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A realistic gauge-higgs unification model on $M^4 \times S^1/Z_2$

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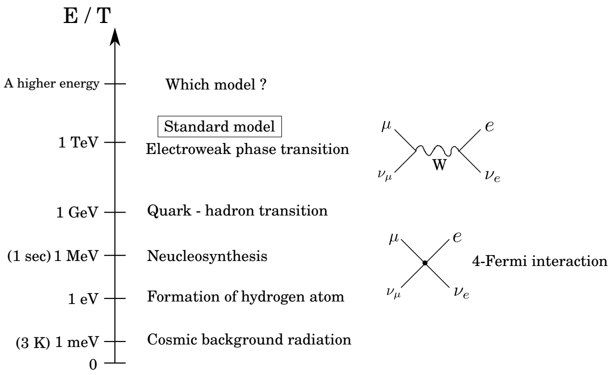
1. Introduction



- What is the model beyond the standard model ?
- What is the origin of the special features ?
  - Chiral gauge symmetry : $\mathrm{SU(2)_L \times U(1)_Y}$
  - Three generation and mass variety
  - Higgs field : Discovered at LHC in 2012, $m_H \simeq 126\,[\mathrm{GeV}]$

Gauge-Higgs unification theory

- Gauge theory on $M^4 \times$ (compact space)
- Extra space component of the gauge field = Higgs field

$$A_y = H$$

Fascination and merit

- Determined by
  (Gauge group, Compact space, Repr. of matter fields)
- Give the origin of the higgs field
- Higgs potential is controlled by gauge symmetry

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

A realistic and simple model

M.Kubo, C.S.Lim and H.Yamashita, Mod.Phys.Lett.A 17,2249(2002)
K.Hasegawa, N.Kurahashi, C.S.Lim and K.Tanabe, Phys.Rev.D87,016011(2013)

(SU(3), S^1/Z_2, $s = 1/2$ in fund.)

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy\ \mathcal{L} \quad \text{and} \quad \boxed{\mathcal{L} = \bar{\psi} i D \psi - \frac{1}{2}\mathrm{Tr}[F^2]}$$

- S^1/Z_2 orbifold

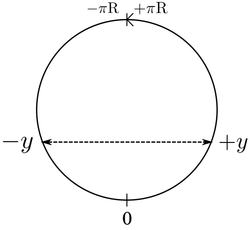
Identify the points $+y$ and $-y$
Same observables at the points, $\pm y$

Z_2-parity assignment : $SU(3) \to SU(2)_L \times U(1)_Y$

$$\psi = \begin{pmatrix} + \\ + \\ - \end{pmatrix} \to \begin{pmatrix} u \\ d \end{pmatrix}_L \text{ : SU(2)-doublet}$$

$$\to \quad d_R \quad \text{ : SU(2)-singlet}$$

$$A_y = \left(\begin{array}{cc|c} - & - & + \\ - & - & + \\ \hline + & + & - \end{array}\right) \to \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \text{ : SU(2)-doublet}$$



- Electroweak symmetry breaking

$$\langle \mathrm{H} \rangle = \mathrm{V} : \ \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \ \rightarrow \ \mathrm{U}(1)_{em}$$

Today's aim

- Mass spectra and (interactions)
- How are they realized ?
  - Chiral gauge theory (Chiral zero modes of $s = 1/2$ fields)
  - $\mathrm{SU}(2)_L$-doublet higgs field
  - Weinberg angle
  - Heavy and light quark masses

Table contents

2. Model definition

- Lagrangian

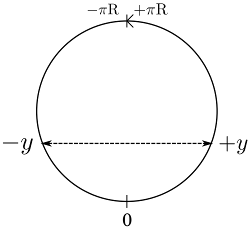
$$\mathcal{L} = \bar{\psi} i \gamma^M D_M \psi(x, y) - \frac{1}{2} Tr[F^{MN} F_{MN}]$$

$$x^M = (x^\mu, x^5 = y) \text{ and } A_M = \sum_{a=1}^{8} A_M^a t^a$$

$$D_M = \partial_M - igA_M$$

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N].$$

- Z_2-parity transformation: $y \to -y$



$$
\begin{array}{lrcl}
s = 1/2 & \psi(\vec{x}, y) & \to & \psi'(\vec{x}, y) = \pm\gamma^5 \psi(\vec{x}, -y) \\
s = 1 & A_\mu(\vec{x}, y) & \to & A'_\mu(\vec{x}, y) = +A_\mu(\vec{x}, -y), \\
 & A_y(\vec{x}, y) & \to & A'_y(\vec{x}, y) = -A_y(\vec{x}, -y).
\end{array}
$$

