Conservation of ζ from holography

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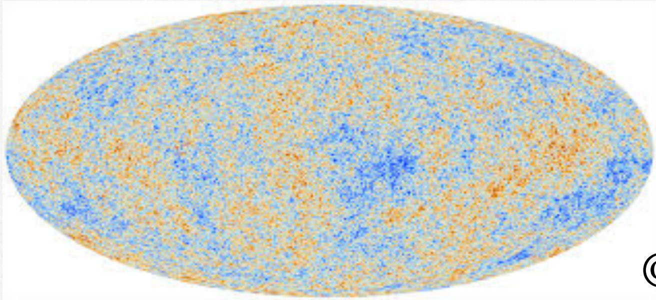
Y.U. & J.G. arXiv:1303.5997, JCAP 1307, 033
arXiv:1403.5497, JHEP 1406, 086

Y.U., J.G.&K.S. arXiv:1410.3290, JCAP 1501,...

Inflation from holography

WMAP, PLANCK, ...



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Inflation is now quite compelling.

- Can we describe inflation holographically?

  If YES, what's the prediction?

  If NO, what's the obstacle?

Lyth bound

- Power spectrums

$$\Delta_\zeta^2 = \frac{1}{2M_{\rm pl}^2}\frac{1}{\varepsilon_*}\left(\frac{H_*}{2\pi}\right)^2$$

$$\Delta_{\rm GW}^2 = \frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2$$

$$r = \frac{\Delta_{\rm GW}^2}{\Delta_\zeta^2} = 16\varepsilon_*$$

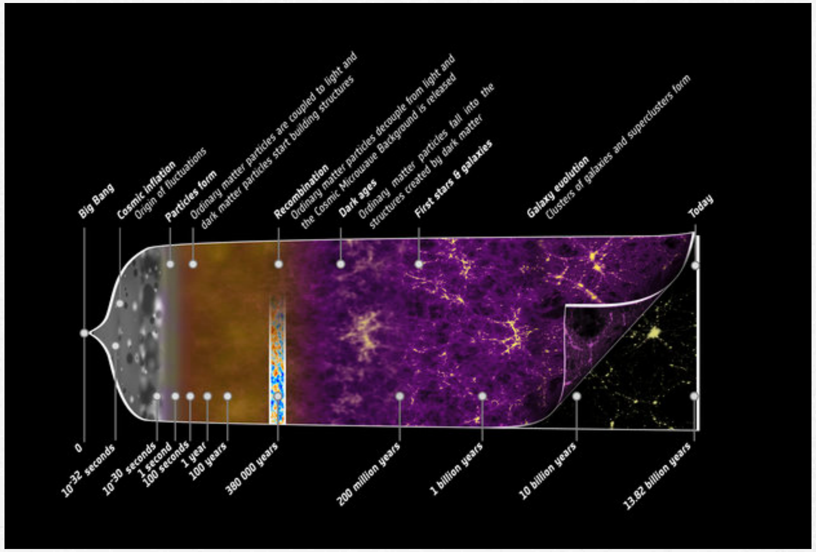
$$N_e = \int d\ln a = \int \frac{H}{\dot\phi}d\phi \sim \sqrt{8}r^{-1/2}\frac{\Delta\phi}{M_{\rm pl}}$$

$$\longrightarrow \quad \frac{\Delta\phi}{M_{\rm pl}} \simeq \left(\frac{r}{0.1}\right)^{1/2}$$

r ≦ 0.12 (Planck + BICEP 2015)

- Planck scale excursion is marginally allowed.

UV sensitivity of inflation



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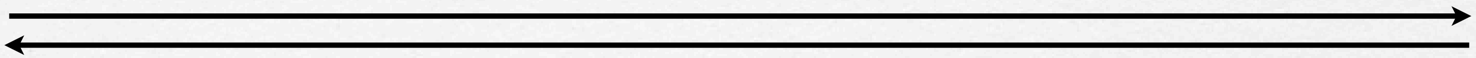
time

energy



- Perturbation with controlled radiative corrections

  - High energy scale near Plank scale

  - Planck scale excursion is marginally allowed.

