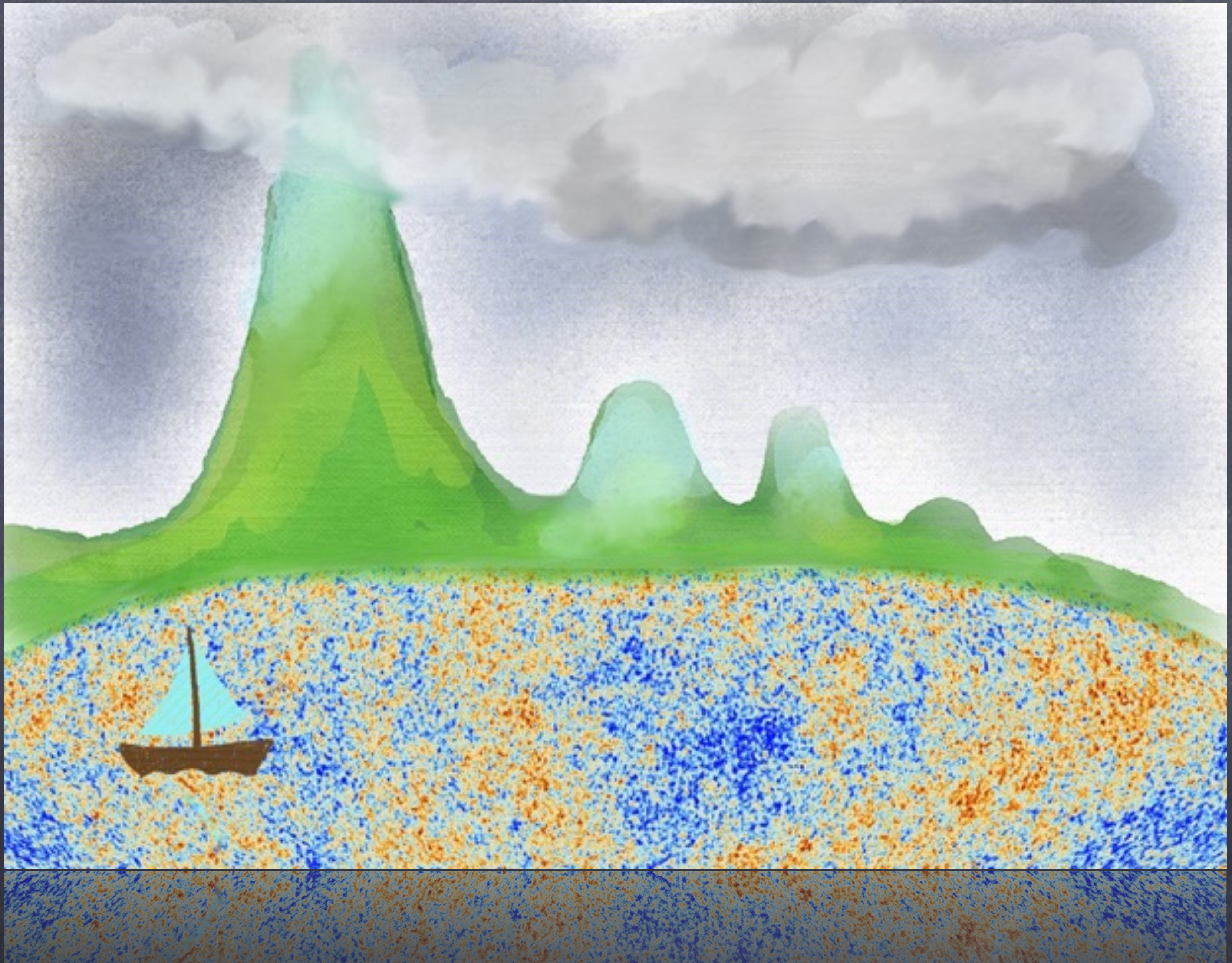
Entanglement and Non-Gaussianity



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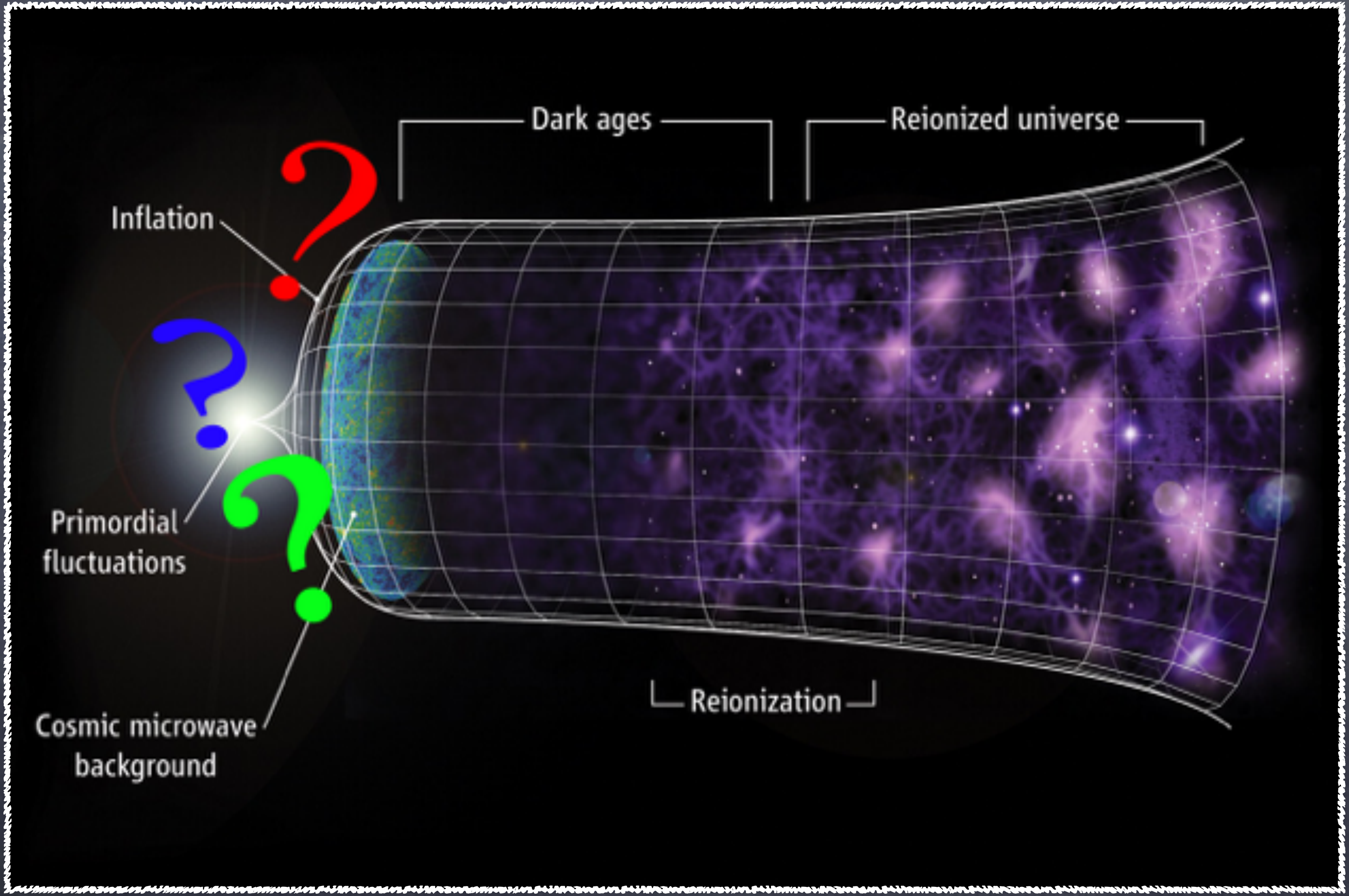
How did the Universe begin?



→ Big Bang (Past), Inflation

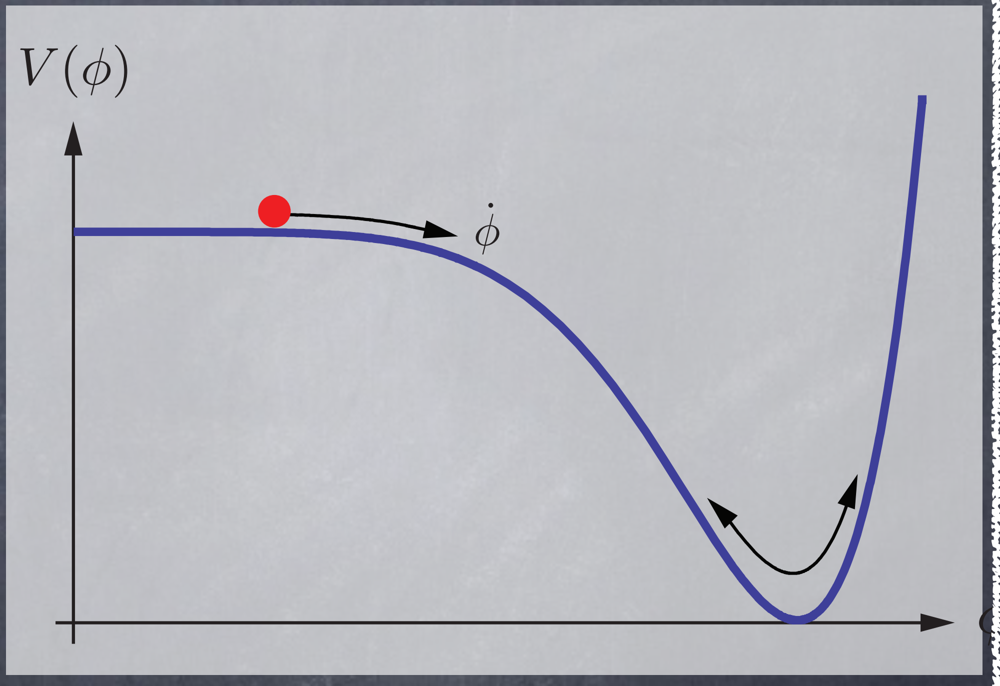
Motivations for Inflation

- Original Motivation: Solving horizon and flatness problems (resolve fine tuning problems).
- Greatest Success: Prediction of temperature fluctuations in CMB, cosmic structure and oscillations in angular power spectra.

Basics of Inflation

- Inflation driven by a scalar field (Inflaton) rolling down a potential.

- Quantum fluctuations get stretched by expansion -> form temperature fluctuations in CMB -> cosmic structure.



Is inflation a special initial condition?

- Big Bang (with no inflation): 'special' initial conditions.
- Inflation -> explains fine tuning + predicts temperature fluctuations in CMB.

Is inflation also a 'special initial state'?

- Usual discussion of inflation starts with Bunch-Davies (BD) vacuum state.
    - BD vacuum state: QFT vacuum state adapted to expanding spacetime.

Is inflation a special initial condition?

- Successes of inflation (solving tuning problems and matching to data) resulted from this choice.
- BUT choosing vacuum initial state –> Free tuning?
  - BD vacuum is a 'special' state.
- Out of all possible states, how do we know nature chose BD vacuum?
- Inflation gives us opportunity to test for deviations from BD vacuum scenario.

Scenarios of Inflation

Eternal Inflation

- Bubbles where inflation stops in fast inflation background.
- Measure problem.
- Push initial conditions back.

Finite Inflation

- Just enough to solve horizon and flatness problem or more.
- What came before that?

Signatures of Finite Inflation

- Inflation generically washes out 'initial conditions'.
- Finite inflation leaves more opportunities for signatures to survive.
- Entanglement during inflation produces observational signatures that survive inflation.

Quick Review of Entanglement and Related Terms

- Pure vs. Mixed States
- Cohered vs. Decohered
- Entanglement vs. No Entanglement

Pure vs. Mixed States

Pure States

$$\mathrm{Tr}\rho^2 = 1$$

Density Matrix:

$$\rho = |\Psi\rangle\langle\Psi|$$

All information known

Mixed States

$$\mathrm{Tr}\rho^2 < 1$$

Density Matrix:

$$\rho = \sum_n p_n |\Psi_n\rangle\langle\Psi_n|$$

Information lost, statistical ensamble

Pure vs. Mixed States

Expand state in basis:

$$|\Psi\rangle = \sum_i \alpha_i |i\rangle$$

Pure density matrix:

$$\rho_P = \sum_{i,j} \alpha_i \alpha_j^* |i\rangle\langle j| = \sum_{i,j} \tilde{\alpha}_{ij} |i\rangle\langle j|$$

Mixed density matrix:

$$\rho_M = \sum_n p_n \left(\sum_{i,j} \alpha_i \alpha_j^* |i\rangle\langle j| \right)_n = \sum_n p_n \sum_{i,j} \left(\tilde{\alpha}_{ij}\right)_n |i\rangle\langle j|$$

Cohered vs. Decohered

Cohered

Exhibits 'quantum behavior'

Cohered - Pure:

$$\rho_P = \sum_{i=j} \tilde{\alpha}_{ii} |i\rangle\langle i| + \sum_{i \neq j} \tilde{\alpha}_{ij}^{\text{off}} |i\rangle\langle j|$$

Off-Diagonal Amplitudes:

$$\tilde{\alpha}_{ij}^{\text{off}} = \alpha_j^* \alpha_i$$

Decohered

'Behaves classically'

Decohered - Pure:

$$\rho_P = \sum_{i} \tilde{\alpha}_{ii} |i\rangle\langle i|$$

Eigen States

Square Probability:

$$\tilde{\alpha}_{ii} = |\alpha_i|^2$$

Cohered vs. Decohered

Decohered - Mixed

$$\rho_M = \begin{pmatrix} p_1\rho_1^{\text{diag}} & 0 & \cdots & 0 \\ 0 & p_1\rho_1^{\text{diag}} & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & p_n\rho_n^{\text{diag}} \end{pmatrix}$$

Simultaneously Diagonalizable

Cohered - Mixed

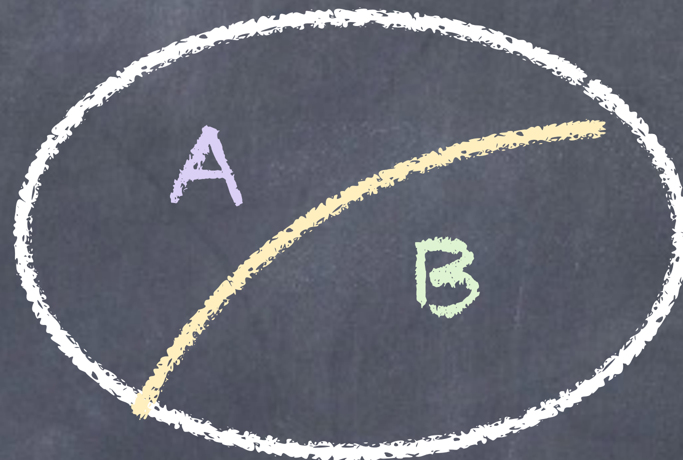
$$\rho_M = \sum_n p_n \rho_n \to \rho_n^{\text{off-diag}\neq 0}$$

