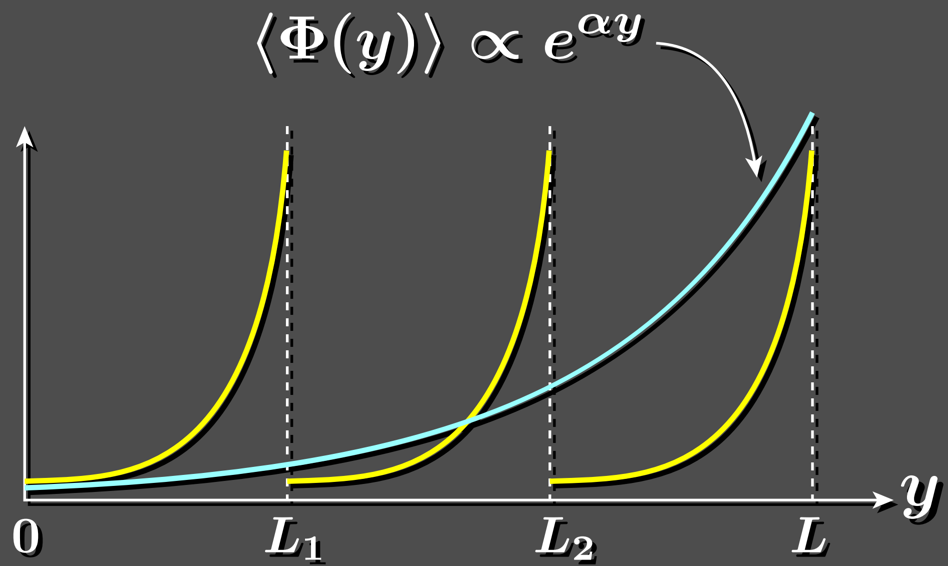
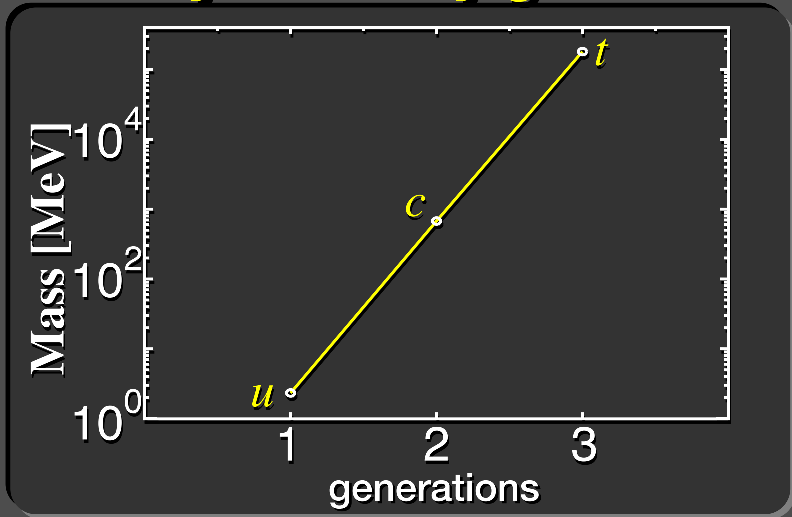


The ***regular intervals*** are a vacuum configuration!

The exponential mass hierarchy can be dynamically generated!





A problem

If the positions of the point interactions are exactly regular intervals, then there are no flavor mixings!

We need some discrepancy from the uniform distribution.

We should analyze our model in more realistic situation.

$\downarrow$

work in progress!

Summary

We have shown that the quark & lepton flavor structure can naturally be explained from the ***geometry*** of an extra dimension with point interactions.

an extra dimension with point interactions

quark & lepton flavor structure

Numerical results

- Quark sector

$$m_{\mathrm{up}} = 2.5\,\mathrm{MeV}$$

$$m_{\mathrm{charm}} = 1.34\,\mathrm{GeV}$$

$$m_{\mathrm{top}} = 173\,\mathrm{GeV}$$

$$m_{\mathrm{down}} = 4.8\,\mathrm{MeV}$$

$$m_{\mathrm{strange}} = 104\,\mathrm{MeV}$$

$$m_{\mathrm{bottom}} = 4.18\,\mathrm{GeV}$$

$$|V_{CKM}| = \begin{pmatrix} 0.971 & 0.238 & 0.00377 \\ 0.237 & 0.971 & 0.0403 \\ 0.00887 & 0.0395 & 0.999 \end{pmatrix}$$