Outline of this talk

1. dS/CFT

2. Inflation/QFT

3. Boundary QFT

4. ζ correlators from boundary

skipped

Gauge/Gravity correspondence

- Holographic principle

't Hooft (92), Susskind (95)

Holographic principle suggests that a gravity theory should be related to a non-gravitational theory in one fewer dimension.

$$\text{d-dim gauge theory} \longleftrightarrow \text{(d+1)-dim gravity theory}$$
$$+ \text{ RG flow}$$

- Non-trivial duality

Maldacena (97)

Boundary CFT		Bulk gravity
'tHooft coupling λ	$\lambda=(r_0/l_s)^4$	Curvature scale r_0
Strong coupling	$\lambda \gg 1,\ r_0 \gg l_s$	Weak coupling
Weak coupling	$\lambda \ll 1,\ r_0 \ll l_s$	Strong coupling

AdS/CFT as H-J formalism

d-dim gauge theory $\longleftarrow$ (d+1)-dim gravity theory
+ RG flow

Recall Hamiltonian-Jacobi formalism....

using equation of motion for (d+1)-dim theory

$$\delta S \sim \mathcal{L} dz \Big|_{z=z_2}^{z=z_1}$$

$z=z_1$ holographic plane $\rightarrow$ CFT

(d+1)dim

$z=z_2$ trivial B.C.

see also... holographic renormalization

S. Haro et al. (00), Skenderis (02), ...

Geometry of AdS and dS

Anti de Sitter (AdS)

Vacuum with $\Lambda < 0$

in $R^{2,3}$ (-,-, +, +, +) SO(2,3)

$$-X_0^2 - X_1^2 + \sum_{a=2,3,4} X_a^2 = -A^2$$

$$ds^2 = l_{\mathrm{AdS}}^2 \left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \right)$$

de Sitter (dS)

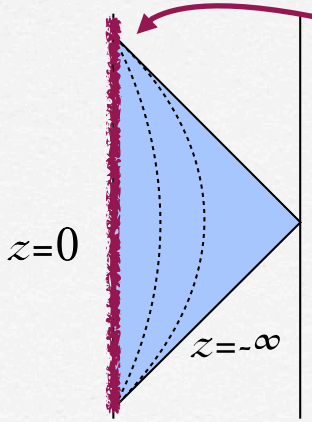
Vacuum with $\Lambda > 0$

in $R^{1,4}$ (-,+, +, +, +) SO(1,4)

$$-X_0^2 + X_1^2 + \sum_{a=2,3,4} X_a^2 = A^2$$

$$ds^2 = l_{\mathrm{dS}}^2 \left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2} \right)$$

Boundary

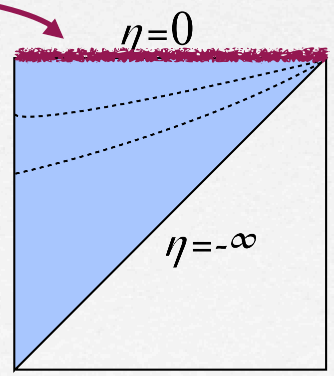


z:const, R^3

$$l_{\mathrm{AdS}} \rightarrow i l_{\mathrm{dS}}$$

$$z \rightarrow i\eta$$

$$t \rightarrow -iw$$

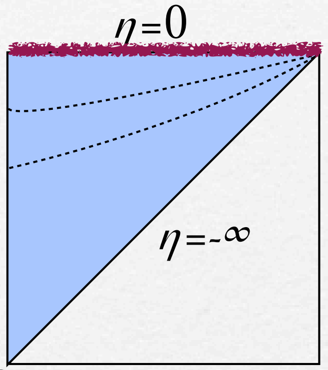


η:const, R^3

dS/CFT

Strominger(01), Witten(01)

- CFT lives on the spacelike boundary at the future infinity of dS.



Maldacena(02)

- Wave function for bulk gravity $\Psi_{\mathrm{dS}}[g]=Z_{\mathrm{CFT}}$

  Probability distribution $P_{\mathrm{dS}}[g]=|Z_{\mathrm{CFT}}|^2$

- Euclidean AdS $$S_{AdS\ \mathrm{ren}} = -\frac{1}{\kappa^2}\int \frac{\mathrm{d}^3\boldsymbol{k}}{2\pi^3}\frac{1}{2}R_{AdS}^2 k^3\phi_0(-k)\phi_0(k)$$

$$\xrightarrow[Z\sim e^{-S}]{} \quad \langle O(k)O(-k)\rangle = -\frac{\delta^2 S_{AdS\ \mathrm{ren}}}{\delta\phi_0(k)\delta\phi_0(-k)} = \frac{1}{2}\frac{R_{AdS}^2}{\kappa^2}k^3$$

- dS $$\langle\phi(k)\phi(-k)\rangle = -\frac{1}{2\mathrm{Re}\langle O(k)O(-k)\rangle}\bigg|_{R_{AdS}=-iR_{dS}} = \frac{\kappa^2}{R_{dS}^2}\frac{1}{k^3}$$