Not Simultaneously Diagonalizable

Entangled vs. Not Entangled

$$\Psi_E = \sum_{a,b} \gamma_{ab} |a\rangle |b\rangle$$

Pure: $\mathrm{Tr}\rho_E^2 = 1.$



From Pure to Mixed State:

$$\mathrm{Tr}_{\,B}\,[\rho_E] = \mathrm{Tr}_{\,B}\left[\sum_{a,b}\sum_{a',b'} \gamma_{ab}\,\gamma^{*}_{a'b'}\,|a,b\rangle\langle a',b'|\right]$$

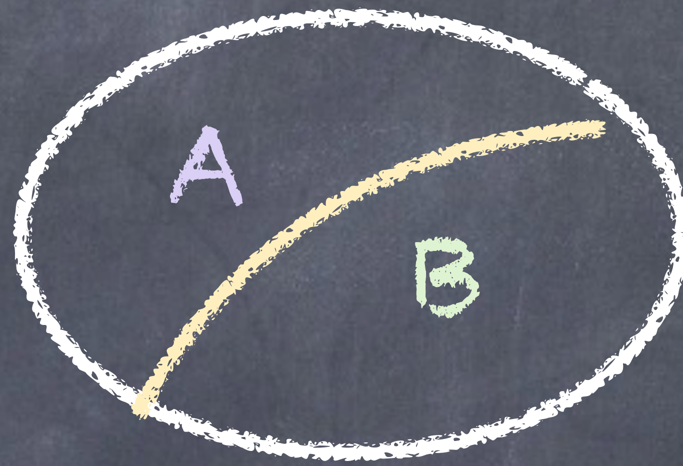
$$= \sum_{a,a'} \gamma_{ab}\gamma^{*}_{a'}{}^{b} |a\rangle\langle a'| = \rho_A$$

Reduced Density Matrix for A

Entangled vs. Not Entangled

$$\Psi_E = \sum_{a,b} \gamma_{ab} |a\rangle |b\rangle$$

Pure: $\mathrm{Tr}\rho_E^2 = 1.$



From Pure to Mixed State:

$$\mathrm{Tr}_B\,[\rho_E] = \mathrm{Tr}_B \left[\sum_{a,b} \sum_{a',b'} \gamma_{ab}\, \gamma^{*}_{a'b'}\, |a,b\rangle\langle a',b'| \right]$$

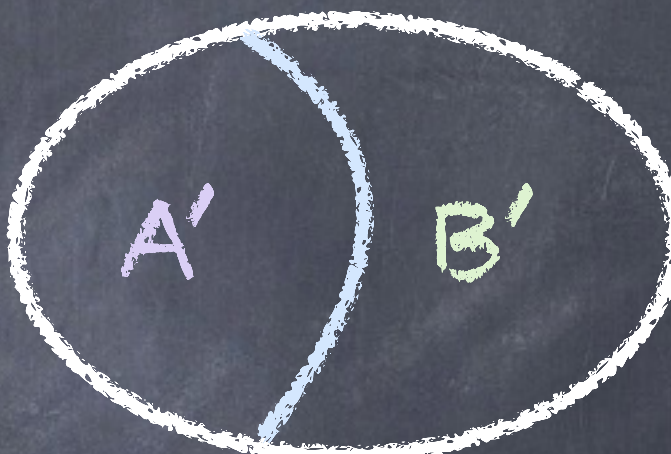
$$= \sum_{a,a'} \gamma_{ab} \gamma^{*}_{a'}{}^{b} |a\rangle\langle a'| = \rho_A$$

$$\boxed{\mathrm{Tr}\rho_A^2 < 1}$$

Entangled vs. Not Entangled

Note: Diagonalizing reduced density matrix does not eliminate entanglement.

Un-entangling the State:



$$|\Psi_E\rangle = \sum_{a,b} \gamma_{ab} |a\rangle |b\rangle$$

$$= \sum_{a,b} \sum_{a',b'} \gamma_{ab} |a'\rangle |b'\rangle \langle a'|a\rangle \langle b'|b\rangle$$

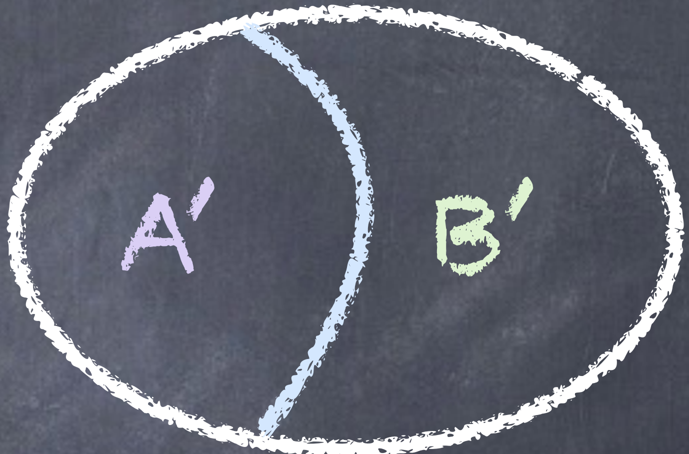
$\tilde{\gamma}_{a'b'}$ is Separable.

$$= \sum_{a',b'} \tilde{\gamma}_{a'b'} |a'\rangle |b'\rangle = \sum_{a',b'} \alpha_{a'} \beta_{b'} \; |a'\rangle |b'\rangle$$

Entangled vs. Not Entangled

Note: Diagonalizing reduced density matrix does not eliminate entanglement.

Un-entangling the State:



$$|\Psi_E\rangle = \sum_{a,b} \gamma_{ab} |a\rangle |b\rangle$$

$$= \sum_{a,b} \sum_{a',b'} \gamma_{ab} |a'\rangle |b'\rangle \langle a'|a\rangle \langle b'|b\rangle \qquad \tilde{\gamma}_{a'b'}$$

$$= \sum_{a',b'} \tilde{\gamma}_{a'b'} |a'\rangle |b'\rangle = \sum_{a',b'} \alpha_{a'} \beta_{b'} \, |a'\rangle |b'\rangle \; = |\Psi_{A'}\rangle \otimes |\Psi_{B'}\rangle$$

Entanglement during Inflation

Inflaton fluctuations entangled with:

- Spectator Scalar Field, χ
- Metric perturbations, $\gamma_{ij} \to h^{+}, h^{\times}$

Ansatz: Entangled initial state at the beginning of inflation (at some finite time τ_0).

arXiv:1408.6859, arXiv:1605.01008

Bunch Davies Vacuum

Bunch Davies is the vacuum state of Field Theory adapted to expanding Universe.

—> Short wavelength modes in ground state

In field space each mode in BD has a Gaussian Wavefunction:

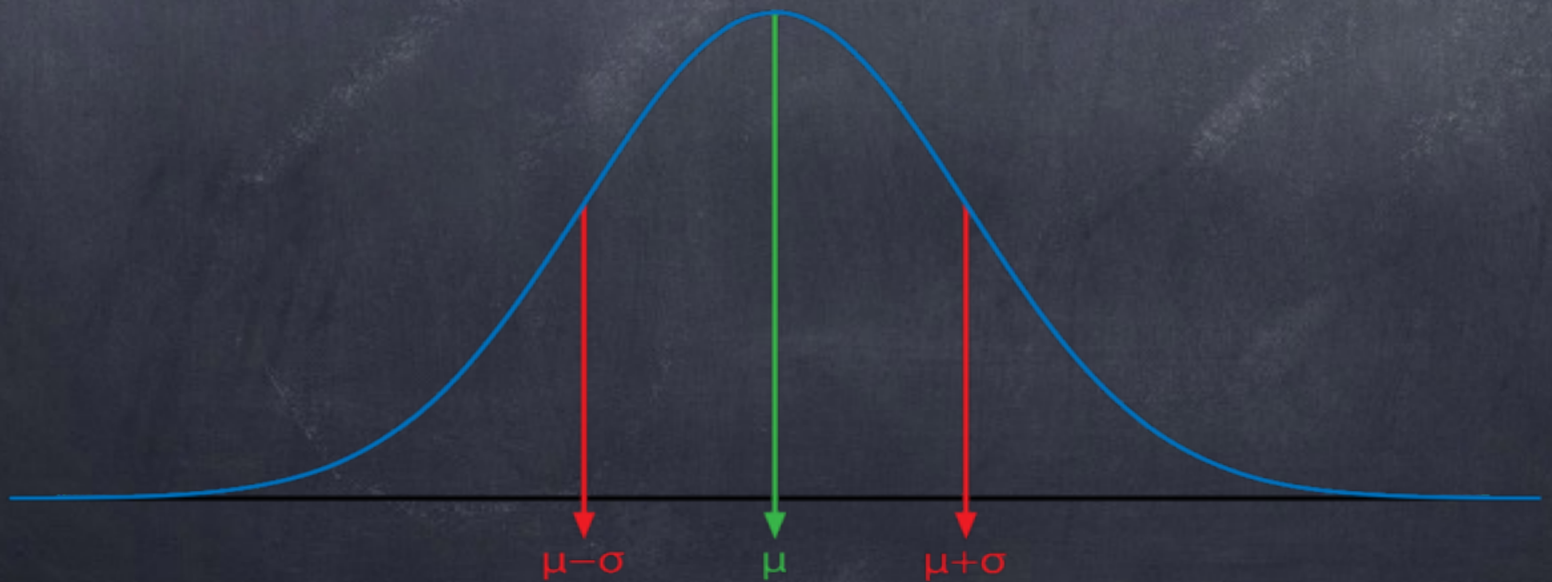
$$\Psi_{\vec{k}}[\varphi_{\vec{k}};\tau] = N_k(\tau)e^{[-\frac{1}{2}A_k(\tau)\varphi_{\vec{k}}\varphi_{-\vec{k}}]}$$

Bunch Davies State of Two Fields

Gaussian state with inflaton φ and χ:

$$\Psi_{\vec{k}}\left[\varphi_{\vec{k}},\chi_{\vec{k}};\tau\right]$$

$$= N_k(\tau)e^{\left[-\frac{1}{2}\left(A_k(\tau)\varphi_{\vec{k}}\varphi_{-\vec{k}}+B_k(\tau)\chi_{\vec{k}}\chi_{-\vec{k}}\right)\right]}$$



Entanglement with Spectator Scalar Field

Gaussian entangled state with inflaton φ:

$$\Psi_{\vec{k}}\left[\varphi_{\vec{k}},\chi_{\vec{k}};\tau\right]$$

$$= N_k(\tau)e^{[-\frac{1}{2}(A_k(\tau)\varphi_{\vec{k}}\varphi_{-\vec{k}}+B_k(\tau)\chi_{\vec{k}}\chi_{-\vec{k}}+C_k(\tau)(\varphi_{\vec{k}}\chi_{-\vec{k}}+\chi_{\vec{k}}\varphi_{\vec{k}}))]}$$

- $C_k(\tau)$: Entanglement Parameter
- When $C_k(\tau)=0$ $\longrightarrow$ recover vacuum (Bunch Davies) solutions

Entanglement with Metric Perturbations

Gaussian entangled state with gauge invariant inflaton ζ:

$$\psi_{\vec{k}}[\zeta_{\vec{k}}, h^{+}_{\vec{k}}, h^{\times}_{\vec{k}}, \tau] = \sqrt{N_k(\tau)} \times$$

$$\exp\left[-\frac{1}{2}\begin{pmatrix} \zeta_{-\vec{k}}, & h^{+}_{-\vec{k}}, & h^{\times}_{-\vec{k}} \end{pmatrix}\begin{pmatrix} A_k(\tau) & C^{+}_k(\tau) & C^{\times}_k(\tau) \\ C^{+}_k(\tau) & b_{0k}(\tau) + b_{3k}(\tau) & b_{1k}(\tau) \\ C^{\times}_k(\tau) & b_{1k}(\tau) & b_{0k}(\tau) - b_{3k}(\tau) \end{pmatrix}\begin{pmatrix} \zeta_{\vec{k}} \\ h^{+}_{\vec{k}} \\ h^{\times}_{\vec{k}} \end{pmatrix}\right]$$

- $C^{+}_k(\tau), C^{\times}_k(\tau)$ Tensor-Scalar Entanglement Parameters
- $b_{3k}(\tau), b_{1k}(\tau)$ + and x Polarization Entanglement Parameters

Entangled State: Closer Look

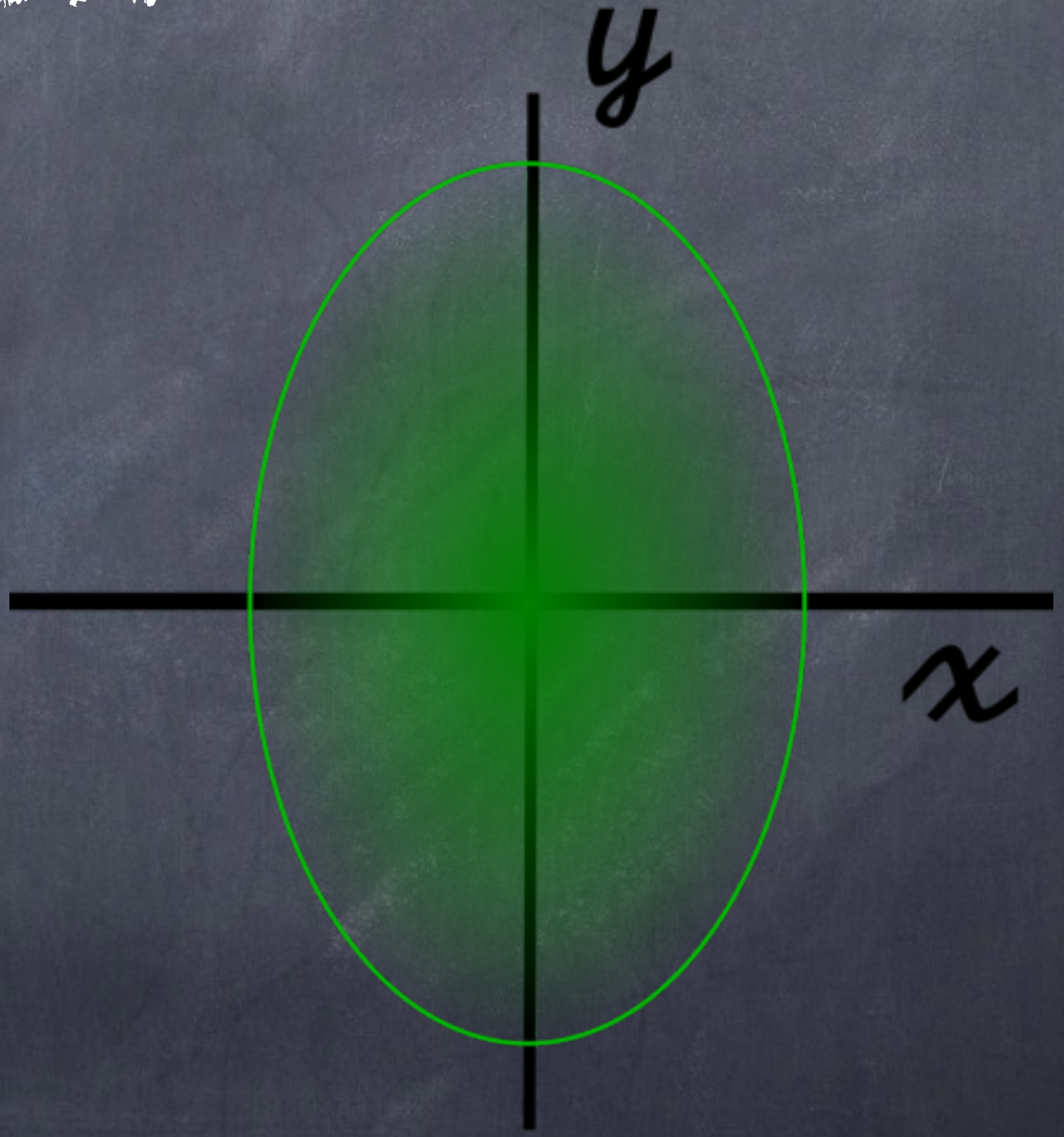
How to think of this state?