$$J_{\mathrm{quark}} = 3.23 \times 10^{-5}$$

Numerical results

- Lepton sector

$$m_{\nu_1} = 0.0092\,\mathrm{eV}$$

$$m_{\nu_2} = 0.013\,\mathrm{eV}$$

$$m_{\nu_3} = 0.018\,\mathrm{eV}$$

$$m_{\mathrm{electron}} = 0.519\,\mathrm{MeV}$$

$$m_{\mathrm{muon}} = 106\,\mathrm{MeV}$$

$$m_{\mathrm{tau}} = 1.778\,\mathrm{GeV}$$

$$\sin^2\theta_{12} = 0.333$$

$$\sin^2\theta_{23} = 0.435$$

$$\sin^2\theta_{13} = 0.0239$$

$$J_{\mathrm{lepton}} = 0.0214 \quad (\sin\delta = 0.607)$$

Neutrino masses

How can tiny neutrino masses be generated in our model?

Answer from our point of view

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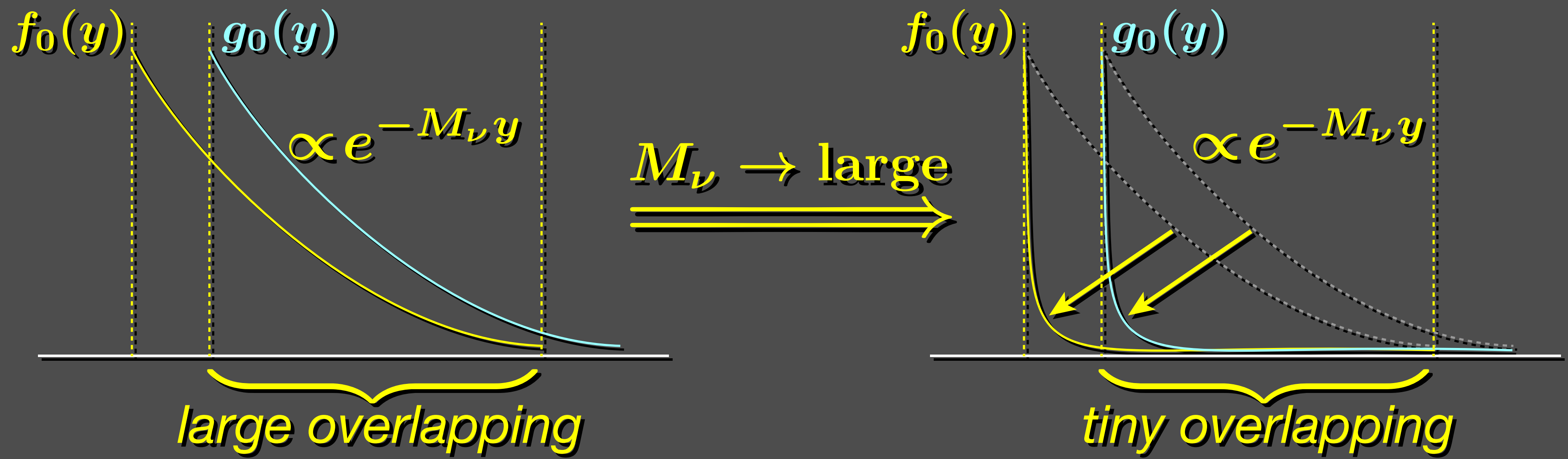

large ***bulk*** neutrino mass $\Longleftrightarrow$ tiny neutrino mass

CP phases

What is the origin of the CP phases in our model?

Answer from our point of view

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one scalar model $\longrightarrow$ no source of CP phases

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⇨ two scalar model

$$\Phi(x,y) \xrightarrow{\text{Robin b.c.}} \langle \Phi(y) \rangle \propto e^{\alpha y}$$

$$H(x,y) \xrightarrow[\text{twsited b.c.: } H(y+L) = e^{i\theta} H(y)]{} \langle H(y) \rangle = v e^{i\theta y/L}$$

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$\Rightarrow$ two scalar model

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The origin of CP phases of both quark and lepton sectors!