- S^1/Z_2 orbifold condition

$$
\begin{array}{rcl}
\psi'(\vec{x}, y) & = & \lambda_\psi \psi(\vec{x}, y) \quad \text{with} \ \ \lambda_\psi = \pm 1 \\
A'_M(\vec{x}, y) & = & \lambda_A A_M(\vec{x}, y) \quad \text{with} \ \ \lambda_A = +1 \ \ (\because g\bar{\psi}\gamma^M A_M \psi)
\end{array}
$$

The Z_2-parity of each field is determined as

$$
\begin{array}{rcl}
\begin{pmatrix} \eta(\vec{x}, y) \\ \chi(\vec{x}, y) \end{pmatrix} & = & \lambda_\psi \begin{pmatrix} \eta(\vec{x}, -y) \\ -\chi(\vec{x}, -y) \end{pmatrix} \\
\begin{pmatrix} A_\mu(\vec{x}, y) \\ A_y(\vec{x}, y) \end{pmatrix} & = & \begin{pmatrix} A_\mu(\vec{x}, -y) \\ -A_y(\vec{x}, -y) \end{pmatrix}
\end{array}
$$

$\to$ Zero modes:

$$
\psi(x)^{(0)} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}^{(0)} \text{for } \lambda_\psi = 1, \quad A_\mu(x)^{(0)} : \mathrm{SU(3)}, \text{ and } A_y(x)^{(0)} = H^{(0)} = 0 : \text{No higgs,}
$$

$\to$ Unlike standard model

- Make Z_2-parity assignment non-trivial (gauge component dependent)

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}'(x,y) = \mathrm{P}\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}(x,y) \ \text{ with } \ \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$A'_M(\vec{x},y) = \mathrm{P}A_M(\vec{x},y)\mathrm{P}^\dagger$$

$$\longrightarrow \quad \psi = \left(\begin{array}{c} + \\ + \\ \hline - \end{array}\right), \ A_\mu = \left(\begin{array}{cc|c} + & + & - \\ + & + & - \\ \hline - & - & + \end{array}\right), \text{ and } \ A_y = \left(\begin{array}{cc|c} - & - & + \\ - & - & + \\ \hline + & + & - \end{array}\right).$$

- The mode expansions of the fields are determined as

$$\eta(x,y) = \begin{pmatrix} \eta_D(x,y) \\ \eta_3(x,y) \end{pmatrix} = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} \sum_{n=0}^{\infty} \eta_D^{(n)}(x)\cos a_n y \\ \sum_{n=1}^{\infty} \eta_3^{(n)}(x)\sin a_n y \end{pmatrix}$$

$$\chi(x,y) = \begin{pmatrix} \chi_D(x,y) \\ \chi_3(x,y) \end{pmatrix} = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} \sum_{n=1}^{\infty} \chi_D^{(n)}(x)\sin a_n y \\ \sum_{n=0}^{\infty} \chi_3^{(n)}(x)\cos a_n y \end{pmatrix}.$$

$$\begin{pmatrix} A_\mu^{sym}(x,y) \\ A_y^{sym}(x,y) \end{pmatrix} = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} \sum_{n=0}^{\infty} A_\mu^{sym,(n)}(x) \ \cos a_n y \\ \sum_{n=1}^{\infty} A_y^{sym,(n)}(x) \ \sin a_n y \end{pmatrix}$$

$$\begin{pmatrix} A_\mu^{bro}(x,y) \\ A_y^{bro}(x,y) \end{pmatrix} = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} \sum_{n=1}^{\infty} A_\mu^{bro,(n)}(x) \ \sin a_n y \\ \sum_{n=0}^{\infty} A_y^{bro,(n)}(x) \ \cos a_n y \end{pmatrix}.$$