- The plot of a 2D Gaussian of the form

$$\psi = N e^{-\frac{1}{2}(Ax^2 + By^2)}$$

is an ellipse with widths determined by A and B.

Here there is no entanglement between x and y coordinates.



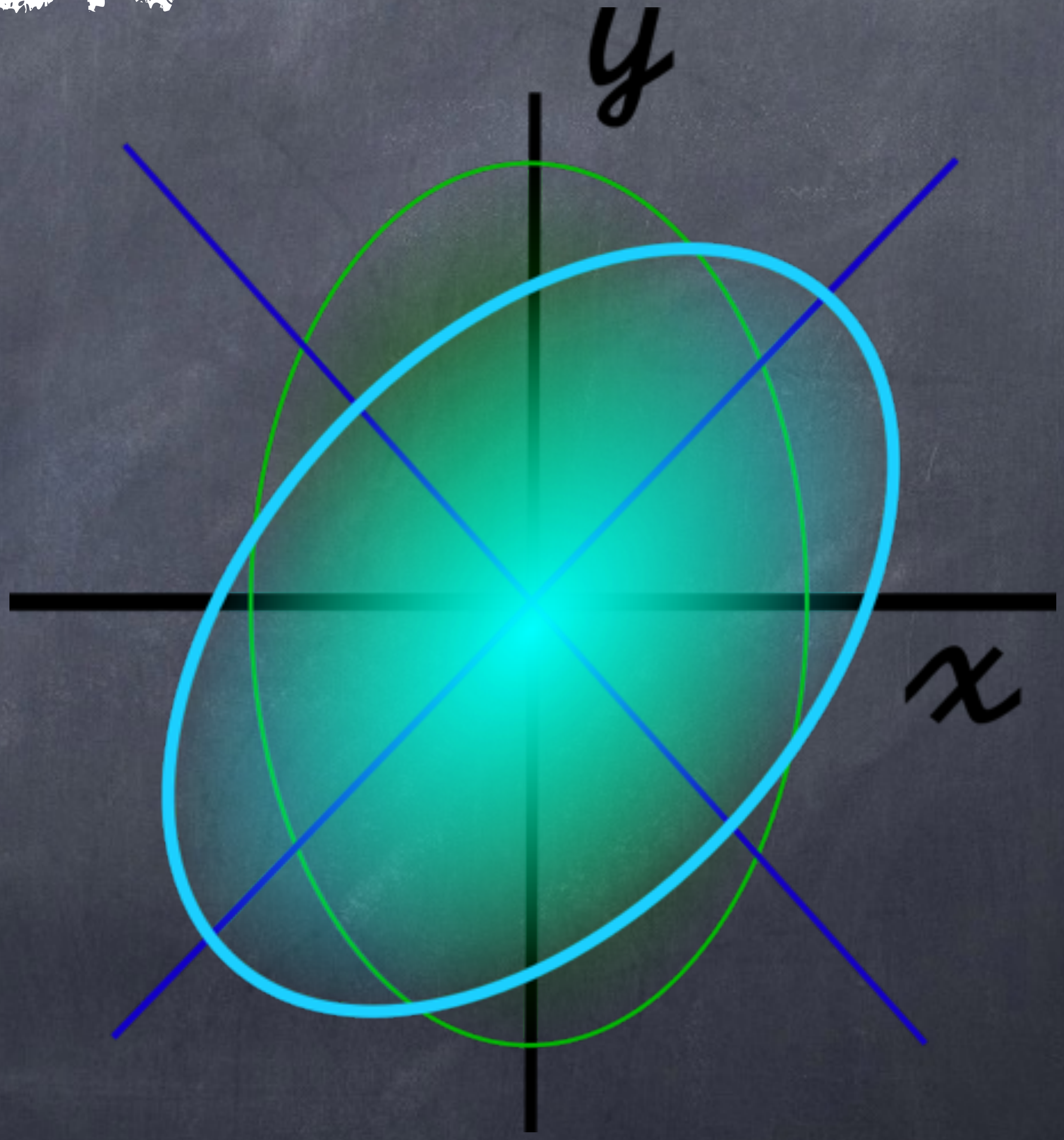
Entangled State: Closer Look

Our state of the form

$$\psi = Ne^{-\frac{1}{2}(Ax^2+By^2+2Cxy)}$$

is a tilted ellipse with respect to the x and y coordinates.

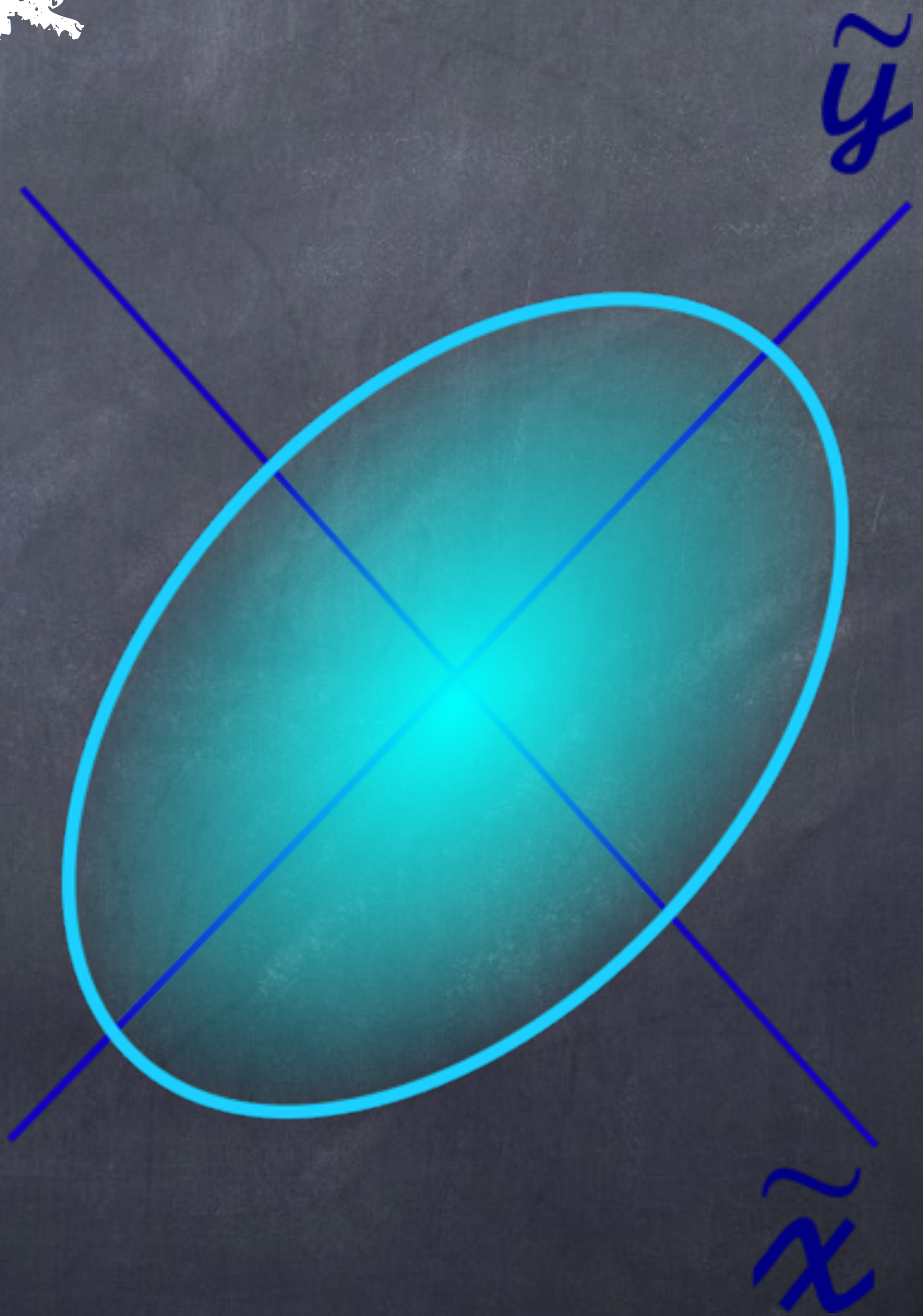
This is an entangled state in the x and y coordinates.



Entangled State: Closer Look

We could redefine the coordinates to $\tilde{x}$ and $\tilde{y}$ such that the ellipse would no longer be tilted.

In these coordinates the state would no longer be entangled; however, the Hamiltonian would have coupling terms between $\tilde{x}$ and $\tilde{y}$.



Schrödinger Picture QFT

Schrödinger Equation:

$$i\frac{\partial}{\partial\tau}\Psi_E = \left(H_{\zeta\vec{k}} + H_{\gamma,\chi\,\vec{k}}\right)\Psi_E$$

Entangled Gaussian State

Quadratic decoupled Hamiltonian for inflaton fluctuation

Quadratic decoupled Hamiltonian for spectator scalar field or metric perturbations

→ Equations of motion for mode functions of inflaton and entangled perturbations

Observables: Primordial

- Two point correlation function

$$\langle\varphi_{\vec{k}}\varphi_{-\vec{k}}\rangle(\tau) \equiv \mathrm{Tr}\left(\rho_{\vec{k}}(\tau)\varphi_{\vec{k}}\varphi_{-\vec{k}}\right)$$

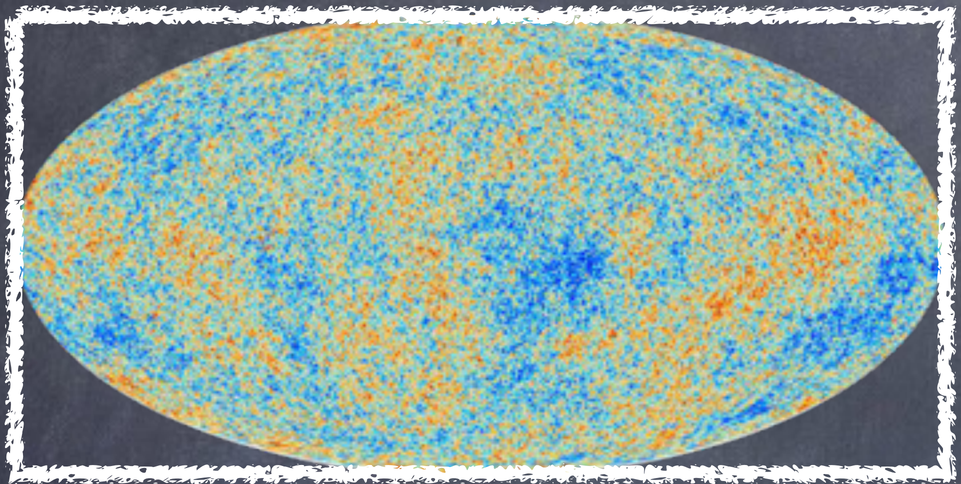
Density Matrix

$$\rho_{\vec{k}}[\varphi_{\vec{k}},\tilde{\varphi}_{\vec{k}};\tau] = \int \mathcal{D}^2\chi_{\vec{k}}\langle\varphi_{\vec{k}},\chi_{\vec{k}}|\Psi(\tau)\rangle\langle\Psi(\tau)|\tilde{\varphi}_{\vec{k}},\chi_{\vec{k}}\rangle$$

- Primordial Power Spectrum

$$P(k) \equiv \frac{k^3}{2\pi^2}\langle\varphi_{\vec{k}}\varphi_{-\vec{k}}\rangle\Big|_{\tau\to 0^-}$$

Observables: Primordial to CMB



$$= \frac{\Delta T(\vec{n})}{T_0}$$

Temperature Fluctuation

$T_0 = 2.7K$

Background Temperature

$$\zeta \to \delta\rho \to \Delta T$$

Harmonic Expansion:

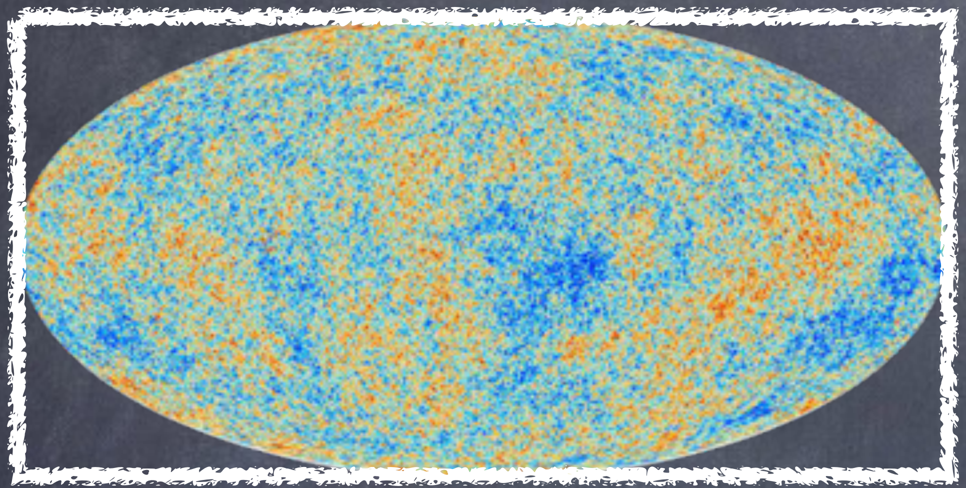
$$\Theta(\vec{n}) \equiv \frac{\Delta T(\vec{n})}{T_0} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\vec{n}) \longrightarrow a_{\ell m} = \int d\Omega Y^*_{\ell m}(\vec{n}) \Theta(\vec{n})$$

Angular Power Spectrum:

$$\langle a^*_{\ell m} a_{\ell' m'} \rangle = C^{TT}_{ll'} \delta_{mm'}$$

$$C^{TT}_{ll'} = \frac{1}{2\ell+1} \sum_m \langle a^*_{\ell m} a_{\ell' m} \rangle$$