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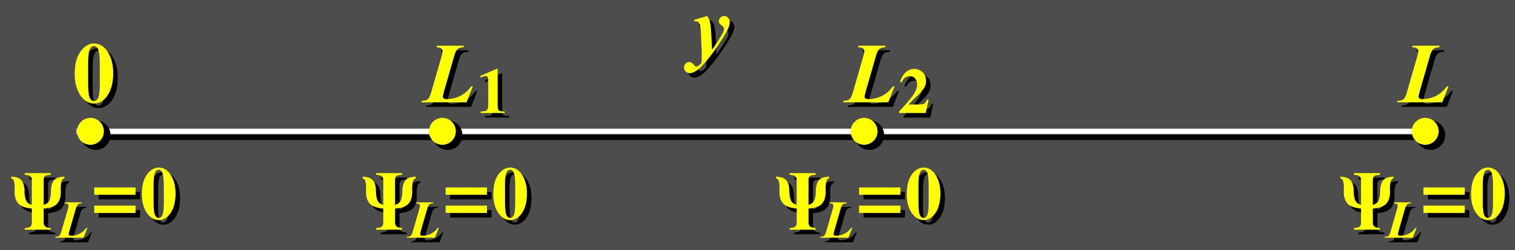
⇨ good news

We have found that the parameter fitting becomes better with the CP phase from 20% to 10%!

Profiles on an extra dimension



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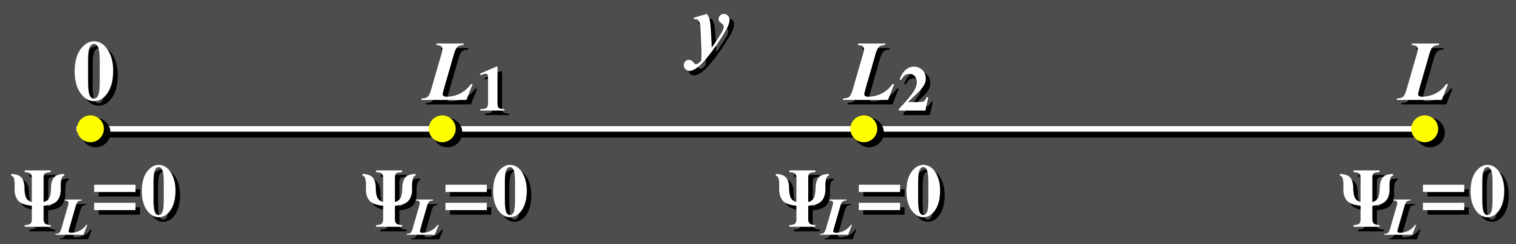
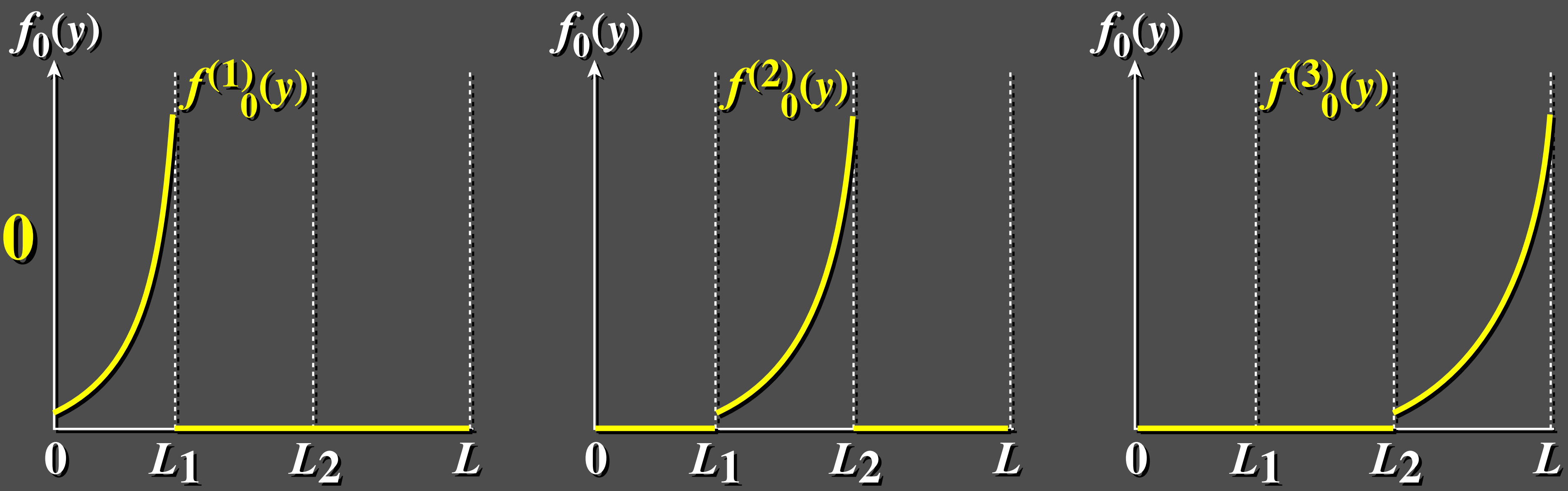