Analytic cont. connects dual boundaries of dS and AdS

Challenges of dS/CFT

- Holographic direction is time like.
  Dual boundary theories to dS are non-unitary.
  Good property?
- Poor understanding on analytic continuation.
  Extendable to a non-perturbative example in 1/N?
- Lack of a concrete example.
  First concrete example of dS/CFT
  Anninos, Hartman, &Strominger(11)

  Vasiliev gravity in dS_4 $\longleftrightarrow$ Sp(N) CFT_3 living at $\mathscr{I}^+$

  $\Lambda \rightarrow -\Lambda$ $\qquad$ $N \rightarrow -N$

Outline of this talk

1. dS/CFT

2. Inflation/QFT

3. Holographic inflation (Simplest setup)

4. Conservation of ζ

Breaking symmetry

<u>de Sitter space</u>

4D hyperboloid:

$$ds_4^2 = \{\eta_{\mu\nu} X^\mu X^\nu = H^{-2}\}$$

in 5D flat spacetime

SO(1,4)

$\longleftrightarrow$

<u>CFT on R^3</u>

- Poincare T.
- Dilatation
- Special C.T.

Cosmological const. Λ

+ inflaton φ

Breaking dS sym.

$\downarrow$

Inflation

CFT

+ φO (ex)mass

Breaking conf. sym.

$\downarrow$

Deformed CFT

Standard lore of inflation

<u>4D bulk</u>

Inflation
= dS + modulation

Given that....

- GR, V(φ)
- GR, V(φ), P(X=$(\partial\varphi)^2$)
- $f(R)$, V(φ)

and so on

local QFT weakly coupled to gravity

$\longrightarrow$ $\varphi(t)$, $<\zeta\zeta....\zeta>$, ...

Holographic inflation

4D bulk

Inflation
= dS + modulation

3D boundary

QFT
CFT+1 deformation

$$\boxed{\Psi_{\text{bulk}}[\varphi] = Z_{\text{QFT}}[g]}$$

$$Z_{\text{QFT}} = \int D\chi \exp\left[-S_{\text{CFT}} - \underbrace{\int g\mathcal{O}[\chi] + \cdots}_{\text{deformation}}\right]$$

Necessary building blocks
- φ & g relation?
- t & μ relation?

Conservation of ζ

From cosmological perturbation

Single clock $\quad \partial_t \zeta = O((k/aH)^2)$

Wands et al. (00), Weinberg (03), Lyth et al (04), Langlois&Vernizzi (05),...

- Energy conservation $\nabla^\mu T^0{}_\mu = 0$
- Holds at full non-linear order

(ex) Single inflaton in Einstein gravity

$$\zeta'' + 2\frac{z'}{z}\zeta' - \cancel{\partial^2\zeta} = 0$$

Holographic inflation

4D bulk

Inflation
= dS + modulation

Conservation
of P_ζ

3D boundary

QFT

CFT+1 deformation

$$\boxed{\Psi_{\rm bulk}[\varphi] = Z_{\rm QFT}[g]}$$

$$Z_{\rm QFT} = \int D\chi \exp\left[-S_{\rm CFT} - \int g\mathcal{O}[\chi]\right]$$

- φ & g relation? Field redefinition
- t & μ relation? $a(t) \propto \mu^C$ C : const J.G.&Y.U.(14)

Outline of this talk

1. dS/CFT

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$$Z_{\rm QFT} = \int D\chi \exp\left[-S_{\rm CFT} - \int g\mathcal{O}[\chi]\right]$$

4. Conservation of ζ

skipped

What's RG flow?

μ_p:Physical scale μ:Cutoff scale

given that $\mu_p << \mu$

$1/\mu_p$

$1/\mu$

(ex) interaction $S = \prod_{i=1,2,3,4} \sum_{p_i<\mu} \varphi(p_i)$

$$\boxed{\text{Physical quantity} \quad F_{\text{phys}}(\mu_p; g(\mu), \mu) = F_{\text{phys}}(\mu_p; g(\mu'), \mu') \qquad g\text{: Physical constant}}$$

$\longrightarrow$ Rrenormalization group (RG) flow

Boundary QFT

Conformal perturbation theory (~ Slow-roll expansion)

$$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \delta S \qquad \delta S = \int \mathrm{d}^3x\, g\mathcal{O}[\chi] \qquad (0 \leqq g << 1)$$