$(a_n = n/R)$

- Gauge symmetry broken by the Z_2-parity assignment

$$\mathrm{SU}(3) \;\; \rightarrow \; \mathrm{SU}(2)_L \; \times \; \mathrm{U}(1)_Y$$

The generators are classified as

$$\begin{aligned} t^{a_{sym}} &= \{ t^{a_{SU(2)}}, t^{a_{U(1)}} \} \\ t^{a_{SU(2)}} &= \{ t^1, t^2, t^3 \} \;\; \text{and} \;\; t^{a_{U(1)}} = t^8 \\ t^{a_{bro}} &= \{ t^4, t^5, t^6, t^7 \}. \end{aligned}$$

Correspondingly the gauge fields also are classified as

$$A_M^{sym} = A_M^{a_{sym}} t^{a_{sym}} \quad \text{and} \quad A_M^{bro} = A_M^{a_{bro}} t^{a_{bro}}.$$

- Zero modes:

	Repr. of $\mathrm{SU}(2)_L$
$\eta_D^{(0)}(x)$	2
$\chi_3^{(0)}(x)$	1
$A_\mu^{sym,(0)}(x)$	3
$A_y^{bro,(0)}(x)$	2

$\rightarrow$ Like standard model

Gauge sector at symmetric phase

$$\mathcal{L}_A = -\frac{1}{2}\mathrm{Tr}[F_{MN}F^{MN}]$$

Extract free part

$$\mathcal{L}_{A,free} = -\frac{1}{2}\mathrm{Tr}[f_{MN}f^{MN}] = -\frac{1}{4}f^a_{\mu\nu}f^{\mu\nu,a} - \frac{1}{2}f^a_{\mu y}f^{\mu y,a} \ \text{ with } \ f_{MN} = \partial_M A_N - \partial_N A_M$$

The Kaluza-Klein(KK) decomposition is done as

$$\begin{aligned}
\mathcal{L}_{A,free,4} &= \int_{-\pi R}^{\pi R} dy \ \mathcal{L}_{A,free} \\
&= -\frac{1}{2}f^{a_{sym},(0)}_{\mu\nu}f^{\mu\nu,a_{sym},(0)} - \frac{1}{4}\sum_{n=1}^{\infty} f^{a,(n)}_{\mu\nu}f^{\mu\nu,a,(n)} \\
&+ (\partial_\mu H^{a_{bro},(0)})(\partial^\mu H^{a_{bro},(0)}) + \frac{1}{2}\sum_{n=1}^{\infty}\Big[\partial_\mu H^{a,(n)}\partial^\mu H^{a,(n)} + a_n^2 A^{a,(n)}_\mu A^{\mu,a,(n)} \\
&+ 2a_n\Big\{(\partial_\mu H^{a_{sym},(n)})A^{\mu,a_{sym},(n)} - (\partial_\mu H^{a_{bro},(n)})A^{\mu,a_{bro},(n)}\Big\}\Big].
\end{aligned}$$

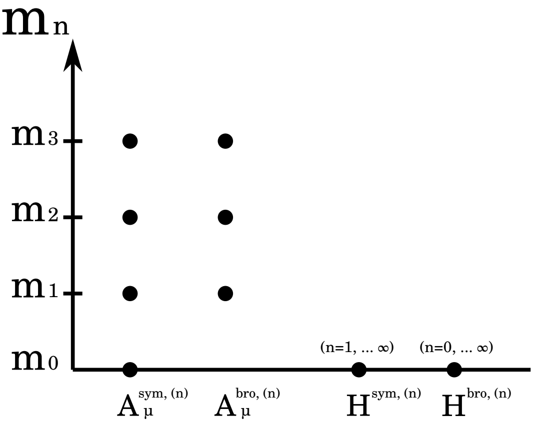


Figure : The mass spectra of the gauge and scalar fields with $m_n = n/R$.