$$\Psi(x,y) = \sum_{i=1}^{3} \psi^{(i)}_{\mathrm{R},0}(x)\, f^{(i)}_0(y) + \left(\text{massive modes}\right)$$

ゼロモード解：$(\partial_y - M)\, f_0(y) = 0 \qquad (0<y<L)$

$M>0$



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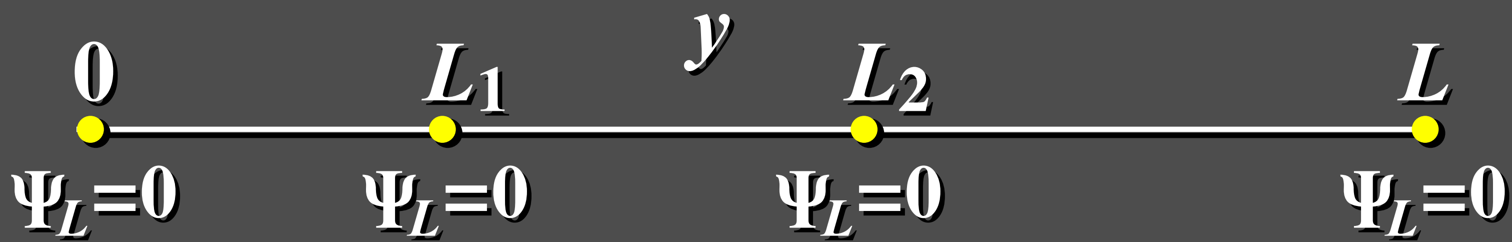

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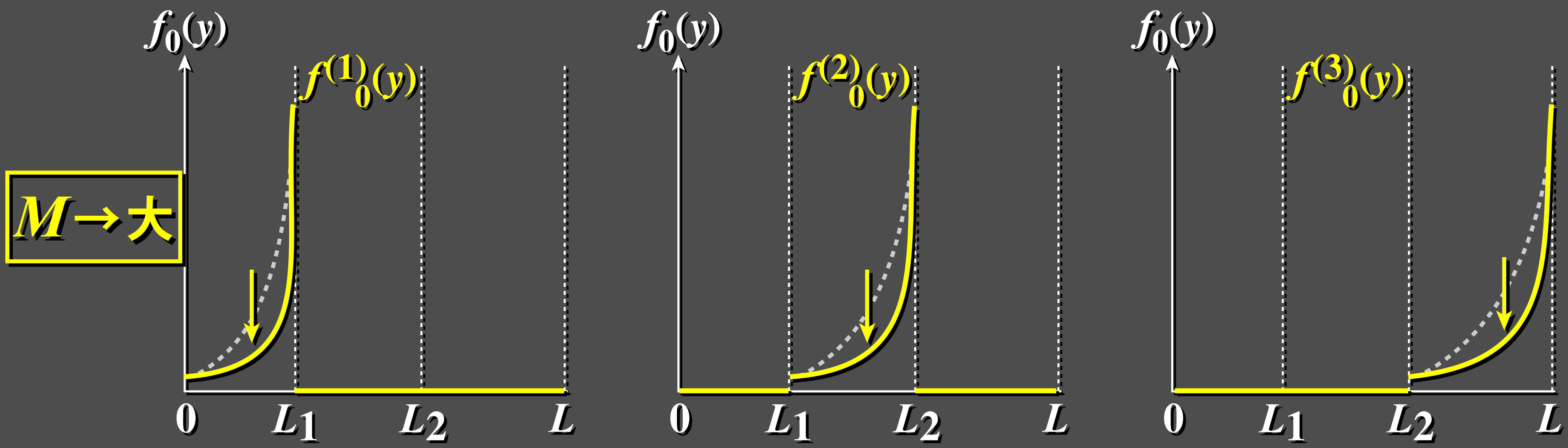