Observables: Primordial to CMB



$$= \frac{\Delta T(\vec{n})}{T_0}$$

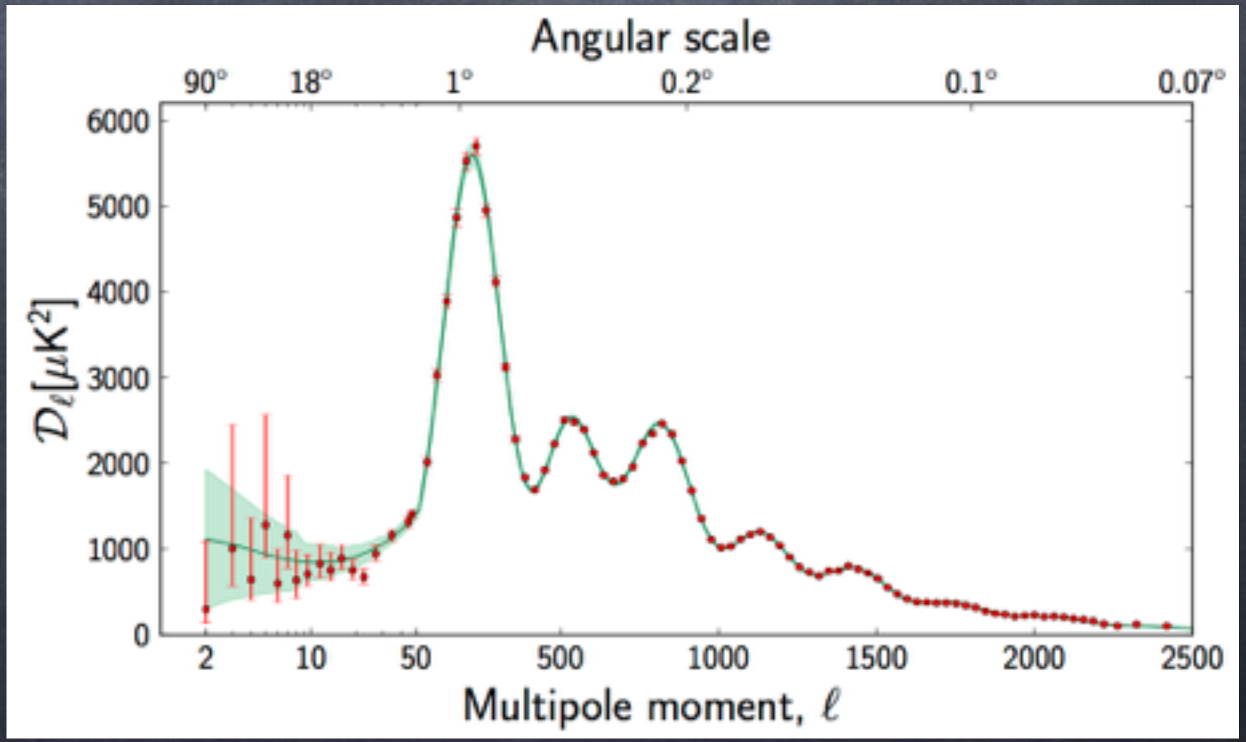
Temperature Fluctuation

$T_0 = 2.7K$

Background Temperature

Angular Power Spectrum:

$$C_\ell^{TT} \longrightarrow$$



E and B Polarizations

CMB polarization decomposed in E (curl-free) and B (divergence-free) modes.

Scalar Perturbations
Tensor (gravity) perturbs
Gravity Lensing

Vector (velocity) perturbs
Tensor (gravity) perturbs

E Modes

B Modes

$E < 0$ $E > 0$ $B < 0$ $B > 0$

E and B Polarizations

More Angular Power Spectra:

TE, EE, BB, $\boxed{\text{TB, EB}}$ Usually Zero

C_ℓ^{TE} C_ℓ^{EE} C_ℓ^{BB} C_ℓ^{TB} C_ℓ^{EB}

E Modes

B Modes

$E < 0$ $E > 0$ $B < 0$ $B > 0$

Observational Effects

Scalar-Scalar Entanglement

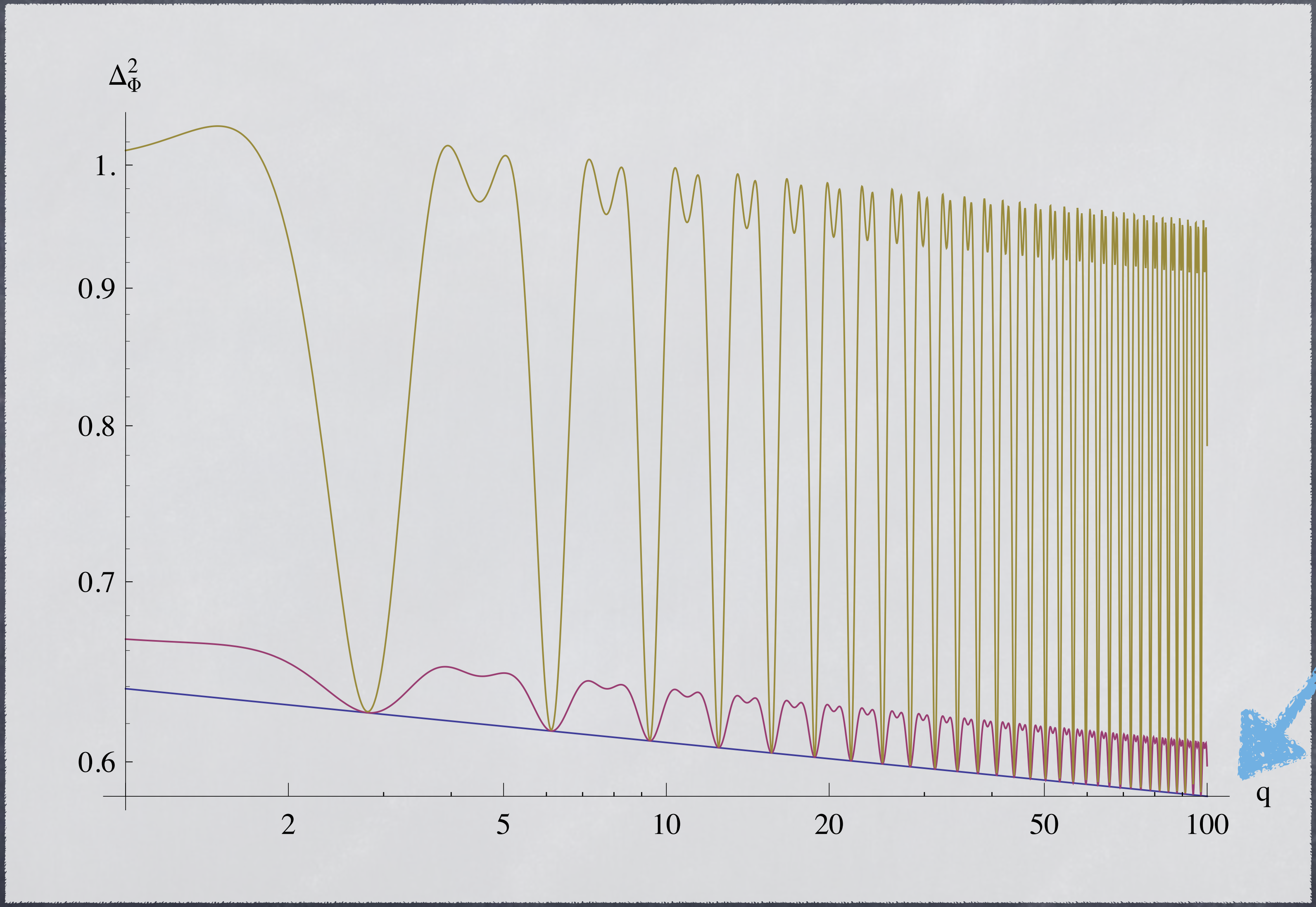
- Small oscillations in the primordial power -> oscillations in angular power

Scalar-Tensor Entanglement

- Same as Scalar-Scalar
- Non-Zero TB and EB power spectra
- Correlation between l multipoles

Primordial Power
Scalar-Scalar

P(k)



$\lambda = 0.3$

$\lambda = 0.1$

$\lambda = 0.0$

No Entanglement

k

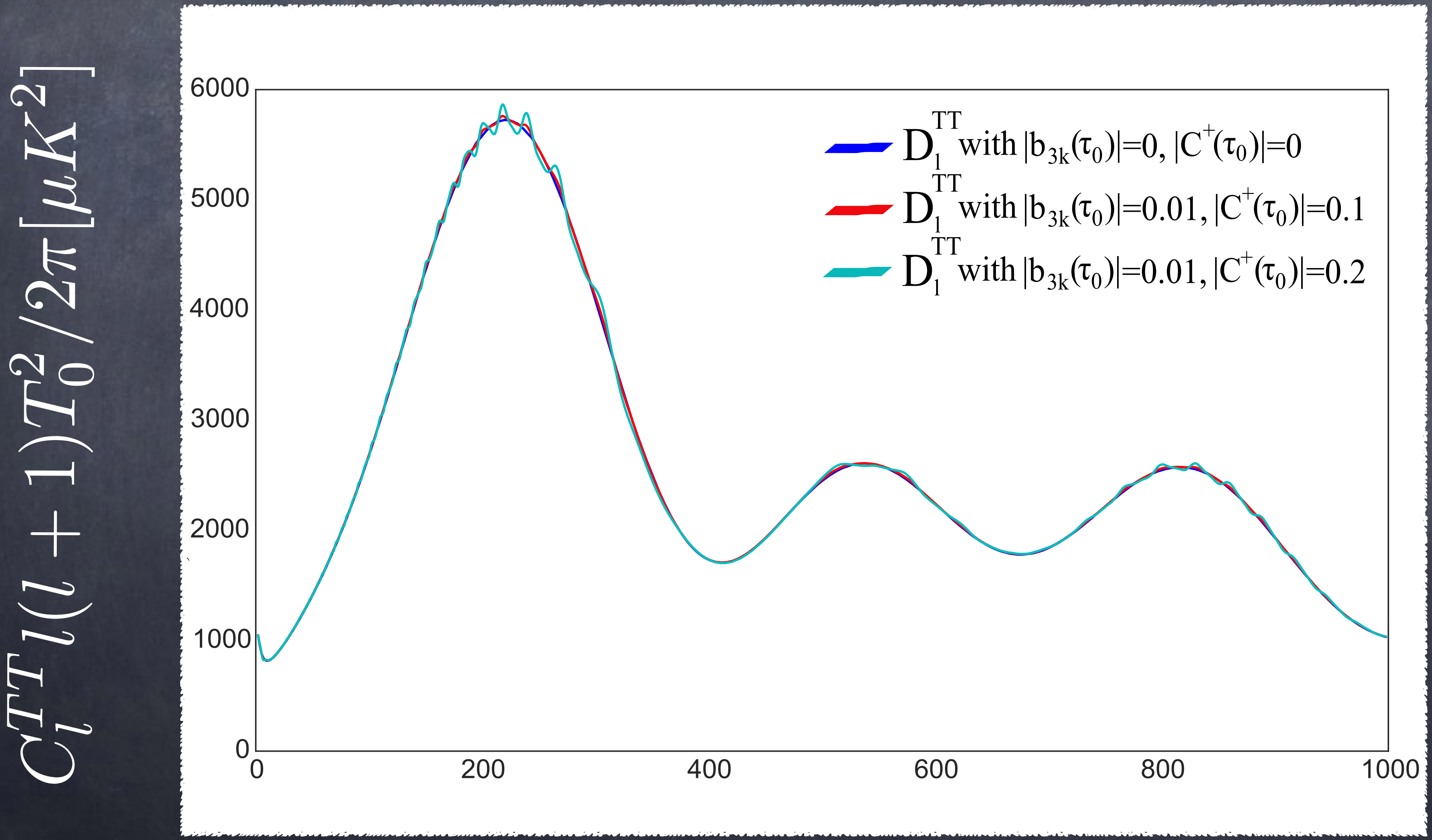
$\boxed{C_k \propto \lambda}$

Entanglement Straight Parameter: λ

Larger λ , larger oscillation amplitudes

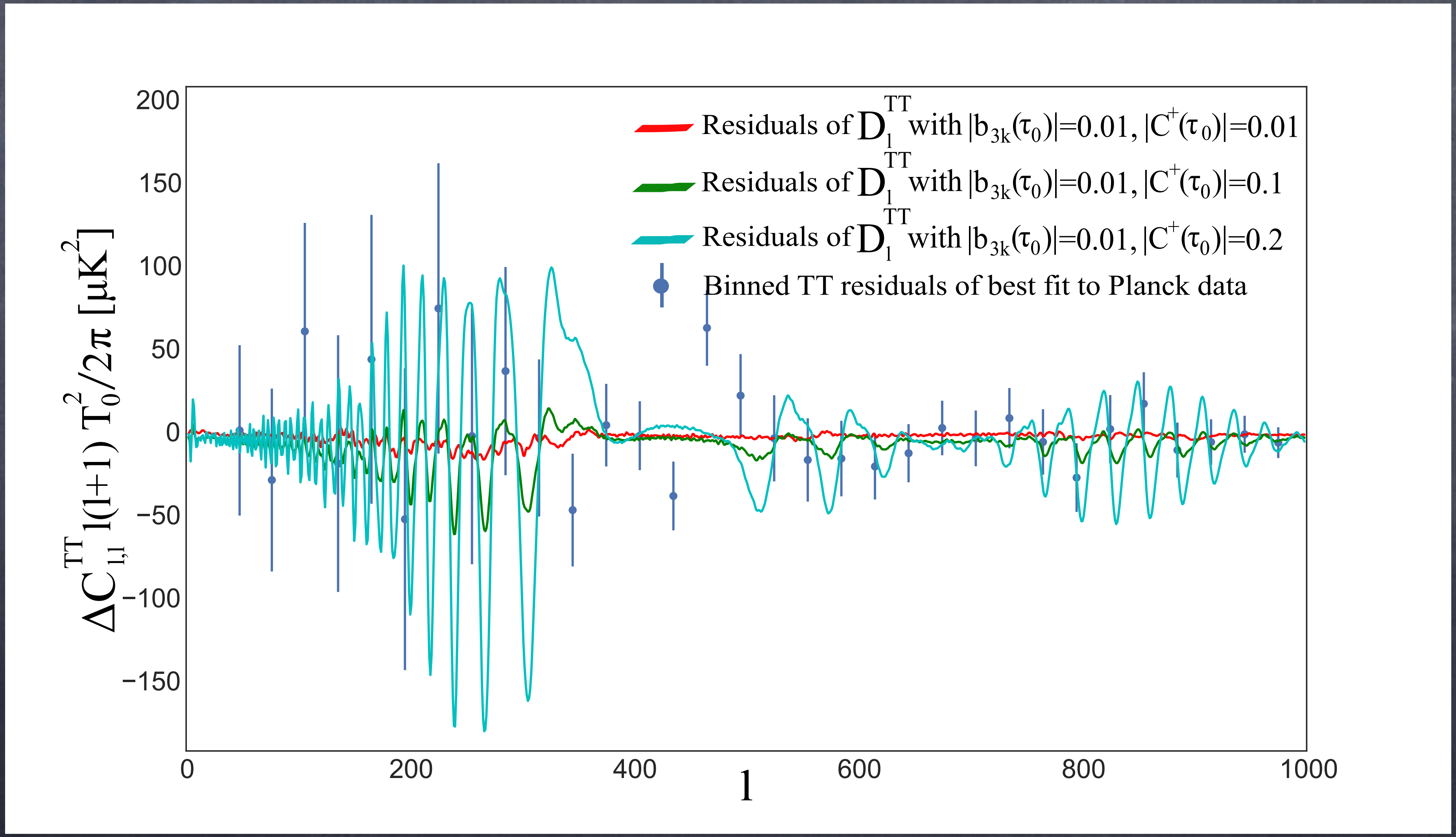
Temperature Angular Power Spectrum: Scalar-Tensor