O : Boundary operator consists of χ

g : Dimensionless coupling

μ : Renormalization scale

- Correlators for CFT

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})\rangle_{\mathrm{CFT}} = \frac{c}{|\boldsymbol{x}-\boldsymbol{y}|^{2\Delta}}$$

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})\rangle_{\mathrm{CFT}} = \frac{C}{|\boldsymbol{x}-\boldsymbol{y}|^{\Delta}|\boldsymbol{y}-\boldsymbol{z}|^{\Delta}|\boldsymbol{z}-\boldsymbol{x}|^{\Delta}}$$

Beta function & FP

ß function $\beta(\mu) \equiv \frac{\mathrm{d}g(\mu)}{\mathrm{d}\ln\mu}$

Klebanov et al. (11)

$$\beta(\mu) = \underbrace{\lambda\, g(\mu)}_{\text{Classical scaling}} + \underbrace{\frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3)}_{\text{Quantum corrections}}$$

$\tilde{C} \sim \frac{C}{c}$

$\lambda = \Delta - 3$

- Fixed point (FP) $\beta=0$

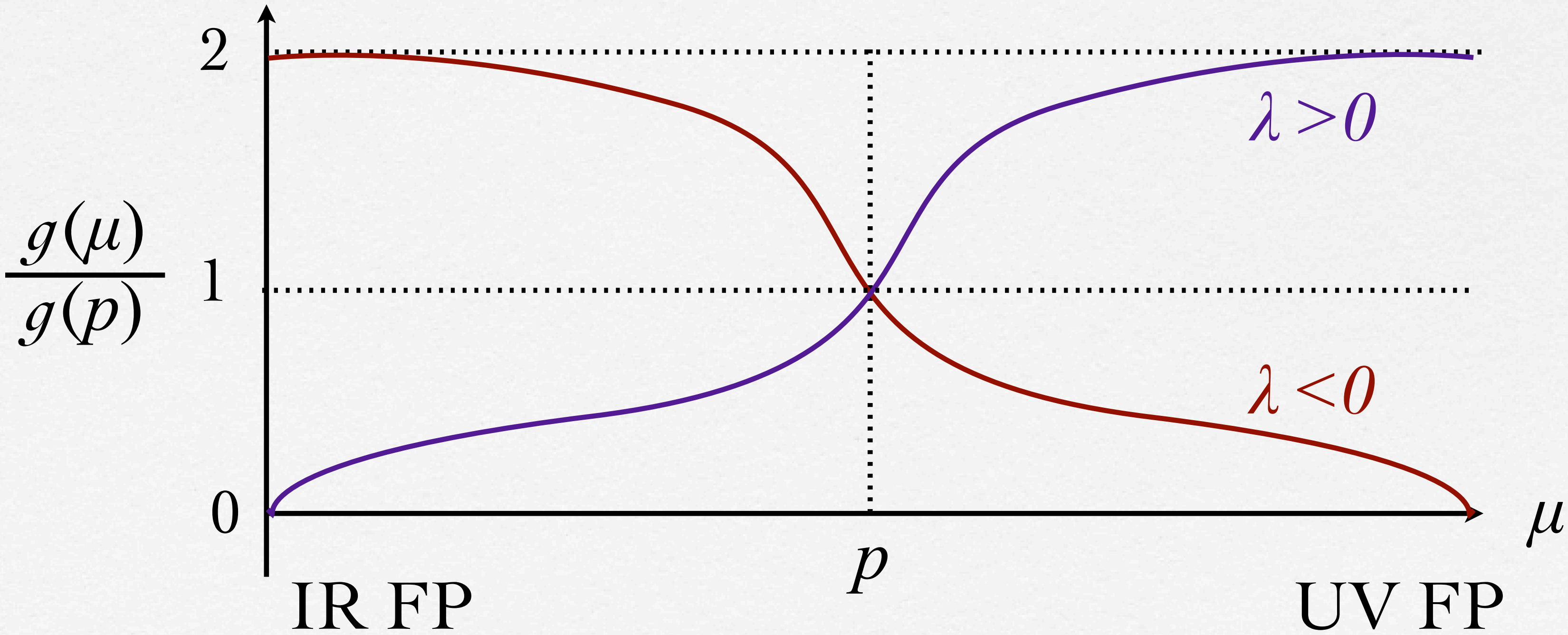
For $\tilde{C}/\lambda < 0$ Two FPs $g=0$, $-2\lambda/\tilde{C}$