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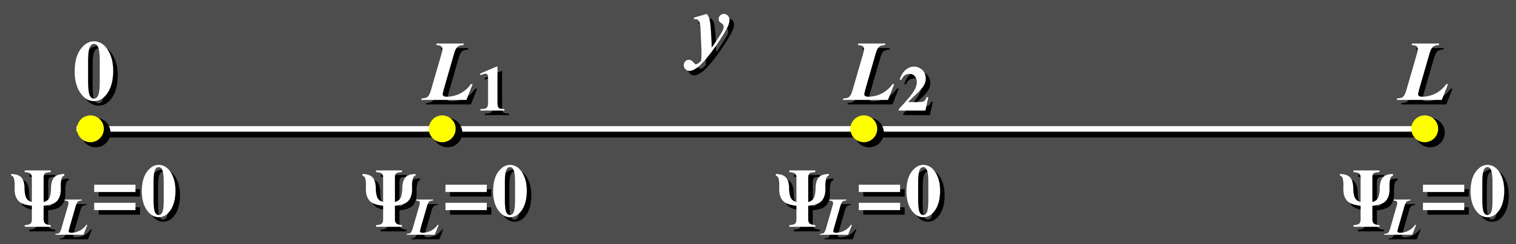

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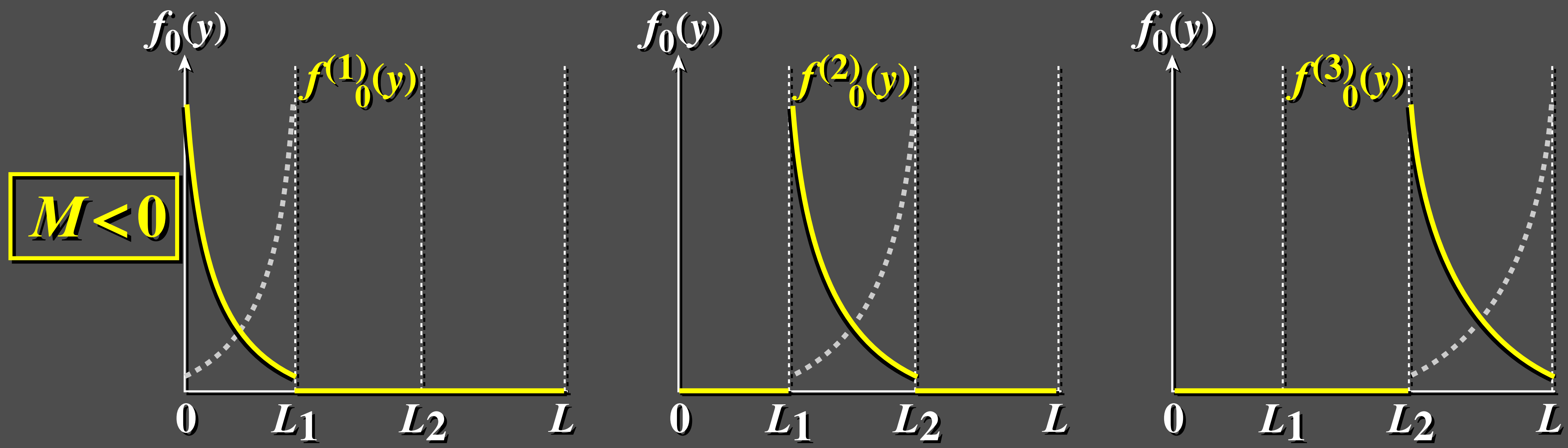

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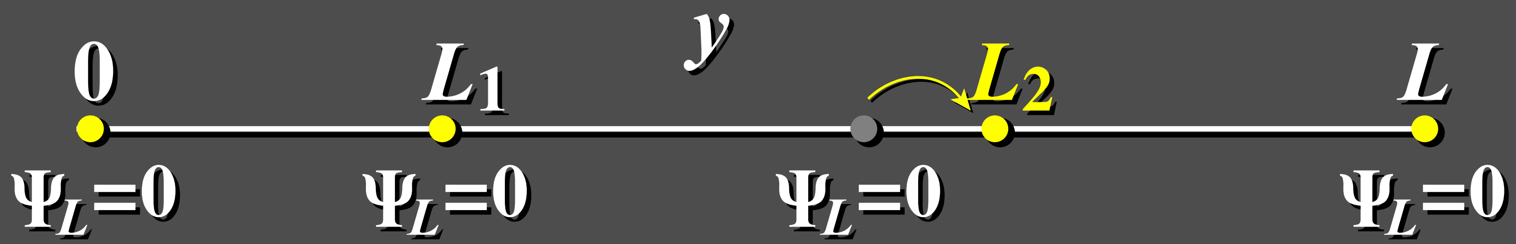