Fermion sector at symmetric phase

$$\mathcal{L}_\psi = \bar{\psi} i D\psi(x,y) \quad \supset \quad \mathcal{L}_{\psi,free} = \bar{\psi} i\gamma^M \partial_M \psi$$

The KK-decomposition is done as

$$\mathcal{L}_{\psi,free,4} = 2(\bar{Q}_L^{(0)} i\gamma^\mu \partial_\mu Q_L^{(0)} + \bar{q}_R^{(0)} i\gamma^\mu \partial_\mu q_R^{(0)}) + \sum_{n=1}^{\infty} \bar{\psi}^{(n)}(i\gamma^\mu \partial_\mu - m_n)\psi^{(n)},$$

where the spinors are introduced after the chiral transformation

$$\psi^{(n)}(x) = \begin{pmatrix} Q^{(n)}(x) \\ q^{(n)}(x) \end{pmatrix} = \begin{pmatrix} \psi_D^{(n)}(x) \\ e^{i\pi\gamma^5/2}\psi_3^{(n)}(x) \end{pmatrix} .$$

The mass of the n-th Kaluza Klein mode :

$$m_n = \frac{n}{R}$$

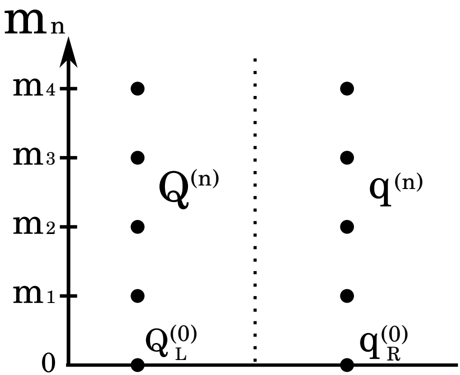


Figure : The mass spectra of the spin 1/2 fields with $m_n = n/R$.

Zero mode sector at symmetric phase

Zero mode sector is extracted as

$$\mathcal{L}_{A,SU(2)} = -\frac{1}{2}\mathrm{Tr}\big[f_{\mu\nu}f^{\mu\nu}\big] + \frac{ig}{\sqrt{\pi R}}\cdot\sqrt{2}\mathrm{Tr}\big[(\partial_\mu A_\nu)[A^\mu, A^\nu]\big] + \frac{g^2}{\pi R}\cdot\frac{1}{4}\mathrm{Tr}\big[[A_\mu, A_\nu][A^\mu, A^\nu]\big]$$

$$\mathcal{L}_{A,U(1)} = -\frac{1}{4}f^{U(1)}_{\mu\nu}f^{\mu\nu,U(1)}.$$

$$\mathcal{L}_{Q_L} = \bar{Q}_L\Big(i\gamma^\mu\partial_\mu + \frac{g}{\sqrt{\pi R}}\cdot\frac{1}{\sqrt{2}}\gamma^\mu A_\mu + \frac{g}{\sqrt{\pi R}}\cdot\frac{1}{4\sqrt{3}}\gamma^\mu B_\mu\Big)Q_L$$

$$\mathcal{L}_{q_R} = \bar{q}_R\Big(i\gamma^\mu\partial_\mu + \frac{g}{\sqrt{\pi R}}\cdot\frac{1}{2\sqrt{3}}\gamma^\mu B_\mu\Big)q_R.$$

$$\mathcal{L}_{higgs} = (\partial_\mu h)^\dagger\partial^\mu h + \frac{ig}{\sqrt{\pi R}}\cdot\frac{1}{\sqrt{2}}(\partial_\mu h^\dagger A^\mu h - h^\dagger A^\mu\partial_\mu h) + \frac{ig}{\sqrt{\pi R}}\cdot\frac{\sqrt{6}}{4}B^\mu(\partial_\mu h^\dagger h - h^\dagger\partial_\mu h)$$
$$+ \frac{g^2}{\pi R}\cdot\frac{1}{2}\ h^\dagger A^\mu A_\mu h + \frac{g^2}{\pi R}\cdot\frac{3}{8}\ B^\mu B_\mu h^\dagger h + \frac{g^2}{\pi R}\cdot\frac{\sqrt{3}}{2}B^\mu h^\dagger A_\mu h.$$

$$\mathcal{L}_{yukawa} = \frac{g}{\sqrt{\pi R}}\cdot 2(\bar{Q}_L h q_R + \bar{q}_R h^\dagger Q_L).$$

