The Weak Gravity Conjecture, Black Holes and Cosmology

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Based on work with:



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J. Brown, W. Cottrell, GS, P. Soler, JHEP **1510**, 023 (2015), JHEP **1604**, 017 (2016), JHEP **1610** 025 (2016).
 M. Montero, GS and P. Soler, JHEP **1610** 159 (2016).
 A. Landete, F. Marchesano, GS, Gianluca Zoccarato, JHEP **1706**, 071 (2017).
 W. Cottrell, GS and P. Soler, arXiv:1611.06270 [hep-th].
 Y. Hamada and GS, arXiv:1707.06326 [hep-th].





Detectable Inflationary Gravity Waves?

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WGC in 3 dimensions: M. Montero, GS and P. Soler, JHEP 1610 159 (2016). Quantum entropy of extremal BHs: W. Cottrell, GS and P. Soler, arXiv:1611.06270 [hep-th].



WGC, Multiple Point Principle, and the Standard Model Landscape

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String Theory Landscape



String Theory Landscape



An even vaster Swampland?

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END OF LANDSCAPE SWAMPLAND BEYOND THIS POINT

All inflationary theories

All inflationary theories

UV consistent theories

All inflationary theories

UV consistent theories

All inflationary theories

UV consistent theories

Understanding the space of UV consistent inflationary theories also helps in assessing how inflation fares with data.

Primordial Gravitational Waves



Many experiments including BICEP/KECK, PLANCK, ACT, PolarBeaR, SPT, SPIDER, QUEIT, Clover, EBEX, QUaD, ... can potentially detect primordial B-mode at the sensitivity r~10⁻².

Further experiments, such as CMB-S4, PIXIE, LiteBIRD, DECIGO, Ali, ... may improve further the sensitivity to eventually reach $r \sim 10^{-3}$.

UV Sensitivity of Large Field Inflation



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Axions & Large Field Inflation

Natural Inflation [Freese, Frieman, Olinto]

Pseudo-Nambu-Goldstone bosons are natural inflaton candidates.



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Axions in String Theory

String theory has many **higher-dimensional form-fields**:



Integrating the 2-form over a 2-cycle gives an **axion**:

$$a(x) \equiv \int_{\Sigma_2} A$$

The gauge symmetry becomes a **shift symmetry**.

Axions with super-Planckian decay constants don't seem to exist in controlled limits of string theory. Banks, Dine, Fox, Gorbatov, '03

The Weak Gravity Conjecture

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

• The conjecture:

"Gravity is the Weakest Force"

• For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m}M_P \ge ``1"$$

 $M_P \equiv 1$

The Weak Gravity Conjecture

U(1) and a single family with $q < m_{A}(k_{A}) \oplus M_{G}$, Motl, Nicolis, Vafa '06



2q

$$\frac{q}{m}M_P \ge ``1" \equiv \frac{Q_{Ext}}{M_{Ext}}$$



 $M_P \equiv 1$

• Take a U(1) and a single family with q < m (WGC)



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• Postulate the existence of a state with ("mild form" of WGC)

Electric WGC:
$$\frac{q}{m} \ge "1" \equiv \frac{Q_{Ext}}{M_{Ext}}$$

Magnetic WGC: $\Lambda \le gM_P$
 $[m_{mag} \sim \Lambda/g^2, q_{mag} \sim 1/g]$



 $M_P \equiv 1$

WGC and Axions

• Formulate the WGC in a duality frame where the axions and instantons turn into gauge fields and particles, e.g.



• The WGC takes the form $f \cdot S_{\text{instanton}} \leq \mathcal{O}(1)M_P$ and generalizes to a convex hull condition for multiple axions.

Multiple Axions and Convex Hull

• Generalization of the WGC to multiple U(1)'s is a convex hull condition [Cheung, Remmen], which has been dualized to the WGC for multiple axions [Brown, Cottrell, GS, Soler];[Rudelius]



Strong vs Mild Form

- Consistencies suggested that the WGC takes stronger forms:
 - Madison Strong Form (1503.04783) [Brown, Cottrell, GS, Soler]:

"The lightest (possibly multi-particle) state in <u>any</u> given direction in charge space satisfies $|z_{lightest}| \ge 1$ "

• Harvard Strong Form (1509.06374) [Heidenreich, Reece, Rudelius]:

Lattice WGC: For every point \vec{Q} on the charge lattice, there is particle of charge \vec{Q} with charge-to-mass ratio at least as large as that of a large, semi-classical, non-rotating extremal black hole with charge $\vec{Q}_{BH} \propto \vec{Q}$.

- The Lattice WGC was shown to be false. There are counter examples where the conjecture holds only by a proper sublattice (sublattice WGC) [Montero, GS, Soler];[Heidenreich, Reece, Rudelius]
- The precise form of the WGC is still being formulated.