For $\tilde{C}/\lambda \geqq 0$ One FP $g=0$,

Solving RG flow

$$\beta(\mu) = \lambda\, g(\mu) + \frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3) \qquad \text{for} \quad \tilde{C}/\lambda < 0$$

$$g(\mu) = \frac{2}{1+\left(\frac{\mu}{p}\right)^{\lambda}} \left(\frac{\mu}{p}\right)^{\lambda} g(p) \qquad g(p) \equiv -\frac{\lambda}{\tilde{C}}$$



Reconstruction of potential

KG equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

$$\downarrow$$

$$\frac{d\phi}{d\ln a} = -\frac{2}{\kappa^2}\frac{1}{W(\phi)}\frac{\partial W(\phi)}{\partial \phi}$$

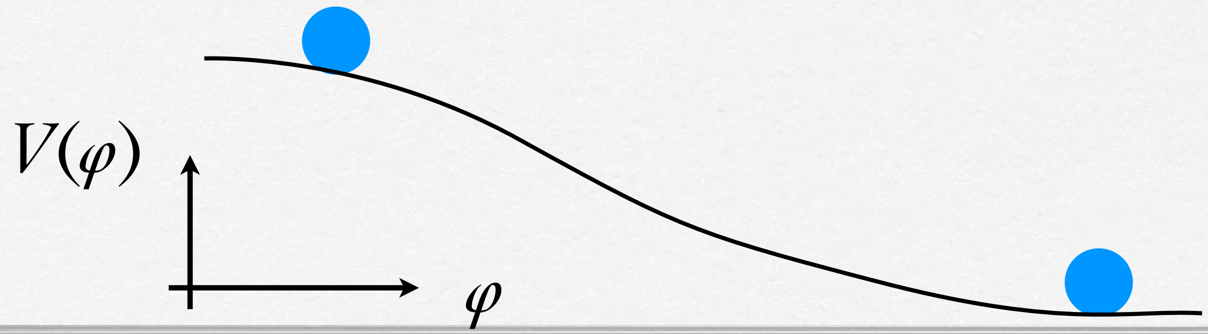
$$V(\phi) = \frac{8}{\kappa^2}\left[\frac{3}{2}W^2(\phi) - \frac{1}{\kappa^2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2\right]$$

RG equation

$$\frac{dg}{d\ln\mu} = \lambda g + \frac{\tilde{C}}{2}g^2 + O(g^3)$$

$$g(\mu, \mathbf{x}) = \kappa\phi(t(\mu), \mathbf{x})$$

$d\ln a = p\, d\ln\mu$



Correlators of O

Expanding by correlators for CFT with cutoff

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle_\mu$$ Bzowski et al. (12)

$$= \langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)e^{-\int \mathrm{d}^3x\, g\mathcal{O}}\rangle_{\mu,\mathrm{CFT}}$$

$\Big\downarrow$ integrating out $k>\mu$, changing μ, using OPE

$$\underbrace{Z^{-n/2}(\mu)\langle \mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_\mu}_{k<\mu} = \underbrace{Z^{-n/2}(\mu_0)\langle \mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_{\mu_0}}_{k<\mu_0}$$

Wave fn. renormalization

$$\sqrt{Z(\mu)} = \mu^{-\lambda}\left[1+\left(\frac{\mu}{p}\right)^\lambda\right]^2 = 4p^{-\lambda}\frac{\beta(p)}{\beta(\mu)}$$ J.G.&Y.U. (14)

Correlators

From boundary QFT to bulk gravity

$$\Psi_{\mathrm{qdS}}[\delta\varphi] = Z_{\mathrm{QFT}}[\delta\varphi] = e^{-W_{\mathrm{QFT}}[\delta\varphi]} \qquad P[\delta\varphi] = \left|\Psi_{\mathrm{qdS}}[\delta\varphi]\right|^2$$

$$\langle \delta\phi(x_1)\cdots\delta\phi(x_n)\rangle = \int D\delta\phi P[\delta\phi]\delta\phi(x_1)\cdots\delta\phi(x_n)$$

* Distribution function $\quad P[\delta\varphi] = e^{-\delta W[\delta\varphi]}$

$$\delta W[\delta\phi] = \sum_{n=1}^{\infty}\int d^d\mathbf{x}_1\cdots\int d^d\mathbf{x}_n W^{(n)}(\mathbf{x}_1,\cdots,\mathbf{x}_n)\delta\phi(\mathbf{x}_1)\cdots\delta\phi(\mathbf{x}_n)$$

$$W^{(n)}(\mathbf{x}_1,\cdots,\mathbf{x}_n) \equiv 2\mathrm{Re}\left[\frac{\delta^n W_{\mathrm{QFT}}[\delta\phi]}{\delta\phi(\mathbf{x}_1)\cdots\delta\phi(\mathbf{x}_n)}\bigg|_{\delta\phi=0}\right]$$