There appears only one gauge coupling constant

$$g$$

Standard model lagrangian at symmetric phase :

$$\begin{aligned}
\mathcal{L}_{A,SU(2)} &= -\frac{1}{2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] \\
\mathcal{L}_{A,U(1)} &= -\frac{1}{4}f^{U(1)}_{\mu\nu}f^{\mu\nu,U(1)} \\
\mathcal{L}_{Q_L} &= \bar{Q}_L i D_{Q_L} Q_L \\
\mathcal{L}_{q_R} &= \bar{q}_R i D_{q_R}\, q_R \\
\mathcal{L}_{higgs} &= (D_{h,\mu}h)^\dagger D^\mu_h h - V(h) \\
\mathcal{L}_{yukawa} &= y_u(\bar{Q}_L\tilde{h}u_R + \bar{u}_R\tilde{h}^\dagger Q_L) + y_d(\bar{Q}_L h d_R + \bar{d}_R h^\dagger Q_L),
\end{aligned}$$

with the covariant derivatives,

$$\begin{aligned}
D_{Q_L,\mu} &= \partial_\mu - ig_2 A_\mu - ig_1 Y_{Q_L} B_\mu \\
D_{q_R,\mu} &= \partial_\mu \qquad\qquad\quad - ig_1 Y_{q_R} B_\mu \\
D_{h,\mu} &= \partial_\mu - ig_2 A_\mu - ig_1 Y_h B_\mu ,
\end{aligned}$$

with the $U(1)_Y$-hypercharges : $(Y_{Q_L}, Y_{q_R}, Y_h) = (1/6, -1/3, 1/2)$.

$$\begin{aligned}
\frac{g}{\sqrt{\pi R}} \cdot \frac{1}{\sqrt{2}} &= g_2 \\
\frac{g}{\sqrt{\pi R}} \cdot \frac{\sqrt{6}}{2} &= g_1 \\
\frac{g}{\sqrt{\pi R}} \cdot 2 &= y_d \\
0 &= y_u .
\end{aligned}$$

Prediction on Weinberg angle : $\tan\theta_W = \frac{g_1}{g_2} = \sqrt{3} \ \rightarrow\ \sin^2\theta_W = \frac{3}{4} = 0.75\,.$

Unfortunately fail to predict the experimental value, $\sin^2\theta_W = 0.231\,.$

3. Gauge sector

- $\mathrm{U}(1)_{em}$ eigenstate

$$\begin{array}{ccccccc} \mathrm{SU}(3) & \to & \mathrm{SU}(2)_L & \times & \mathrm{U}(1)_Y & \to & \mathrm{U}(1)_{em} \\ & & \{t^1, t^2, t^3\} & & \{t^8\} & & \{t_{em}\} \end{array}$$

Vacuum expectation value (VEV) : $\langle A_y \rangle = \langle H \rangle = V = v \cdot t^6 = v \cdot \frac{1}{2} \begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}$.

Symmetry of vacuum

$$\delta V \simeq iv[\theta^a t^a, t^6] = 0$$

$$\begin{pmatrix} t_Z \\ t_{em} \end{pmatrix} = R(\theta_W) \begin{pmatrix} t^3 \\ t^8 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} d(0, -1, 1) \\ \frac{1}{2\sqrt{3}} d(2, -1, -1) \end{pmatrix} \quad \text{where} \quad \theta_W = \pi/3$$

EM charges

$$\psi' = U_{em}\psi \quad \text{with} \quad U_{em} = e^{i\theta_{em} Q_{em}} \quad : \quad Q_{em} = Q_{em}^{\psi_1} \begin{pmatrix} 1 & & \\ & -1/2 & \\ & & -1/2 \end{pmatrix} \to Q_{em}^{\psi_1} = +\frac{2}{3}$$

$$\vec{A}_M' = U_{em}\vec{A}_M \quad \text{with} \quad U_{em} = e^{i\theta_{em} Q_{em}^{adj}} \quad : \quad Q_{em}^{adj} = (\text{non-diagonal}) \to \hat{Q}_{em}^{adj} = Q_{em}^{\psi_1}(3/2) \cdot diag(1, 1, 0, -1, -1, 0, 0, 0)$$

$$A_M = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c|c} 0 & W_{12}^+ & W_{13}^+ \\ \hline W_{12}^- & 0 & (A^6 - iA^7)/\sqrt{2} \\ \hline W_{13}^- & (A^6 + iA^7)/\sqrt{2} & 0 \end{array} \right)_M + Z_M \cdot t_Z + \gamma_M \cdot t_{em}$$