Entanglement Strength

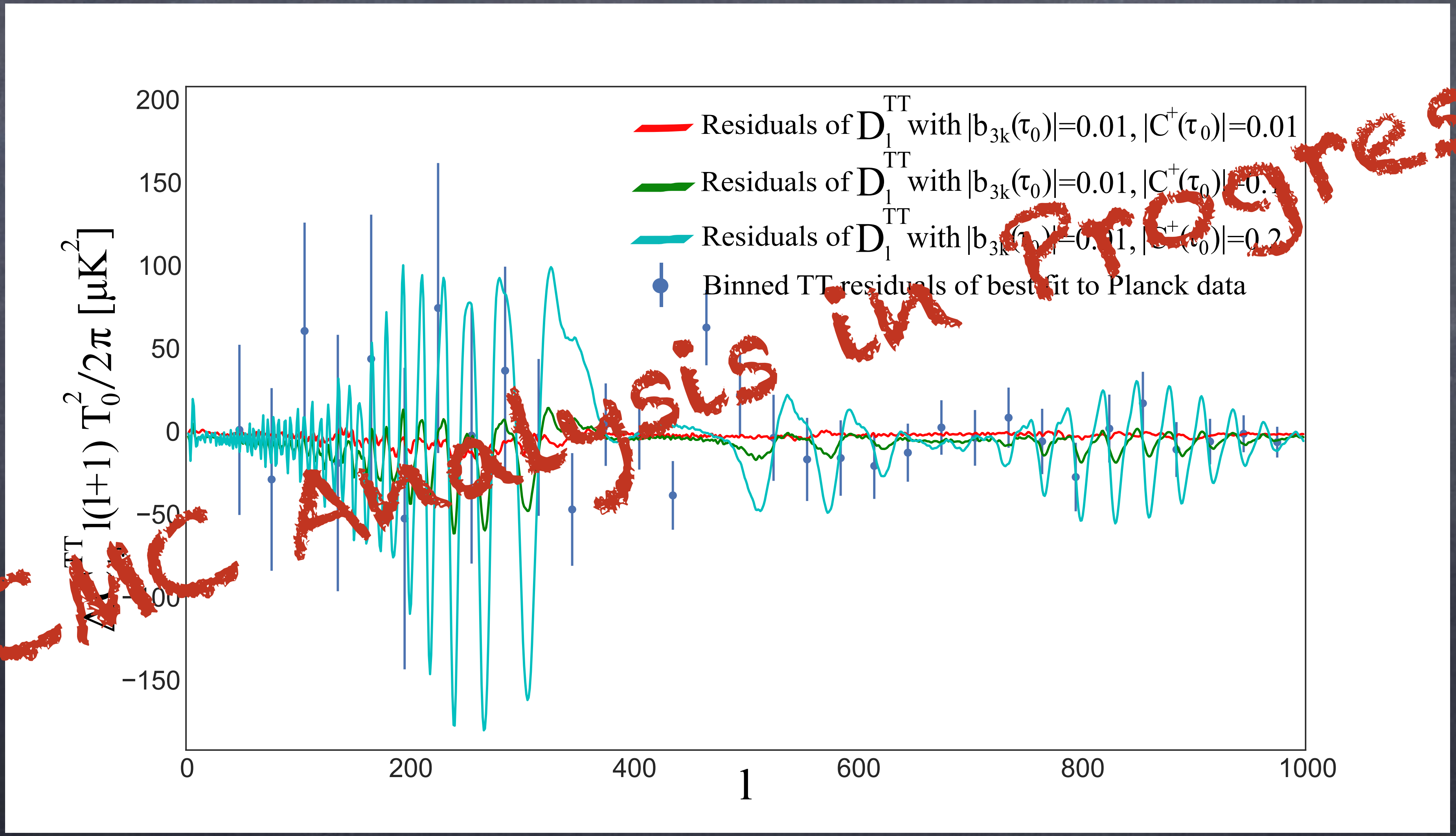


l multipoles

Angular Power Spectrum Difference with Zero Entanglement Angular Power



Angular Power Spectrum Difference with Zero Entanglement Angular Power



MCMC Analysis in Progress

TB and EB Power Spectra
Tensor-Scalar Entanglement

Correlation of the temperature and the E, B polarizations:

$$C_{ll'}^{TB,EB} = 4\pi \int \frac{dk}{k} \left\{ \Delta_{l2}^{T,E}(k) \Delta_{l'2}^{B}(k) P^{+\times}(k) \delta_{ll'} \right\}$$

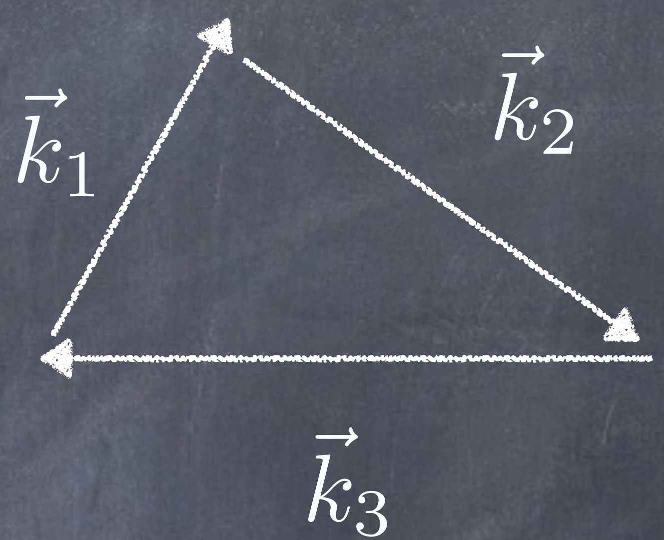
$\langle h^{+}_{\vec{k}} h^{\times}_{-\vec{k}} \rangle \neq 0$ thanks to b_{k1}, b_{k3}, the + and x polarization entanglement parameters.

→ "Parity Violation" in CMB

Primordial Non-Gaussianity

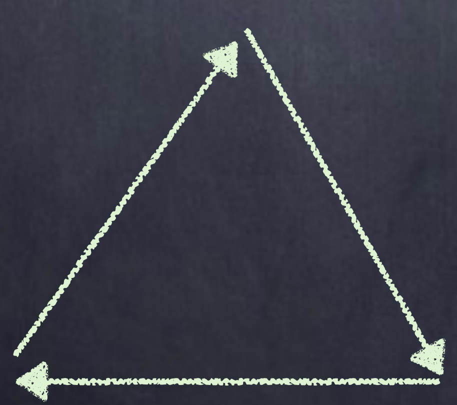
Three point correlation function

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta(k_1, k_2, k_3)$$



Shapes

$k_1 = k_2 = k_3$ $\qquad$ $k_1 \ll k_2 = k_3$ $\qquad$ $k_1 \approx k_2 + k_3$



Equilateral

Squeezed

Folded/Flattened

Primordial Non-Gaussianity

Higher derivative correlations in inflation

Multi-field inflation

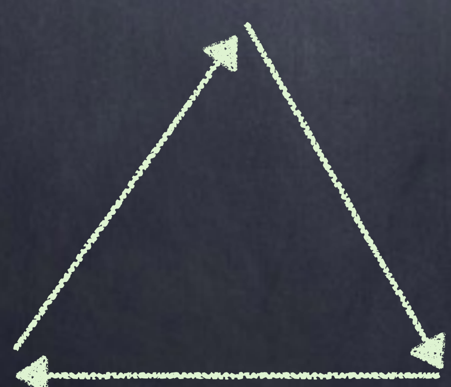
(cannot be high in single field)

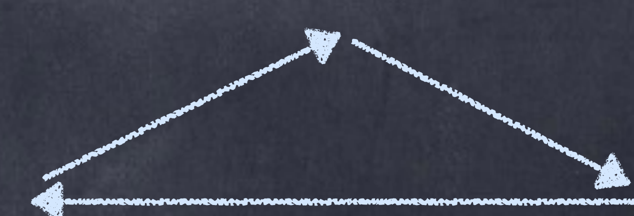
Non Bunch-Davies inflation models

$k_1 = k_2 = k_3$

$k_1 \ll k_2 = k_3$

$k_1 \approx k_2 + k_3$





Equilateral

Squeezed

Folded/Flattened

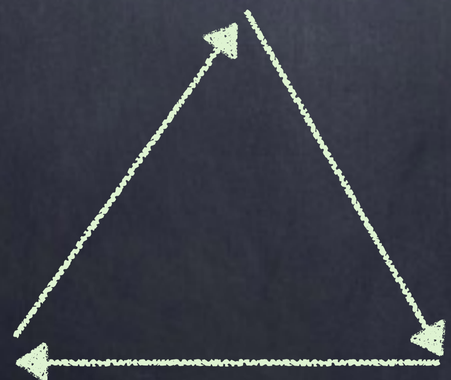
Primordial Non-Gaussianity from Scalar-Scalar Entanglement

(Preliminary result - not finalized)

Entanglement strength parameter << 0.5

$$\langle \zeta^3 \rangle = \langle \zeta^3 \rangle_{BD}(1 + 3\lambda) + \lambda \langle \zeta^3 \rangle_{NON-BD}$$

$k_1 = k_2 = k_3$

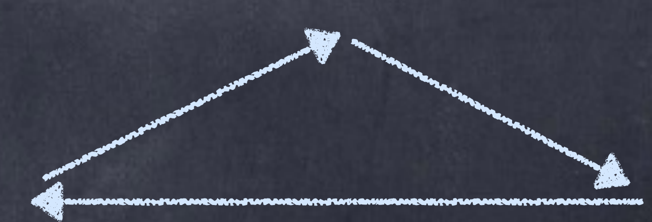


Equilateral

$k_1 \ll k_2 = k_3$

Squeezed

$k_1 \approx k_2 + k_3$



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What do we hope to learn from the bispectrum?

- Will it distinguish entanglement of 2 fields during inflation from multi-field inflation?
- Another bound on entanglement strength parameter.
- Understand what characteristic bispectrum shape to expect from entanglement.

Schrödinger Picture Bispectrum Setup

Cubic order expanded state:

$$\Psi(\zeta,\chi,\tau) = \int \prod_{i=1}^{3} \left(\frac{d^3\vec{k}_i}{(2\pi)^3}\right)(2\pi)^3\delta\left(\sum_{j=1}^{3}\vec{k}_j\right)\Big[1+\mu\Big\{Z^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}$$

$$+Y^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}+\sum_{ijl}\big(\epsilon_{ijl}W_i^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_i}\zeta_{\vec{k}_j}\zeta_{\vec{k}_l}$$

$$+\epsilon_{ijl}X_i^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_i}\chi_{\vec{k}_j}\chi_{\vec{k}_l}\big)\Big\}\Big]\Psi_G(\tau)$$

Schrödinger Picture Bispectrum Setup

Expansion parameter

$$\Psi(\zeta,\chi,\tau) = \int \prod_{i=1}^{3}\left(\frac{d^3\vec{k}_i}{(2\pi)^3}\right)(2\pi)^3\delta\left(\sum_{j=1}^{3}\vec{k}_j\right)\Big[1+\mu\Big\{Z^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}$$
$$+Y^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}+\sum_{ijl}\left(\epsilon_{ijl}W_i^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_i}\zeta_{\vec{k}_j}\zeta_{\vec{k}_l}\right.$$
$$\left.+\epsilon_{ijl}X_i^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_i}\chi_{\vec{k}_j}\chi_{\vec{k}_l}\right)\Big\}\Big]\Psi_G(\tau)$$

Schrödinger Picture Bispectrum Setup

$$\Psi(\zeta,\chi,\tau) = \int \prod_{i=1}^{3}\left(\frac{d^3\vec{k}_i}{(2\pi)^3}\right)(2\pi)^3\delta\left(\sum_{j=1}^{3}\vec{k}_j\right)\Big[1+\mu\Big\{Z^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}$$

$$+Y^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}+\sum_{ijl}\big(\epsilon_{ijl}{W_i}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_i}\zeta_{\vec{k}_j}\zeta_{\vec{k}_l}$$

$$+\epsilon_{ijl}{X_i}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_i}\chi_{\vec{k}_j}\chi_{\vec{k}_l}\big)\Big\}\Big]\Psi_G(\tau)$$

Cubic order coefficients

Schrödinger Picture Bispectrum Setup

$$\Psi(\zeta,\chi,\tau) = \int \prod_{i=1}^{3}\left(\frac{d^3\vec{k}_i}{(2\pi)^3}\right)(2\pi)^3\delta\left(\sum_{j=1}^{3}\vec{k}_j\right)\Big[1+\mu\Big\{Z^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}$$

$$+Y^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}+\sum_{ijl}\big(\epsilon_{ijl}W_i{}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_i}\zeta_{\vec{k}_j}\zeta_{\vec{k}_l}$$

$$+\epsilon_{ijl}X_i{}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_i}\chi_{\vec{k}_j}\chi_{\vec{k}_l}\big)\Big\}\Big]\Psi_G(\tau)$$

All combinations of fields at cubic order

Schrödinger Picture Bispectrum Setup

$$\Psi(\zeta,\chi,\tau)=\int\prod_{i=1}^{3}\left(\frac{d^3\vec{k}_i}{(2\pi)^3}\right)(2\pi)^3\delta\left(\sum_{j=1}^{3}\vec{k}_j\right)\Big[1+\mu\Big\{Z^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}$$
$$+Y^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_1}\chi_{\vec{k}_2}\chi_{\vec{k}_3}+\sum_{ijl}\Big(\epsilon_{ijl}W_i{}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\chi_{\vec{k}_i}\zeta_{\vec{k}_j}\zeta_{\vec{k}_l}$$
$$+\epsilon_{ijl}X_i{}^{\vec{k}_1\vec{k}_2\vec{k}_3}(\tau)\zeta_{\vec{k}_i}\chi_{\vec{k}_j}\chi_{\vec{k}_l}\Big)\Big\}\Big]\Psi_G(\tau)$$

Quadratic entangled Gaussian state

Schrödinger Picture Bispectrum Setup

Schrödinger Equation:

$$i\frac{\partial}{\partial \tau}\Psi = (H^{(2)} + \mu H^{(3)})\Psi$$

For each order of μ s.t. $\Psi = \Psi^{(2)} + \mu\Psi^{(3)}$

$$\mathcal{O}(\mu^0) : \frac{\partial}{\partial \tau}\Psi^{(2)} = H^{(2)}\Psi^{(2)}$$

$$\mathcal{O}(\mu^1) : \frac{\partial}{\partial \tau}\Psi^{(3)} = H^{(2)}\Psi^{(3)} + H^{(3)}\Psi^{(2)}$$

Bispectrum: Perturbative Solution

1) $i\frac{\partial}{\partial \tau}\Psi = (H^{(2)} + \mu H^{(3)})\Psi$ $\longrightarrow$ Equations of motion for cubic state coefficients (Z, Y, W, X).