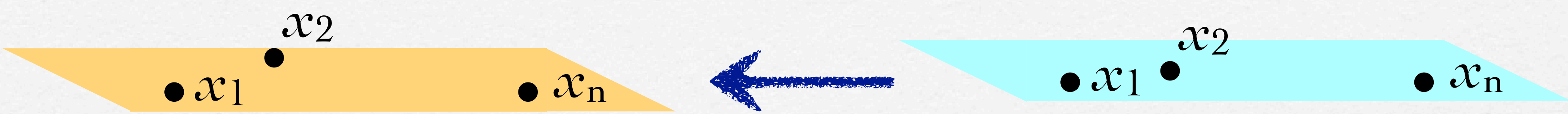
ζ Correlators

in cosmology

$$\langle \zeta(x_1)\zeta(x_2)\cdots\zeta(x_n)\rangle$$

in boundary QFT

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)\rangle$$



$\delta\phi(t, \mathbf{x}) = 0$

flat space R=0

at large scales

$$\zeta = -\frac{H}{\dot\phi}\delta\phi + \frac{\varepsilon_2}{4}\left(\frac{H}{\dot\phi}\right)^2\delta\phi^2 + \cdots$$

$$\varepsilon_1 \equiv \frac{1}{2}\frac{\dot\phi^2}{H^2}$$

$$\varepsilon_2 \equiv \frac{\mathrm{d}\ln\varepsilon_2}{\mathrm{d}\ln a}$$

Vertex function

$$W^{(n)}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \equiv 2\mathrm{Re}\left[\left.\frac{\delta^n W_{\mathrm{QFT}}[\zeta]}{\delta\zeta(\boldsymbol{x}_1)\cdots\delta\zeta(\boldsymbol{x}_n)}\right|_{\zeta=0}\right] \longleftarrow \langle\mathcal{O}(x_1)\cdots\mathcal{O}(x_m)\rangle_\mu$$

$$\frac{\delta}{\delta\zeta} = \frac{\delta\phi}{\delta\zeta}\frac{\delta}{\delta\phi} \sim \frac{\delta\phi}{\delta\zeta}\mathcal{O}$$

Conservation of ζ

Power spectrum

J.G.&Y.U.(14)

$$\langle \zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\rangle_{\rm conn} = W^{(2)\,-1}(\boldsymbol{x}_1,\,\boldsymbol{x}_2)$$

$$B_1(\mu) = -\left.\frac{\partial\delta\phi}{\partial\zeta}\right|_{\zeta=0}$$

$$W^{(2)}(\boldsymbol{x}_1,\,\boldsymbol{x}_2) = -2\mathrm{Re}\left[B_1^2(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\rangle_\mu\right]$$

Conservation $\longrightarrow$ $$\frac{\mathrm{d}}{\mathrm{d}\mu}\left[B_1(\mu)\sqrt{Z(\mu)}\right] = 0$$

Gauge transformation $$B_1 = \frac{\dot\phi}{H} = \frac{\mathrm{d}\phi}{\mathrm{d}\ln a}$$

RG flow $$\sqrt{Z(\mu)} = 4p^{-\lambda}\frac{\beta(p)}{\beta(\mu)}$$

$\longrightarrow$ Identification between t & μ

$$\boxed{\ln(\mu/\mu_0) = \mathcal{C}\ln(a/a_0)}$$

$\mathcal{C}$: const

$\mathcal{C}=1+\mathrm{O}(\varepsilon)$

Conserved Power spectrum

$$P(k) = -\frac{3}{8\pi}\frac{1}{c\beta^2(p)}\frac{1}{k^3}\left(\frac{k}{p}\right)^{-2\lambda}\left[1+\left(\frac{k}{fp}\right)^{\lambda}\right]^4 \sim -\frac{1}{c\beta^2(k)}$$

cf Agrees with the result of Bzowski+ (12) in $\mu \to \infty$

Remarks

1.Amplitude

$$\beta = \frac{dg}{d\ln\mu} \sim \frac{d(\phi/M_{pl})}{d\ln a} = \sqrt{2\varepsilon}$$

$$c \simeq (M_{\rm pl}/H_{\rm dS})^2$$ Strominger(01)

$$\longrightarrow \quad \frac{1}{c\beta^2} \sim \frac{1}{\varepsilon}\left(\frac{H}{M_{pl}}\right)^2$$