• Mass eigenstate

$$\mathcal{L}_A = -\frac{1}{2}\mathrm{Tr}[F_{MN}F^{MN}] = -\frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] - \mathrm{Tr}[F_{\mu y}F^{\mu y}]$$

Extract mass terms with VEV

$$A_y \to V + A_y : \quad \mathcal{L}_{A,mass} = \mathrm{Tr}\big[\ (\partial_y A_\mu)(\partial_y A^\mu) + 2ig(\partial_y A_\mu)[A^\mu, V] + g^2[A_\mu, V][A^\mu, V]\ \big]$$

KK-decomposition

$$\begin{aligned}\mathcal{L}_{A,mass,4} = {} & T_R\frac{g^2v^2}{2}\left((A^{1,(0)})^2 + (A^{2,(0)})^2 + 4(Z^{(0)})^2\right) \\ & + \sum_{n=1}^{\infty}\Big[\ a_n^2(A_\mu^{a,(n)})^2 + 2gva_n\left(A^{4,(n)}A^{2,(n)} - A^{5,(n)}A^{1,(n)} + 2A^{7,(n)}Z^{(n)}\right) \\ & + \frac{g^2v^2}{4}\Big((A^{1,(n)})^2 + (A^{2,(n)})^2 + 4(Z^{(n)})^2 + (A^{4,(n)})^2 + (A^{5,(n)})^2 + 4(A^{7,(n)})^2\Big)\Big]\end{aligned}$$

- Diagonalize
- $U(1)_{em}$ eigenstate
- Proper normalization

$$\begin{aligned}\mathcal{L}_{A,mass,4} = {} & \frac{1}{2}M_Z^{(0)\ 2}(Z^{(0)})^2 + M_W^{(0)\ 2}|W^{+(0)}|^2 \\ & + \sum_{n=1}^{\infty}\Big[\ \frac{1}{2}\Big\{M_\gamma^{(n)\ 2}(\gamma^{(n)})^2 + M_N^{(n)\ 2}(N^{(n)})^2 + M_{Z_1}^{(n)\ 2}(Z_1^{(n)})^2 + M_{Z_2}^{(n)\ 2}(Z_2^{(n)})^2\Big\} \\ & \qquad + M_{W_1}^{(n)\ 2}|W_1^{+(n)}|^2 + M_{W_2}^{(n)\ 2}|W_2^{+(n)}|^2\Big]\end{aligned}$$

where the masses are denoted as

$$(M_\gamma^{(0)}, M_Z^{(0)}, M_W^{(0)}) = \left(0, gv, \frac{gv}{2}\right),$$

$$(M_\gamma^{(n)}, M_N^{(n)}, M_{Z_1}^{(n)}, M_{Z_2}^{(n)}, M_{W_1}^{(n)}, M_{W_2}^{(n)}) = \left(a_n, a_n, |a_n + gv|, |a_n - gv|, \left|a_n + \frac{gv}{2}\right|, \left|a_n - \frac{gv}{2}\right|\right).$$

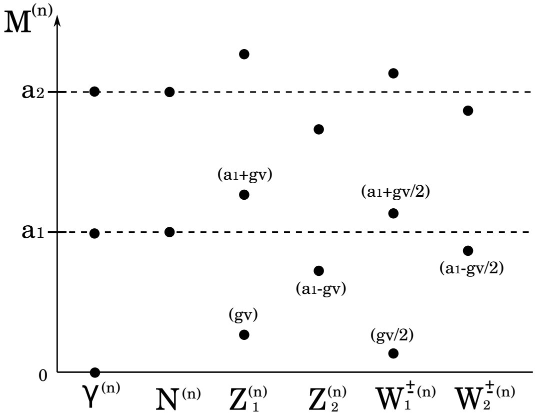
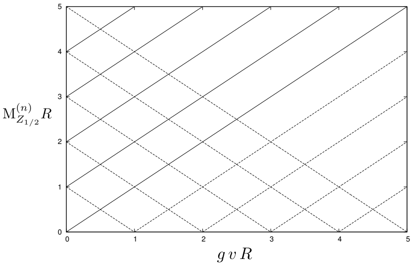
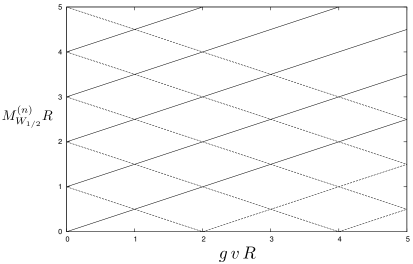