2) Find perturbative solutions to these equations (expanding in powers of small entanglement strength parameter).

3) Use the solutions of cubic state coefficients (Z, Y, W, X) to calculate bispectrum.

Entangled Bispectrum

Calculating the entangled bispectrum in terms of the cubic and quadratic coefficients:

$$\langle\Psi|\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}|\Psi\rangle = \int\int \mathcal{D}^2\zeta_{\vec{q}}\mathcal{D}^2\chi_{\vec{q}}\Psi^*(\zeta,\chi,\tau)\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\Psi(\zeta,\chi,\tau)$$

$$\Big\downarrow$$

$$\langle\Psi|\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}|\Psi\rangle = 12\delta(\sum_i \vec{k}_i)\frac{1}{A_{1R}B_{1R}-C_{1R}{}^2}\frac{1}{A_{2R}B_{2R}-C_{2R}{}^2}\frac{1}{A_{3R}B_{3R}-C_{3R}{}^2}$$

$$\frac{1}{2^3}[Z_R B_{1R}B_{2R}B_{3R} + 2^3 Y_R C_{1R}C_{2R}C_{3R} + 2\sum_{ijl} W_{lR}B_{iR}B_{jR}C_{lR} + 2^2\sum_{ijl} X_{lR}C_{iR}C_{jR}B_{lR}]$$

Contributions of Different Orders to the Bispectrum

Looking at orders of entanglement strength parameter λ:

$$C_{kR} \propto \lambda, \quad Z_R, Y_R \propto 1 + \lambda(...) + \mathcal{O}(\lambda^2), \quad W_R, X_R \propto \lambda(...) + \mathcal{O}(\lambda^2)$$

$$\langle\Psi|\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}|\Psi\rangle = 12\delta(\sum_i \vec{k}_i)\frac{1}{A_{1R}B_{1R} - C_{1R}{}^2}\frac{1}{A_{2R}B_{2R} - C_{2R}{}^2}\frac{1}{A_{3R}B_{3R} - C_{3R}{}^2}$$

$$\frac{1}{2^3}[\underbrace{Z_R}_{\propto 1 + \lambda(...)} B_{1R}B_{2R}B_{3R} + 2^3 \underbrace{Y_R}_{\propto \lambda^3(...) + ...} C_{1R}C_{2R}C_{3R} + 2\sum_{ijl} \underbrace{W_{lR}}_{\propto \lambda^2(...) + ...} B_{iR}B_{jR}C_{lR} + 2^2\sum_{ijl} \underbrace{X_{lR}}_{\propto \lambda^3(...) + ...} C_{iR}C_{jR}B_{lR}]$$

Contributions of Different Orders to the Bispectrum

Looking at orders of entanglement strength parameter λ:

To lowest order in λ

$$\langle\Psi|\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}|\Psi\rangle = 12\delta(\sum_i \vec{k}_i)\frac{1}{A_{1R}B_{1R}-C_{1R}{}^2}\frac{1}{A_{2R}B_{2R}-C_{2R}{}^2}\frac{1}{A_{3R}B_{3R}-C_{3R}{}^2}$$

$$\frac{1}{2^3}[\boxed{Z_R B_{1R}B_{2R}B_{3R}} + 2^3 Y_R C_{1R}C_{2R}C_{3R} + 2\sum_{ijl} W_{lR}B_{iR}B_{jR}C_{lR} + 2^2\sum_{ijl} X_{lR}C_{iR}C_{jR}B_{lR}]$$

$$\propto 1+\lambda(\ldots) \qquad \propto \lambda^3(\ldots)+\ldots \qquad \propto \lambda^2(\ldots)+\ldots \qquad \propto \lambda^3(\ldots)+\ldots$$

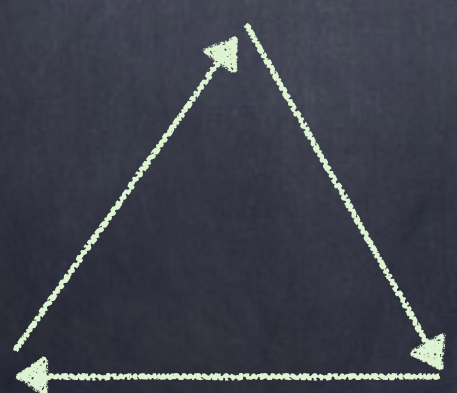
First Order Bispectrum

(Preliminary result - not finalized)

Entanglement strength parameter << 0.5

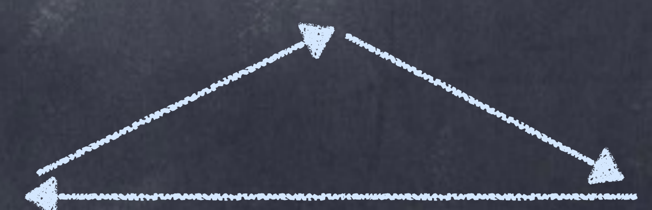
$$\langle \zeta^3 \rangle = \langle \zeta^3 \rangle_{BD}(1 + 3\lambda) + \lambda \langle \zeta^3 \rangle_{NON-BD}$$

$k_1 = k_2 = k_3$ $\quad$ $k_1 \ll k_2 = k_3$ $\quad$ $k_1 \approx k_2 + k_3$



Equilateral

Squeezed



Folded/Flattened

First Order Bispectrum

(Preliminary result - not finalized)

Entanglement strength parameter << 0.5

$$\langle \zeta^3 \rangle = \langle \zeta^3 \rangle_{BD}(1+3\lambda) + \lambda \langle \zeta^3 \rangle_{NON-BD}$$

$$\propto \frac{\text{func}(k_1, k_2, k_3)}{k_i + k_j - k_l} \xrightarrow{k_i + k_j = k_l} \langle \zeta^3 \rangle_{\text{NON-BD}} \to \infty$$

Artificial divergence!

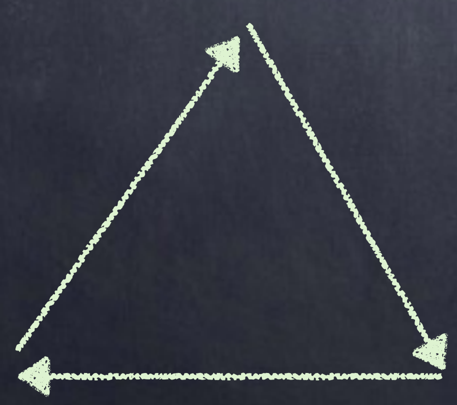
Present because assumed non-BD at infinite past. → Realistically there would be cutoff at large momenta.

Finding Bound on Entanglement Strength Parameter

Taking the equilateral and squeezed (local) limits of the first order bispectrum ->can put a rough upper limit on λ.

$$\langle \zeta^3 \rangle = \langle \zeta^3 \rangle_{BD}(1 + 3\lambda) + \lambda \langle \zeta^3 \rangle_{NON-BD}$$

$$k_1 = k_2 = k_3$$



Equilateral

Using Planck non-Gaussianity limits.

$$k_1 \ll k_2 = k_3$$

Squeezed

What do we hope to learn from the bispectrum?

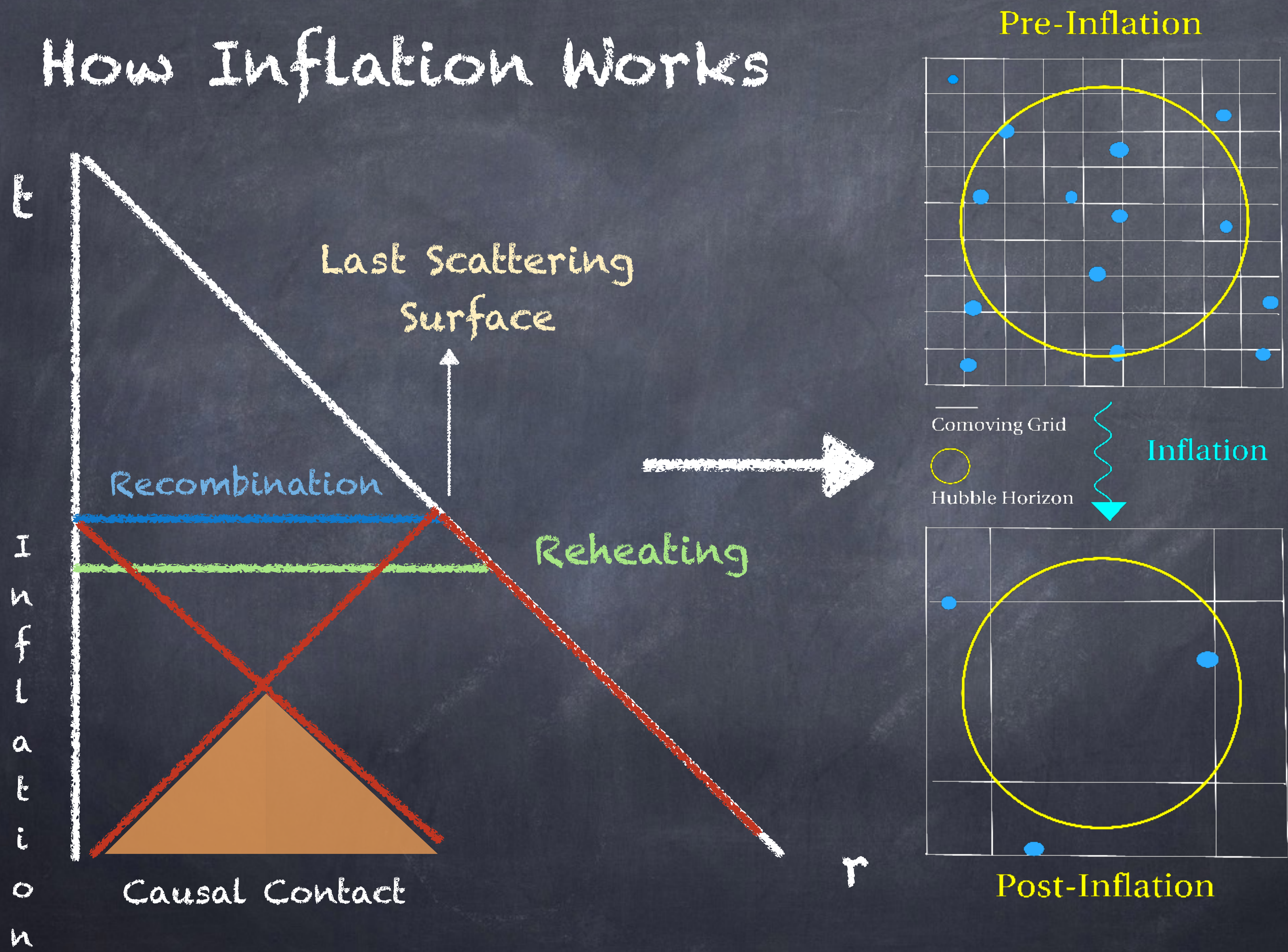
- Will it distinguish entanglement of 2 fields during inflation from multi-field inflation? Different shape produced.
- Another bound on entanglement strength parameter. Can find rough upper bound.
- Understand what characteristic bispectrum shape to expect from entanglement. Good indicator for non-BD vacuum initial state.

Final Entangled Remarks

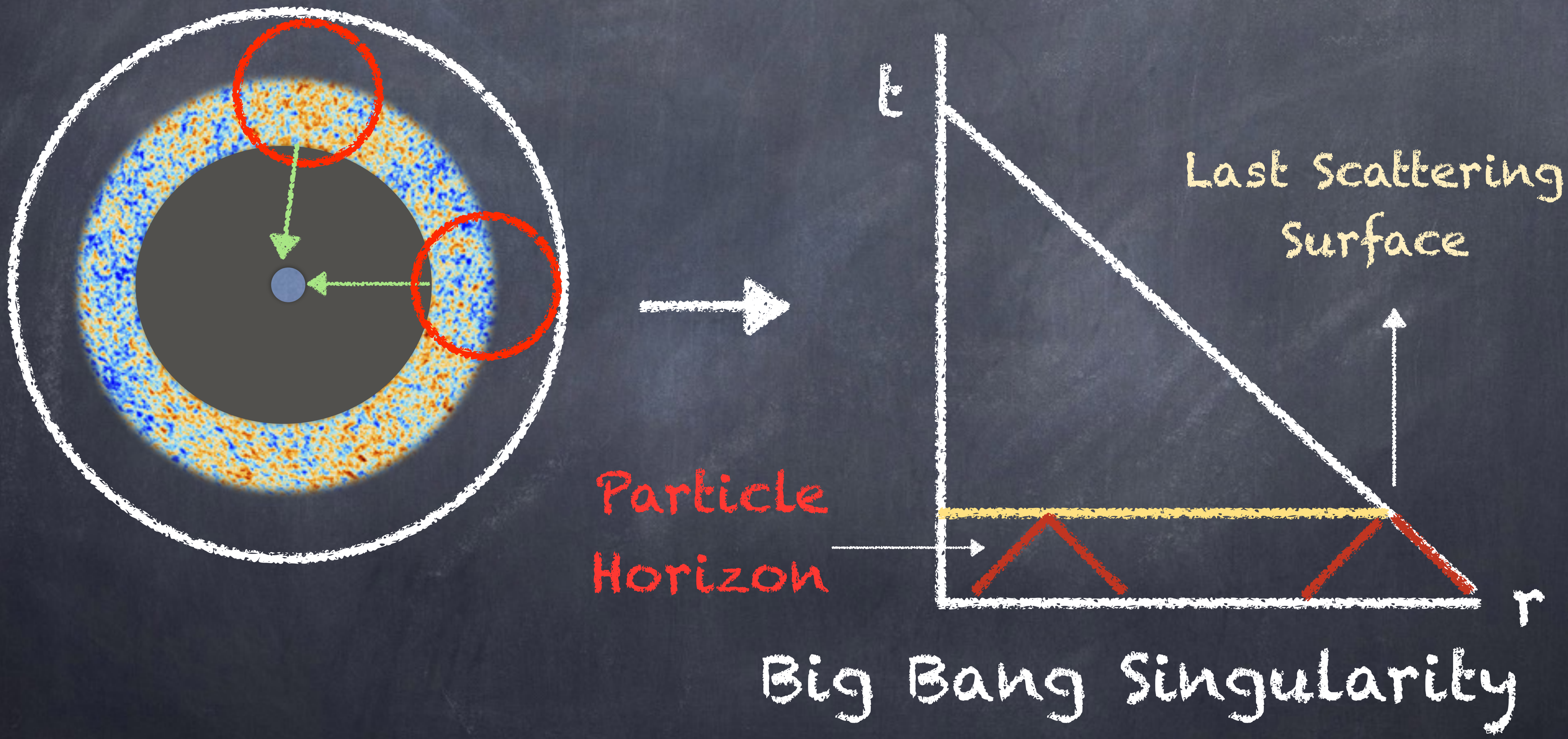
- Several distinguishing observational features of entanglement.
- These can help us constrain or rule out entanglement further validating standard picture -> MCMC analysis in progress.
- If signatures of entanglement are observed, this might point to finite inflation and or the mechanism that started it!