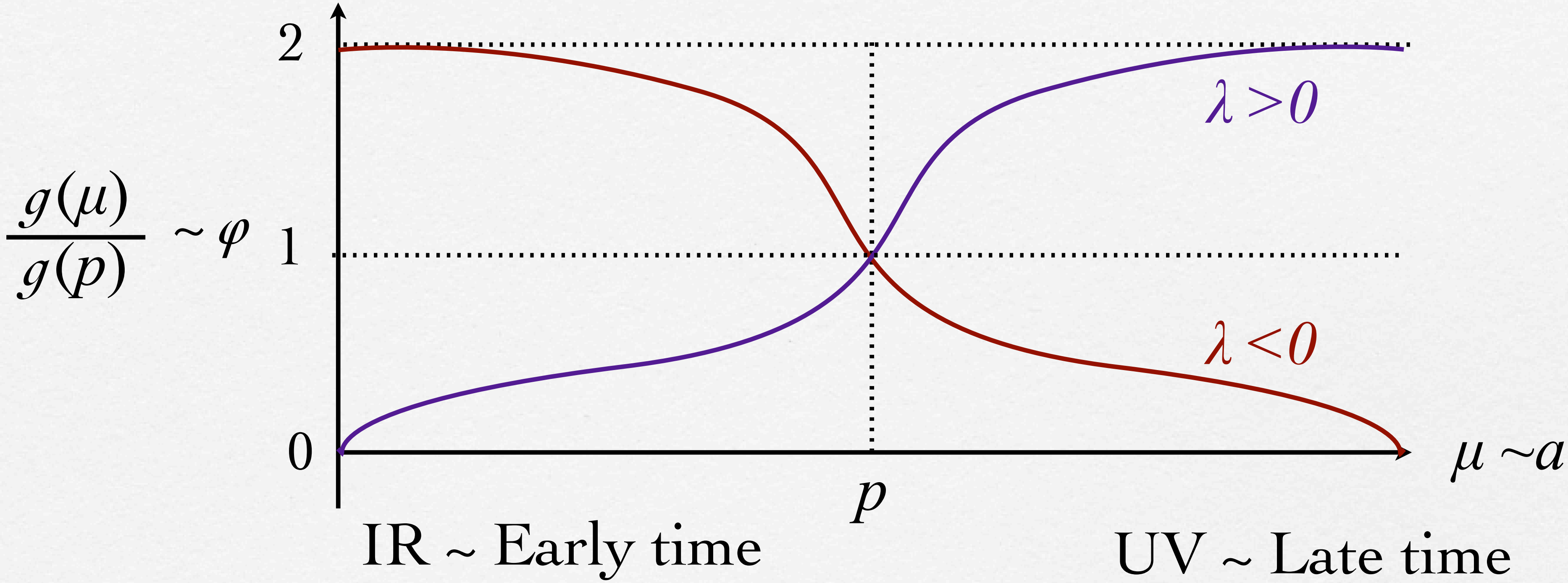
Maldacena(02)

2. Spectral index

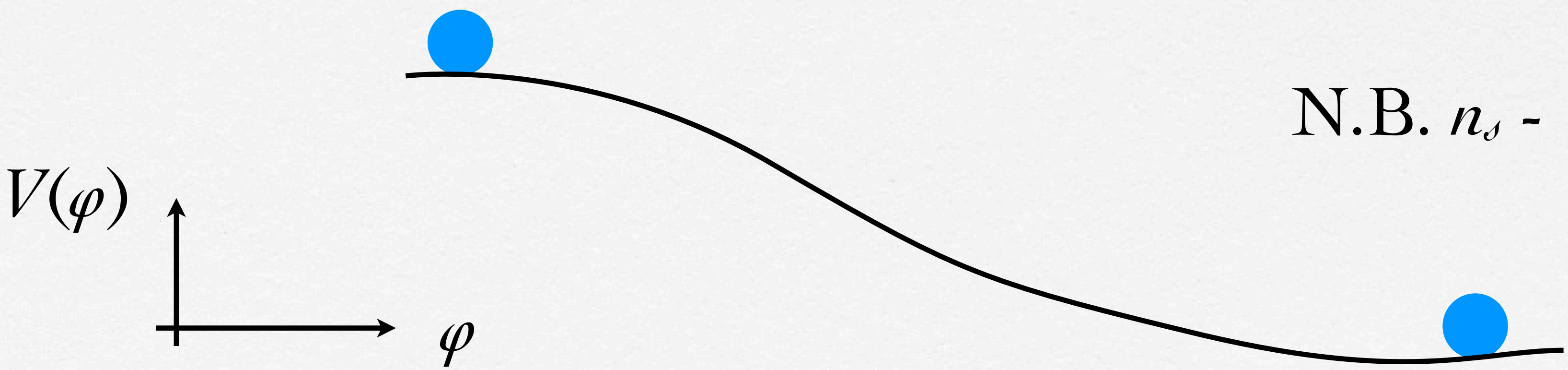
For $k >> fp$ $\quad n_s\text{-}1=2|\lambda|$ $\quad$ Blue-tilted