Figure : Mass spectra of gauge fields: The zero modes are denoted as $\gamma^{(0)}, Z_1^{(0)} = Z^{(0)}$ and $W_1^{(0)} = W^{(0)}$.





$$M^{(n)}_{Z_{1/2}} R = |n \pm gvR|$$

$$M^{(n)}_{W_{1/2}} R = |n \pm gvR/2|$$

- Higgs field
  - All scalars are massless $\rightarrow$ Prediction: $m_h = 0$ at tree level
  - Express in $\mathrm{U}(1)_{em}$ eigenstates

$$\mathcal{L}_{H,free,4} = (\partial_\mu h^{(0)})^\dagger (\partial_\mu h^{(0)}) + \sum_{n=1}^{\infty} \Big[(\partial_\mu h^{(n)})^\dagger (\partial_\mu h^{(n)}) + \frac{1}{2} (\partial_\mu H^{3(n)'})^2 + \frac{1}{2} (\partial_\mu H^{8(n)'})^2 + |\partial_\mu H_{12}^{+(n)}|^2 \Big] ,$$

$$\text{where} \quad h^{(n)} = \begin{pmatrix} H_{13}^{+(n)} \\ \frac{1}{\sqrt{2}} (H^{6(n)} - iH^{7(n)}) \end{pmatrix} .$$

4. Fermion sector

$$\mathcal{L}_\psi = \bar{\psi} i D \psi \quad \supset \quad \mathcal{L}_{\psi,free} = \bar{\psi} i \big(\gamma^\mu \partial_\mu + \gamma^y (\partial_y - igV)\big)\psi$$

KK-decomposition:

$$\mathcal{L}_{\psi,free,4} = \mathcal{L}^{(0)}_{\psi,free,4} + \sum_{n=1}^{\infty} \mathcal{L}^{(n)}_{\psi,free,4}$$

$$\mathcal{L}^{(0)}_{\psi,free,4} = \eta_1^{(0)\dagger} i\bar{\sigma}^\mu \partial_\mu \eta_1^{(0)} + \bar{\psi}_m^{(0)} (i\gamma^\mu \partial_\mu - m^{(0)}) \psi_m^{(0)} ,$$

where the Dirac spinor and mass are denoted as

$$\psi_m^{(0)} = \begin{pmatrix} \eta_2^{(0)} \\ -i\chi_3^{(0)} \end{pmatrix} \text{ and } m^{(0)} = \frac{gv}{2}$$

The zero mode mass is W/Z boson mass scale as

$$m^{(0)} = \frac{gv}{2} \simeq M_{W/Z} \simeq \mathcal{O}(100\text{GeV}) ,$$

which is preferred for top quark, while it can not produce the light quark masses.

$$
\mathcal{L}^{(n)}_{\psi,free,4} = (\eta_1^\dagger, \eta_2^\dagger, \eta_3^\dagger)^{(n)}\, i\bar{\sigma}^\mu \partial_\mu \mathbf{1}_3 \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}^{(n)} + (\chi_1^\dagger, \chi_2^\dagger, -\chi_3^\dagger)^{(n)}\, i\sigma^\mu \partial_\mu \mathbf{1}_3 \begin{pmatrix} \chi_1 \\ \chi_2 \\ -\chi_3 \end{pmatrix}^{(n)}
$$

$$
- (\eta_1^\dagger, \eta_2^\dagger, \eta_3^\dagger)^{(n)} \left[a_n \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{igv}{2} \begin{pmatrix} & & \\ & & 1 \\ & -1 & \end{pmatrix} \right] \begin{pmatrix} \chi_1 \\ \chi_2 \\ -\chi_3 \end{pmatrix}^{(n)}
$$

$$
- (\chi_1^\dagger, \chi_2^\dagger, -\chi_3^\dagger)^{(n)} \left[a_n \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{igv}{2} \begin{pmatrix} & & \\ & & 1 \\ & -1 & \end{pmatrix} \right] \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}^{(n)}
$$