Thank you

How Inflation Works



Horizon Problem



Observables: CMB

- Angular Power Spectrum -> CMB

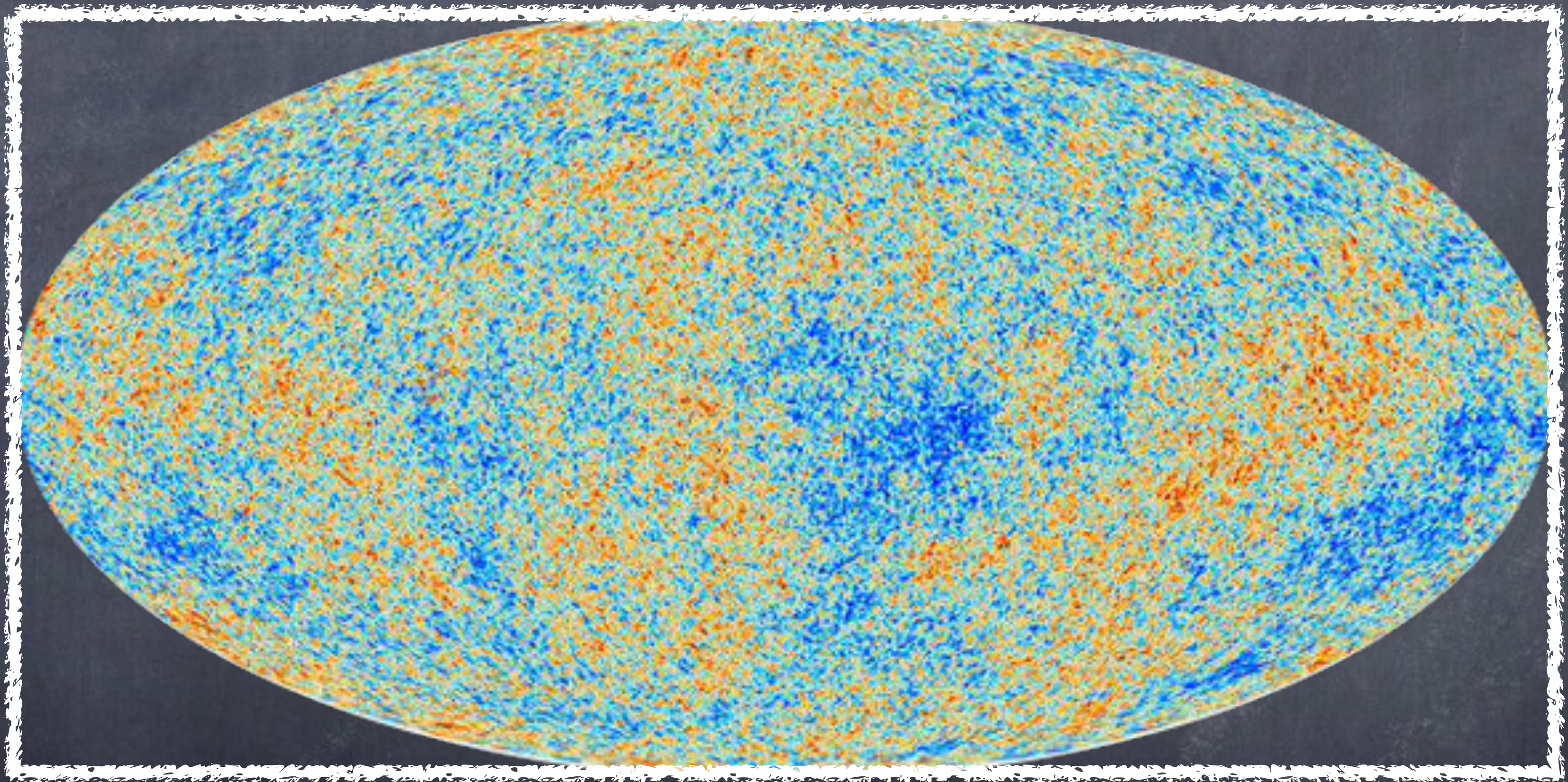
$$C^{XX'}_{l,l',m,m'} = \sum_{s,s'} \mathcal{I}_{ss'} = 4\pi \int \frac{dk}{k} \sum_{s,s'} \Delta^X_{l,s}(k,\eta_0)\Delta^{X'}_{l',s'}(k,\eta_0)$$

$$\int d\Omega_{\hat{\mathbf{k}}} P^{ss'}(\mathbf{k})\, {}_{-s}Y^*_{lm}(\hat{\mathbf{k}},\mathbf{e})\, {}_{-s'}Y_{l'm'}(\hat{\mathbf{k}},\mathbf{e})$$

- $s = 0, \pm 2$: spin of the perturbation
- $X, X' = T, E, B$ (Temperature, E-mode, B-mode)
- $P^{ss'}$: primordial power spectrum
- $\Delta^X_{l,s}(k,\eta_0)$: transfer function

What does the CMB tell us? (Cosmic Microwave Background)

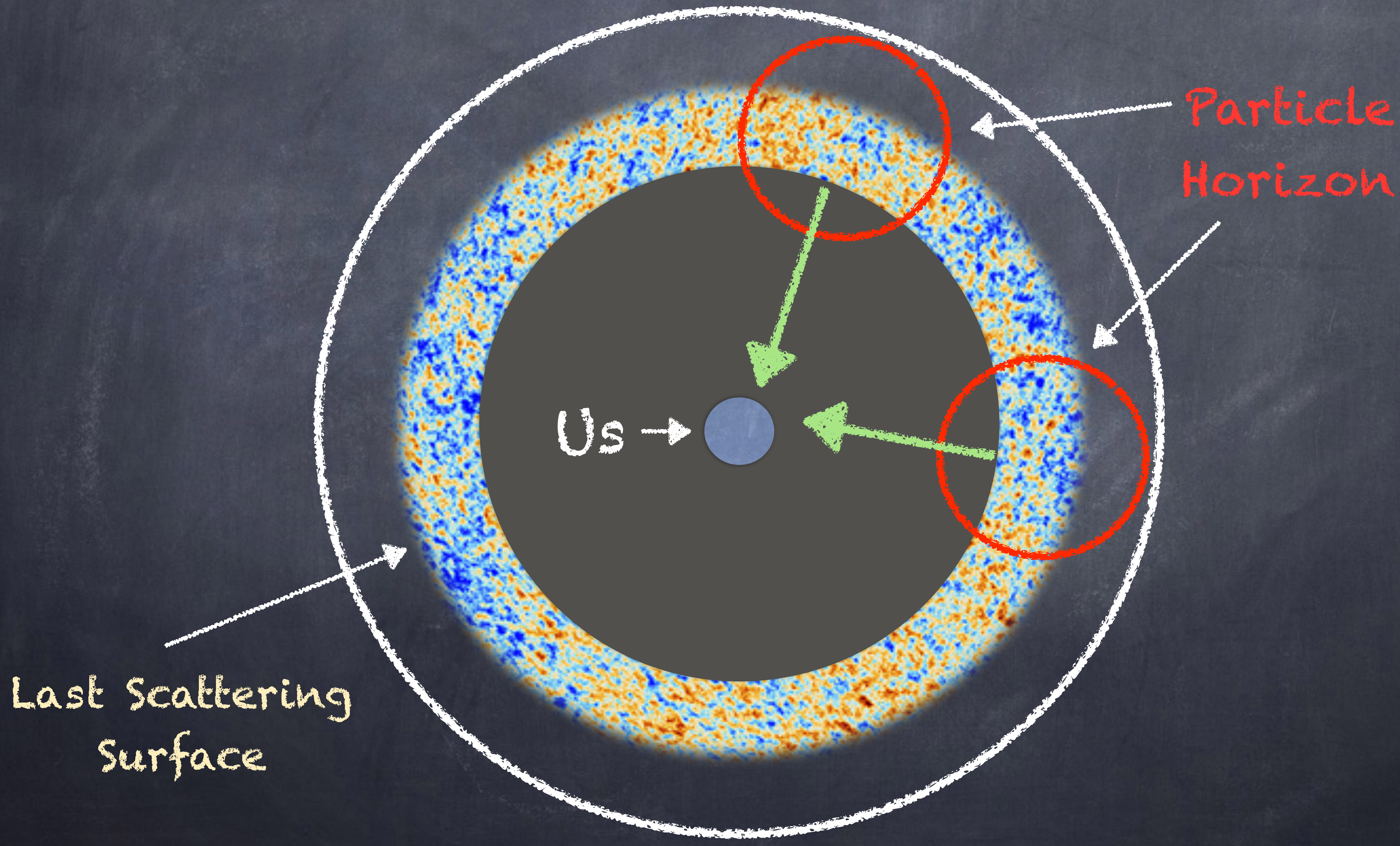
- Mostly homogeneous and isotropic



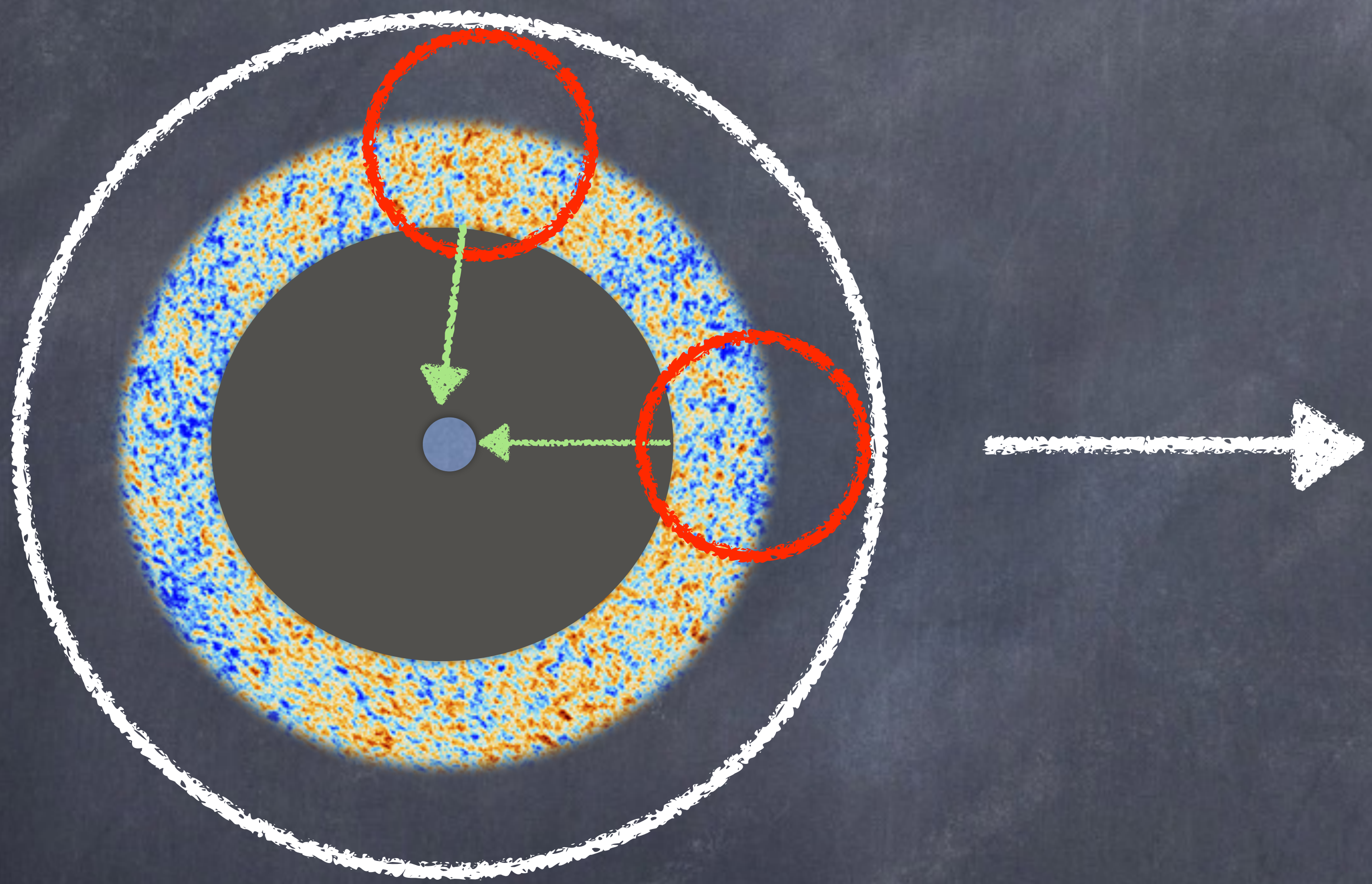
Planck CMB 2015

- Small inhomogeneity ~ $\frac{\delta T}{T} \approx 10^{-5}$

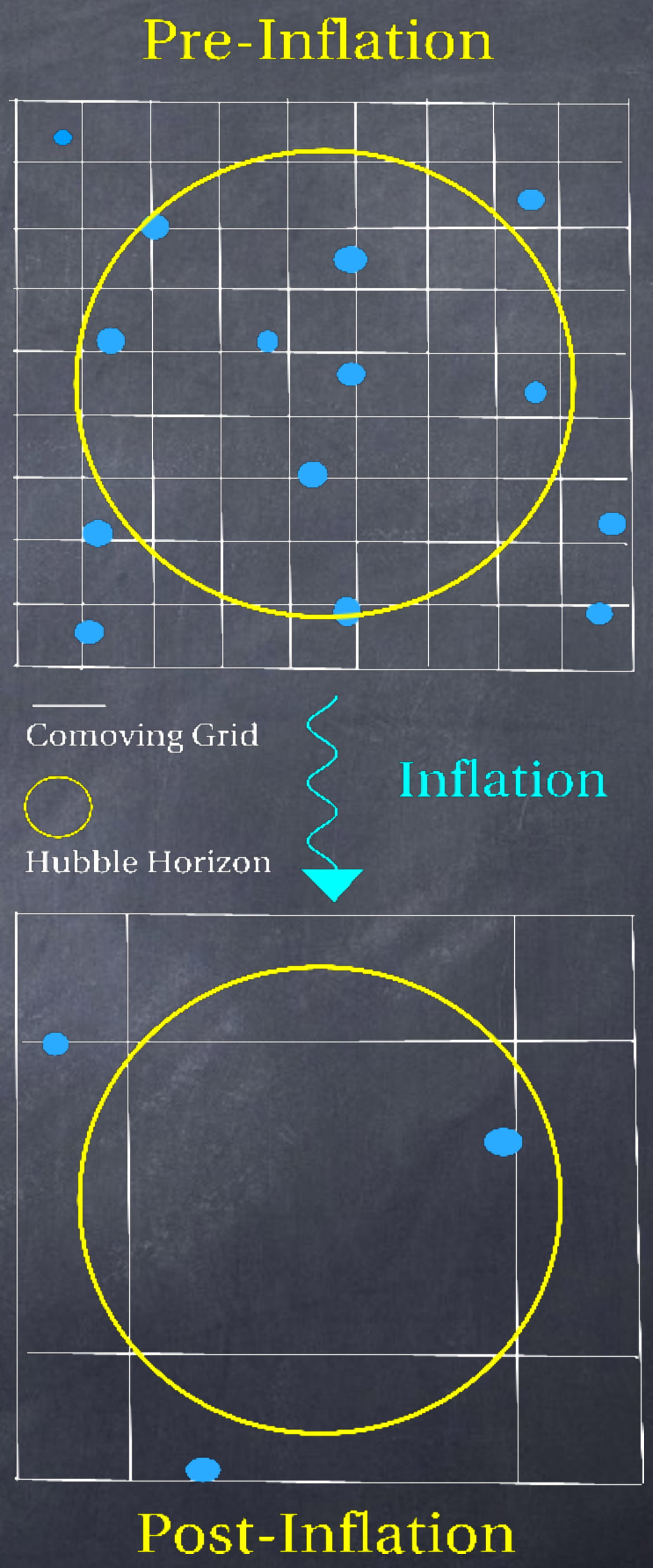
Horizon Problem



How Inflation Works



Regions < 2 degrees apart
never causally connected



Flatness Problem

Why is the spatial curvature of the universe so small?