WGC in 3 dimensions: M. Montero, GS and P. Soler, JHEP 1610 159 (2016). Quantum entropy of extremal BHs: W. Cottrell, GS and P. Soler, arXiv:1611.06270 [hep-th]. Arguments for the Weak Gravity Conjecture

- Heuristic argument suggests a state w/
- $\frac{q}{m} \ge ``1" \equiv \frac{Q_{Ext}}{M_{Ext}}$
- One often invokes the remnants argument [Susskind] for the WGC but the situations are different (finite vs infinite mass range).



- Perfectly OK for some extremal BHs to be stable [e.g., Strominger, Vafa] as q ∈ central charge of SUSY algebra.
 - No q>m states possible (.: BPS bound).
 - BPS BHs *are* the WGC states (boring option)
 - More subtle for theories with some q ∉ central charge
- The WGC is a conjecture on the *finiteness of the # of stable* states that are <u>not</u> protected by a symmetry principle.

Evidences for the Weak Gravity Conjecture

Several lines of argument have been taken (so far):

- Holography [Nakayama, Nomura, '15];[Harlow, '15];[Benjamin, Dyer, Fitzpatrick, Kachru, '16]; [Montero, GS, Soler, '16]
- IR Consistencies (unitarity & causality) [Cheung, Remmen, '14];[Andriolo,Junghans, Noumi, GS,'17, to appear].
- Cosmic Censorship [Horowitz, Santos, Way, '16];[Crisford, Horowitz, Santos, '17]
- Axion Black Holes [Hebecker, Soler, '17]; [Montero, Uranga Valenzuela, '17]

Evidences for *stronger* versions of the WGC:

- Consistencies with T-duality [Brown, Cottrell, GS, Soler, '15] and dimensional reduction [Heidenreich, Reece, Rudelius '15].
- Modular invariance + charge quantization suggest a sub-lattice WGC [Montero, GS, Soler, '16] (see also [Heidenreich, Reece, Rudelius '16])

Further evidence based on entropy considerations [Cottrell, GS, Soler, '16]. (I'll comment on some recent erroneous claims in [Fisher, Mogni, '17])

Back to the Basic



What's wrong if the WGC is violated in the 4D Einstein-Maxwell theory?

Microscopic Intuition

• In the semi-classical, Newton limit, the microcanonical entropy for a system of N stable particles with $\Delta m^2 = m^2 - q^2 > 0$ is **unbounded**.



'gravo-thermal catastrophe' [Antonov, '62]; [Padmanabhan, '89]

- A divergence in entropy, if real, would undermine the consistency of the theory, but an upgrade of this analysis to include GR + quantum is hindered by the presence of horizons.
- We cannot exclude a UV completion saving us from this catastrophe but the WGC suggests that no such consistent UV framework exists.

Horizon Entropy

- No reason a priori for it to agree with the microcanonical entropy but the equivalence was shown in some cases [Lewkowycz, Maldacena '13].
- We computed the 1-loop corrected BH geometry and entropy using the **quantum entropy function** formalism [Sen, '05-'12].
- The Wald formula [Wald, '93] computes the horizon entropy for an arbitrary *local* Lagrangian, e.g.,

$$S = 2\pi \int_{\rho^2} \frac{\delta I}{\delta R_{\mu\nu\alpha\beta}} \epsilon^{\mu\alpha} \epsilon^{\nu\beta} \sqrt{h} d^2 \Omega \qquad \text{for} \qquad I = \frac{1}{16\pi} \int (R + R^2 + R^4 F^4 + \cdots)$$

- Sen's entropy function formalism instructs us to apply Wald's formula to the quantum corrected 1PI effective action, which is not necessarily local.
- For a near horizon geometry that approaches AdS₂ x X, we can rewrite Wald's formula in terms of a Legendre transform of the near-horizon Lagrangian density. This method applies even to non-local Lagrangians.

Summary of Findings

 While corrections from <u>neutral particles</u> have been obtained previously, integrating out <u>charged particles</u> introduce some new features:

Loops of massive charged particles can induce 'unexpected' contributions to the horizon entropy of extremal black holes.

- Our previous paper (1611.06270) established this result for N=0,1 BHs.
- In a forthcoming paper, we demonstrate that this feature persists even with the full structure of N=2 SUGRA.
- This finding is puzzling because:
 - Intuitively, we don't expect loops of massive particles could alter the area law of a *macroscopic BH*.
 - How do we reconcile this finding w/ the results on the entropy of N≥2 BHs in string theory?