For $k << fp$ $\quad n_s\text{-}1=\text{-}2|\lambda|$ $\quad$ Red-tilted

Evolution of "inflaton"



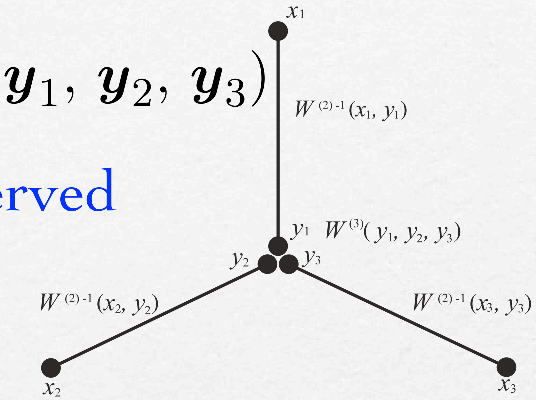
N.B. $n_s - 1 = -6\varepsilon + 2\eta$



Conservation of bi-spectrum

$$\langle \zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\zeta(\boldsymbol{x}_3)\rangle_{\text{conn}} = -\int \prod_{i=1}^{3} \mathrm{d}^d \boldsymbol{y}_i \boxed{W^{(2)\,-1}(\boldsymbol{x}_i\,,\boldsymbol{y}_i)} W^{(3)}(\boldsymbol{y}_1,\,\boldsymbol{y}_2,\,\boldsymbol{y}_3)$$

conserved if P_ζ conserved



$$W^{(3)}(\mathbf{y}_1,\,\mathbf{y}_2,\,\mathbf{y}_3) = -2\mathrm{Re}\Big[\beta^3(\mu)\langle\mathcal{O}(\mathbf{y}_1)\mathcal{O}(\mathbf{y}_2)\mathcal{O}(\mathbf{y}_3)\rangle$$
$$-\frac{d\beta(\mu)}{d\ln\mu}\beta(\mu)\delta(\mathbf{y}_1-\mathbf{y}_2)\langle\mathcal{O}(\mathbf{y}_2)\mathcal{O}(\mathbf{y}_3)\rangle_\mu - (2\,\text{perms})\Big]$$

If the correlators of O are given by

$$Z^{-n/2}(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_\mu = Z^{-n/2}(\mu_0)\langle\mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_{\mu_0} \qquad \sqrt{Z(\mu)} \propto 1/\beta(\mu)$$

Conservation requires

$$\frac{d\ln\beta}{d\ln\mu} = \text{const.} \qquad \text{cosmologically} \quad \varepsilon_2 = \frac{d\ln\varepsilon_1}{d\ln a} = \text{const.}$$

Be more careful....

$$Z^{-n/2}(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_\mu = Z^{-n/2}(\mu_0)\langle\mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_{\mu_0}$$

- RG solution does not apply to coincidence limit (CDL).
- We need regularization to compute CDL.

N.B. In CFT, symmetry argument does not specify the CDL.

$$S_{\rm QFT} = S_{\rm CFT} + \int d^d x g\mathcal{O}(x) \ + \int d^d x g_n \mathcal{O}^n(x)?$$

multi-trace operators

AdS/CFT

- Wilsonian RG, Bulk → Bdry QFT with multi-trace op.

  Heemskerk&Polchinski(10), Faulkner, Liu, & Rangamani(11)
- Bdry QFT with multi-trace op. → GR (+ Λ)

  S.S.Lee(13)

Conclusion

Holographic description of inflation scenario

- We computed the primordial spectrum holographically, and the result may apply to strong/weak gravity regimes (large N, arbitrary 'tHooft coupling).

- The conservation of ζ power spectrum determines t & μ relation as $a(t) \propto \mu^C$.

- A subtle issue on the conservation of bi-spectrum $<\zeta\zeta\zeta>$

  Yet, if we determine the CSL such that the consistency relation is fulfilled, the bispectrum is conserved.