Diagonalize and express in Dirac spinors:

$$
\mathcal{L}^{(n)}_{\psi,free,4} = \sum_{i=1}^{3} \bar{\psi}^{(n)}_{i,m} \left(i\gamma^\mu \partial_\mu - m_i^{(n)} \right) \psi^{(n)}_{i,m} \,,
$$

where

$$
(m_1^{(n)}, m_2^{(n)}, m_3^{(n)}) = \left(a_n,\ a_n - \frac{gv}{2},\ a_n + \frac{gv}{2} \right) ,
$$

$$
\psi^{(n)}_{1,m} = \psi^{(n)}_1 \ \text{ and } \ \begin{pmatrix} \psi_{2,m} \\ \psi_{3,m} \end{pmatrix}^{(n)} = V^\dagger \begin{pmatrix} \psi_2 \\ \psi_3' \end{pmatrix}^{(n)} ,
$$

with the definitions,

$$
V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} .
$$

and

$$
\psi^{(n)}_{1/2} = \begin{pmatrix} \eta_{1/2} \\ \chi_{1/2} \end{pmatrix}^{(n)} \ \text{ and } \ \psi_3^{(n)\prime} = \begin{pmatrix} \eta_3 \\ -\chi_3 \end{pmatrix}^{(n)} ,
$$

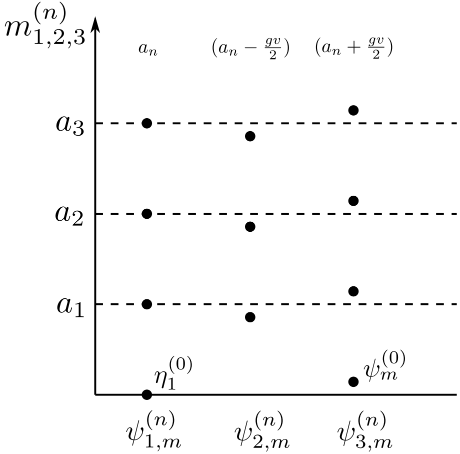


Figure : The mass spectra of the spin-1/2 fields are plotted.
$(m_1^{(n)}, m_2^{(n)}, m_3^{(n)}) = \left(a_n,\, a_n - \frac{gv}{2},\, a_n + \frac{gv}{2}\right)$.

Inclusion of bulk mass term

$$\mathcal{L}_\psi = \bar{\psi}(iD - \epsilon(y)M)\psi$$

Mass eigenvalue equation:

$$\sin^2\left(\frac{g_4}{2}v\pi R\right) = \frac{m_n^2}{m_n^2 - M^2}\sin^2\left(\sqrt{m_n^2 - M^2}\pi R\right).$$

For the light zero mode: $m_0 \ll M$,

$$m_0^2 \simeq \frac{M^2}{\sinh^2(\pi MR)}\sin^2\left(\frac{g_4}{2}v\pi R\right)$$

The light quark masses can be realized.

5. Summary

- Gauge-higgs unification model
  (SU(3), S^1/Z_2, $s = 1/2$ in fund.)

- Mass spectra at symmetric and broken phase

- How are they realized ?
  - Chiral gauge theory (Chiral zero modes of $s = 1/2$ fields)
  - $SU(2)_L$-doublet higgs field
  - Weinberg angle
  - Heavy and light quark masses

- Unsatisfactory points
  - No Lepton sector
  - No QCD interaction
  - Large Weinberg angle, $\sin^2\theta_W = 0.751$
  - Vanishing Yukawa coupling, $y_u = 0$
  - No second and third generation

Appendix

SU(3) generators

$$t^1 = \frac{1}{2}\begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix}, \qquad t^2 = \frac{1}{2}\begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix}, \quad t^3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}$$

$$t^4 = \frac{1}{2}\begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix}, \qquad t^5 = \frac{1}{2}\begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix},$$

$$t^6 = \frac{1}{2}\begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}, \qquad t^7 = \frac{1}{2}\begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix}, \quad t^8 = \frac{1}{\sqrt{12}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$