Summary of Findings

- A resolution to this puzzle: we should *not* integrate out these extremal particles to begin with. For RR U(1)'s in string theory, they are the *D-brane states* that have *already been integrated out*.
- This is how the conifold singularity is resolved [Strominger, '95]. At special points in the moduli space (e.g., conifold), these D-brane states are massless, hence the effective action exhibits singularity.
- This gives evidence for the magnetic WGC which identifies the UV cutoff to the mass scale of the extremal particles:

$$\Lambda \lesssim q M_P$$

• A corollary is that in any UV complete theory of quantum gravity, an extremal particle cannot be fundamental, rather it must be a soliton.

Sketch of the Argument

• The near-horizon geometry is AdS₂ x S₂

$$ds^{2} \equiv g_{\mu\nu}dx^{\mu}dx^{\nu} = a^{2}\left(-r^{2}dt^{2} + \frac{dr^{2}}{r^{2}}\right) + b^{2}\left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right)$$
$$F = Edt \wedge dr$$

• The **heat kernel** is defined by:

$$(\partial_s - D)K(x, y; s) = 0 \qquad K(x, y; 0) = \delta^4(x - y)$$

where D is a generalized laplacian of the field to be integrated out.

- The 1-loop correction: $\mathcal{L}^{(1)} = \frac{1}{2} \int_{\epsilon^2}^{\infty} \frac{ds}{s} K(s)$ where $\epsilon = \text{UV cutoff}$.
- Quantum corrected entropy can be obtained by extremizing:

$$\mathcal{E}(Q; E, a, b) = 2\pi \left[QE - 4\pi a^2 b^2 \mathcal{L}_{GR+EM}(E, a, b) \right]$$

Heat Kernel

- Since the near horizon is AdS₂ x S₂ the heat kernel factorizes. We can apply results of [Banerjee, Gupta, Sen];[Comtet, Houston];[Pioline, Troost]:
- Charged Scalars: $K_s(s) = \frac{e^{-s\Delta m^2}}{4\pi^2 a^2 b^2} \sum_{l=0}^{\infty} (2l+1) \int_0^{\infty} d\lambda \,\lambda \,\rho_s(\lambda) e^{-s\left[\left(\lambda^2 + \frac{1}{4}\right)/a^2 + l(l+1)/b^2\right]}$ $\rho_s(\lambda) = \frac{\sinh(2\pi\lambda)}{\cosh(2\pi\lambda) + \cosh(2\pi qE)}$
- Chiral Fermions: $K_f(s) = \frac{e^{-s\Delta m^2}}{4\pi^2 a^2 b^2} \sum_{l=0}^{\infty} (2l+2) \int_0^{\infty} d\lambda \,\lambda \,\rho_f(\lambda) \, e^{-s\left[\lambda^2/a^2 + (l+1)^2/b^2\right]}$ $\rho_f(\lambda) = \frac{\sinh(2\pi\lambda)}{\cosh(2\pi qE) - \cosh(2\pi\lambda)}$

where $\Delta m^2 = m^2 - \frac{q^2 E^2}{a^2} \rightarrow m^2 - 2q^2 M_P^2$ (classical value)

 The heat kernel is IR divergent for Δm²<0, signaling an instability to Schwinger pair production of superextremal particles.

(Sub)Extremal Particles

• One has to be careful in expanding the heat kernel:



• A qE expansion is *only valid* for **intermediate BHs** where:

$$\Lambda_{WGC} = qM_P << 1/a << M_P$$

even both intermediate and large BHs have a >> $1/M_P$, so a semiclassical treatment of gravity should remain valid.

Relation to the WGC

- See [Cottrell, GS, Soler, '16] for results of various cases (intermediate/ large BHs, loops of (sub)extremal bosons/fermions, SUSY or not).
- As an example, we found that for an intermediate BH, including loop corrections from an extremal scalar:

$$\mathcal{S}_s \approx \mathcal{E}(Q; E_0, a_0, b_0) = \frac{Q^2}{4} - \left(\frac{1}{90} + \frac{q^2 Q^2}{192\pi^2} + \frac{q^4 Q^4}{1024\pi^4}\right) \ln(q^2 Q^2) + \mathcal{O}(Q^0)$$

- For large BHs, loops of (sub)extremal particles do not induce corrections to the entropy, other than renormalizing the couplings.
- {Fisher, Mogni, '17] recently repeated our computations for an extremal scalar and confirmed our formulae in the valid region.
- However they made an erroneous claim of proving the WGC: they found the second law of thermodynamics is violated for large Q, but this is the regime where the qE expansion breaks down.



WGC, Multiple Point Principle, and the Standard Model Landscape

Y. Hamada and GS, arXiv:1707.06326 [hep-th].

WGC for Branes

• We have seen the evidences for and applications of the WGC for particles (and instantons). Analogously for branes, the WGC is:

"
$$T_p \le Q_p$$
"

where = applies only to BPS, otherwise <

- This led [Ooguri,Vafa, '16] to conjecture that non-SUSY AdS vacua supported by fluxes are unstable (AdS fragmentation).
- This conjecture is best supported by the lack of counter examples in string theory, but is supposed to hold more generally.
- A stronger form of their conjecture:

"all non-SUSY AdS (in theories whose low energy description is Einstein gravity coupled to a finite # of fields) are unstable"

How do we test this conjecture?