Friedmann Equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\left(\underbrace{\overset{\propto a^{\approx 0}}{\rho_I}}_{\text{Inflation}} + \underbrace{\overset{\propto a^{-2}}{\rho_k}}_{\text{Curvature}} + \underbrace{\overset{\propto a^{-4}}{\rho_r}}_{\text{Relativistic Matter}} + \underbrace{\overset{\propto a^{-3}}{\rho_m}}_{\text{Non-Relativistic Matter}} + \underbrace{\overset{\propto a^{\approx 0}}{\rho_{DE}}}_{\text{Dark Energy}}\right)$$

Flatness Problem

Why is the spatial curvature of the universe so small?

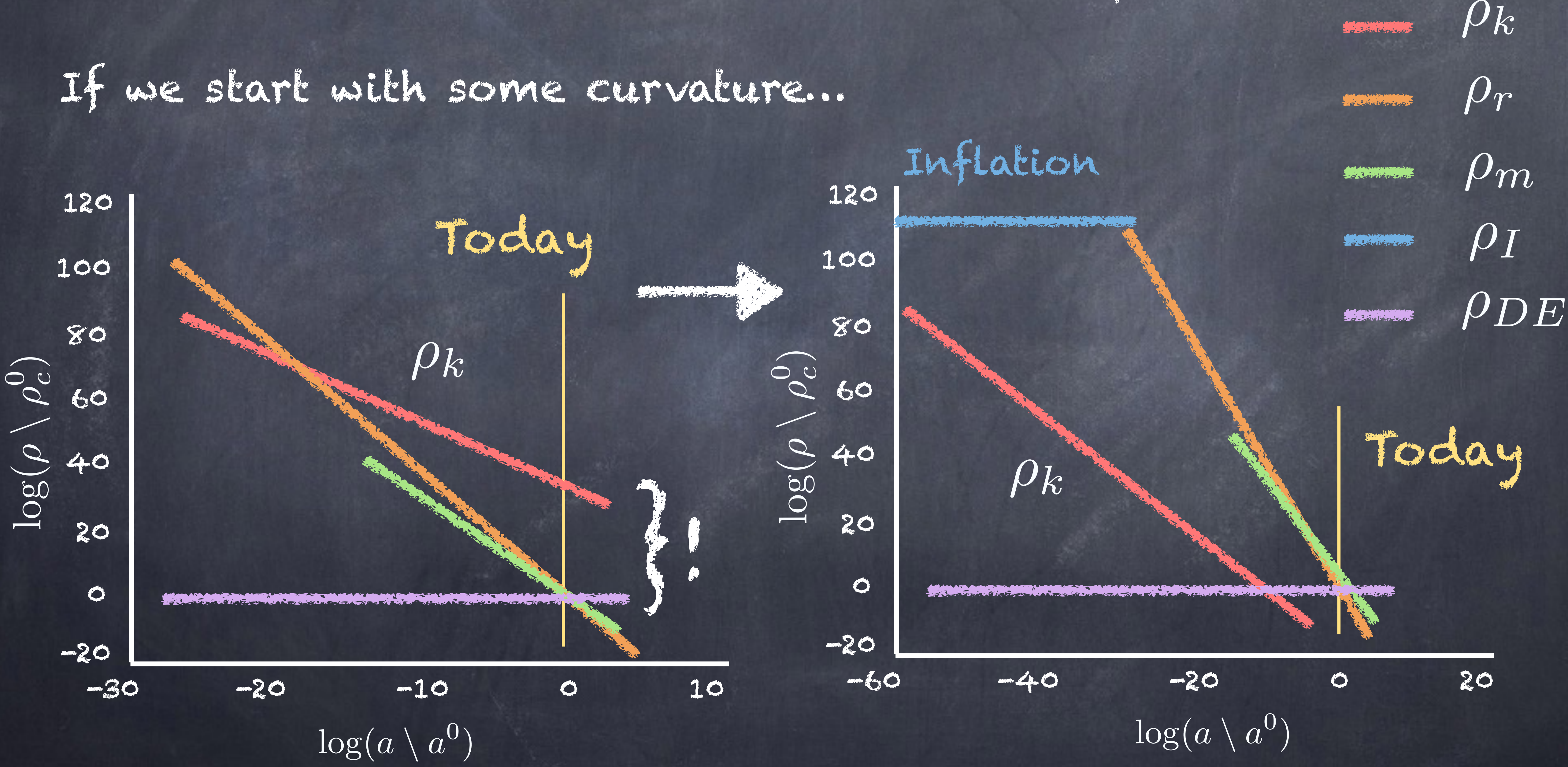
Friedmann Equation:

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\left(\underset{\text{Inflation}}{\overset{\propto a^{\approx 0}}{\rho_I}} + \underset{\text{Curvature}}{\overset{\propto a^{-2}}{\rho_k}} + \underset{\text{Relativistic Matter}}{\overset{\propto a^{-4}}{\rho_r}} + \underset{\text{Non-Relativistic Matter}}{\overset{\propto a^{-3}}{\rho_m}} + \underset{\text{Dark Energy}}{\overset{\propto a^{\approx 0}}{\rho_{DE}}}\right)$$

Flatness Problem

Why is the spatial curvature of the universe so small?

If we start with some curvature...



We would observe high curvature

Low curvature

How Inflation Works: Set Up

Flat FLRW Background - maximally symmetric, homogeneous and isotropic:

$$ds^2 = a^2(\tau)[-d\tau^2 + dr^2 r^2 d\Omega]$$

Scale factor

Physical to Comoving Time: $dt = a(\tau)d\tau$

Hubble Parameter: $H = \frac{1}{a}\frac{da}{dt} = \frac{\dot{a}}{a}$

Comoving Hubble Radius: $(aH)^{-1}$

How Inflation Works: Conditions

Inflation satisfied 2 conditions:

1. Shrinking comoving Hubble radius –> accelerated expansion <–> slowly varying Hubble parameter.

$$\frac{d}{dt}(aH)^{-1} < 0 \longrightarrow \frac{d^2a}{dt^2} > 0 \Leftrightarrow \epsilon \equiv \frac{\dot{H}}{H^2} < 1$$

2. Expansion is long enough ~ 60 e-folds.

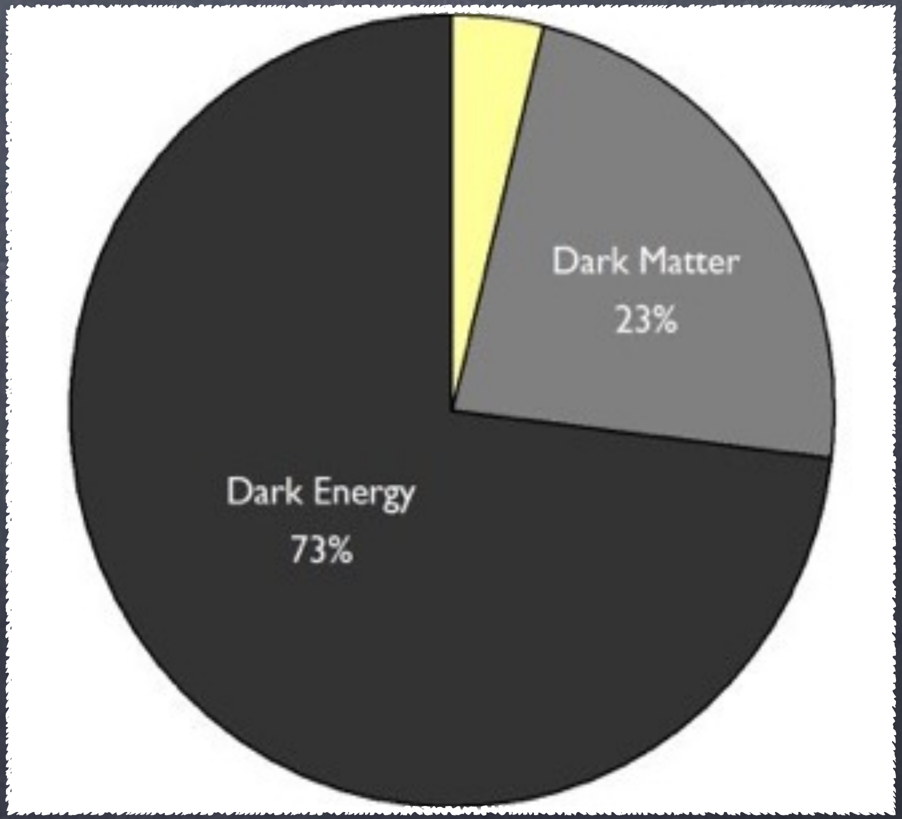
$$\eta \equiv \frac{|\dot{\epsilon}|}{H\epsilon} << 1$$

What we want to know

- Dark Matter
- Dark Energy

(or Modified Gravity?)

- What is the Universe composed of?
- What is the 'fundamental' theory?
- How did the Universe begin?



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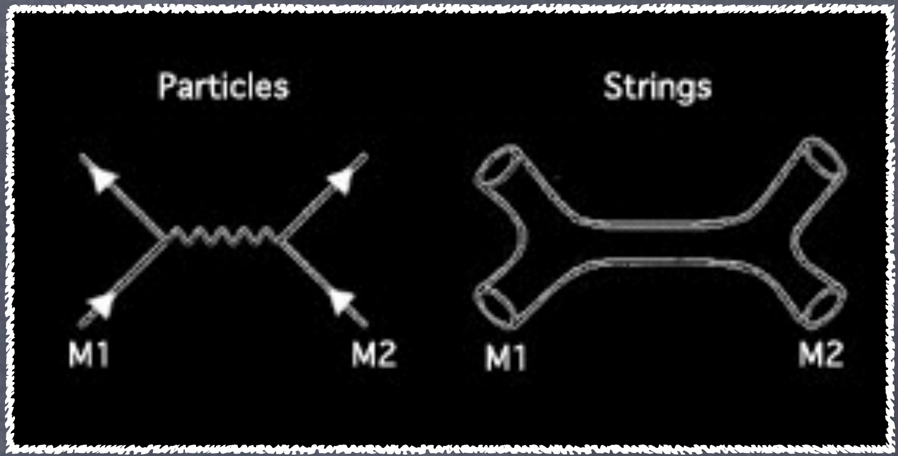
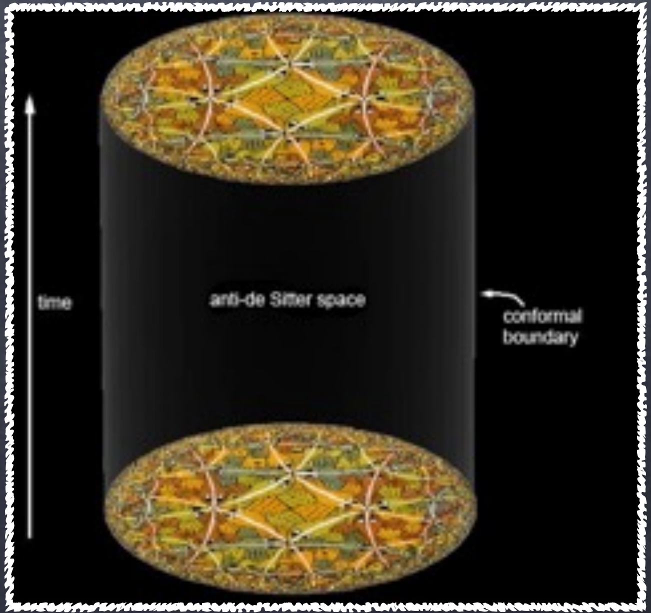


- String Theory ?
- Quantum Gravity ?
- AdS/CFT ?



Schrödinger Picture Bispectrum Setup

Schrödinger Equation:

$$i\frac{\partial}{\partial\tau}\Psi = (H^{(2)} + \mu H^{(3)})\Psi$$

Hamiltonian from quadratic action

$$S_{\text{quad}} = \int d^4x a^3(t)\left[\frac{\epsilon M_{pl}^2}{2c_s^2}(\partial_\mu\zeta\partial^\mu\zeta) + \frac{M_{pl}^2}{2c_s^2}(\partial_\mu\chi\partial^\mu\chi)\right]$$

Schrödinger Picture Bispectrum Setup

Schrödinger Equation:

$$i\frac{\partial}{\partial\tau}\Psi = (H^{(2)} + \mu H^{(3)})\Psi$$

Hamiltonian from cubic action

$$S^{(3)} = \int dx^{(3)}dt\Big[-\frac{2\lambda_c}{\Sigma}\frac{a^3\epsilon}{c_s^2 H}\dot{\zeta}^3 - \tilde{g}\dot{\zeta}(\partial_i\zeta)^2 + \frac{a^3\epsilon}{c_s^2}(2s+\epsilon-\eta)\zeta\dot{\zeta}^2$$

$$+a\epsilon(\epsilon+\eta)\zeta(\partial_i\zeta)^2 - 2\frac{a^3\epsilon^2}{c_s^4}\dot{\zeta}\partial_i\zeta\partial^i\partial^{-2}\dot{\zeta}\Big]$$