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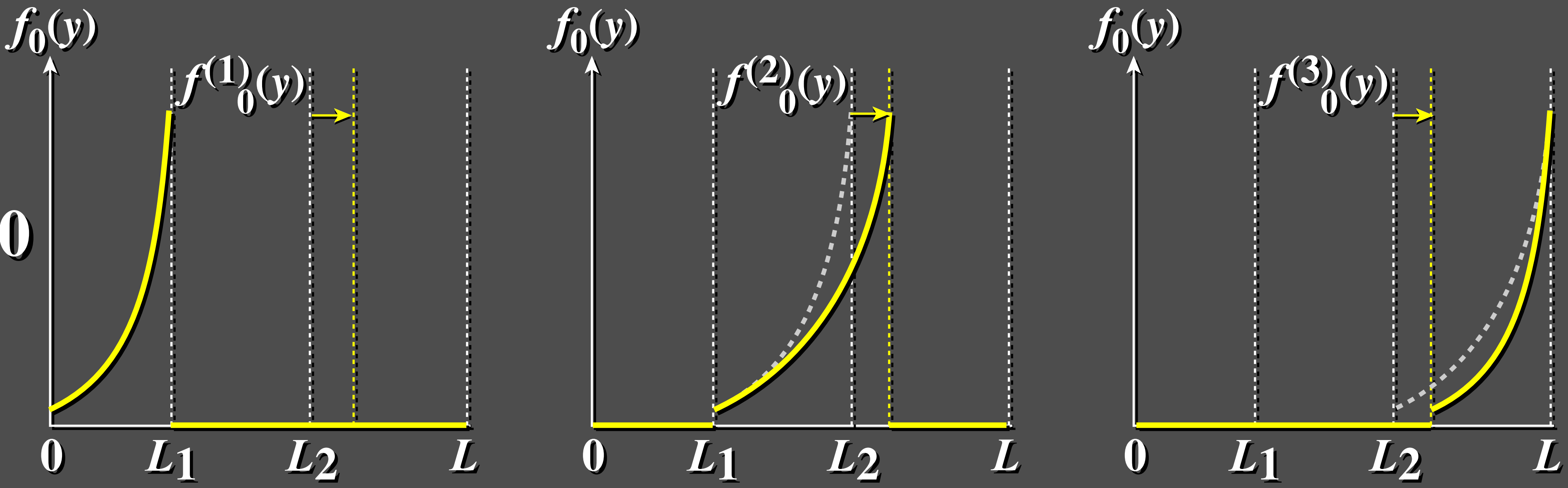
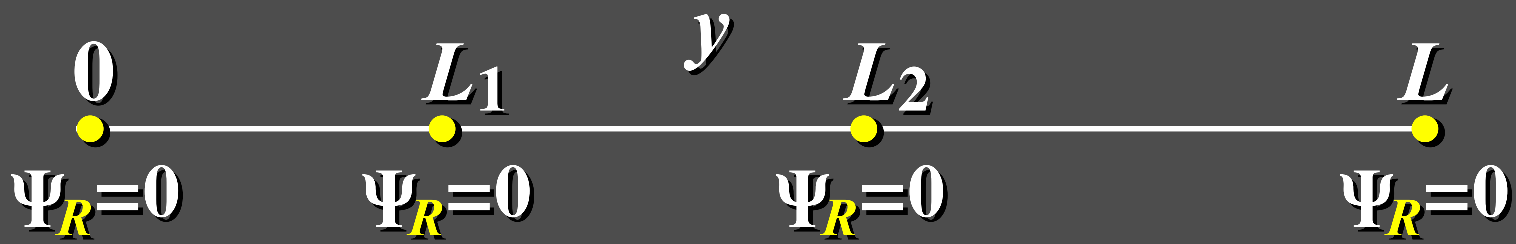
$M > 0$



Profiles on an extra dimension



$$\Psi(x,y)=\sum_{i=1}^{3}\psi^{(i)}_{\mathrm{L},0}(x)\,g^{(i)}_{0}(y)\;+\Big(\text{massive modes}\Big)$$

ゼロモード解：$\left(-\partial_y-M\right)g_0(y)=0 \qquad (0<y<L)$

$M>0$