Standard Model Landscape

 After the Higgs discovery, we know that there is an additional Higgs vacuum at high scale, other than the EW vacuum:



- This high scale vacuum can be AdS₄, M₄, or dS₄ depending on the top quark mass and the higher-dimensional operators.
- Applying this conjecture to the SM landscape, we can constrain the Higgs potential and BSM physics. [Hamada, GS].

Standard Model Landscape

- The SM gives rise to a rich landscape of vacua in 2d & 3d upon compactification, dependent on the type (Majorana or Dirac) and masses of the neutrinos [Arkani-Hamed, Dubovsky, Nicolis, Villadoro].
- The SM with minimal Majorana neutrino masses seems to give rise to a non-SUSY AdS vacuum [Arkani-Hamed et al]. This led [Ooguri, Vafa, '16] to conjecture that this model is in the swampland.
- We carried out a systematic study of the SM landscape in 2d and 3d, including more general BCs and Wilson lines [Hamada, GS].
- We found a runaway behavior at small compactification radii (≤ GeV⁻¹). These candidate non-SUSY AdS neutrino vacua are subject to quantum tunneling instabilities, a possibility overlooked in [Arkani-Hamed, Dubovsky, Nicolis, Villadoro]; [Ibanez, Martin-Lozano, Valenzuela]
- Our result is consistent with the OV conjecture.

Multiple Point Criticality Principle

- There may nonetheless be an interesting correlation between the neutrino mass and the 4d cosmological constant scale,
- The Multiple Point Criticality principle [Froggatt, Nielsen, '96]; [Bennett, Rigg: which demands the coexistence of degenerate phases had some successes in predicting the Higgs mass.



Standard model criticality prediction top mass 173 ± 5 GeV and Higgs mass 135 ± 9 GeV

C.D. Froggatt ^a, H.B. Nielsen ^b

Show more

- Applying the multiple point criticality principle to 2/3d and 4d vacua, we predicted that the νs are Dirac w/ mass of lightest $\nu \simeq \mathcal{O}$ (1-10) meV.
- Our predictions can be tested by future CMB, large-scale structure, and 21cm line observations.





String Theory

String Theory Constructions



Axion Monodromy



Monodromy by brane coupling
[Silverstein, Westphal, '08];
[McAllister, Silverstein, Westphal, 08]
F-term axion monodromy
(embeddable in SUGRA of string theory)

[Marchesano, GS, Uranga '14]

See also [Blumenhagen, Plauschinn '14]; [Hebecker, Kraus, Witowski, '14]

- Axion is mapped to a *massive* gauge field.
- Gauge symmetry:

$$\phi \to \phi + 2\pi f$$
$$F \to F - n$$



Effective 4d Description

• Coupling the axion to a 4-form field strength $F_4 = dC_3$

$$L = -\frac{1}{2}(\partial_{\mu}\phi)^{2} - \frac{1}{2}|F_{4}|^{2} + g\phi F_{4},$$

Upon integrating out C₃

$$*F_4 = f_0 + g\phi, \qquad f_0 = ne \quad \text{where } n \in \mathbb{Z}$$

one finds a quadratic potential

$$V = \frac{1}{2} \left(f_0 + g\phi \right)^2$$

with a shift symmetry:

$$\phi o \phi + rac{\mathsf{e}}{\mu} \qquad \mathsf{n} o \mathsf{n} - 1 \qquad \mathsf{n} \in \mathbb{Z}$$

Planck-suppressed Corrections

• Gauge symmetry \Rightarrow UV corrections only depend on F₄



Axion Monodromy Inflation

$$\mathcal{L} = \frac{1}{2} \left(\partial \phi \right)^2 - \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right) - \mu^{4-p} \phi^p$$



Axion Monodromy Inflation



Axion Monodromy Inflation $\mathcal{L} = \frac{1}{2} \left(\partial \phi \right)^2 - \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right) - \mu^{4-p} \phi^p$



Current bound combining Planck+BICEP2/KECK+BAO: r < 0.07

Inflationary Observables



• Taking into account constraints from moduli stabilization:



Flux flattening generates a *family* of $m^2\phi^2$ inflation with:

$$n_s \simeq 0.96 - 0.97$$

 $r \simeq 0.04 - 0.14$

[Landete, Marchesano, GS, Zoccarato, '17]

Conclusions

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- Progress in experimental cosmology and string theoretical considerations may help narrow down the range of r.
- We have formulated the WGC for (a large class of) axions which can be dualized to U(1) gauge fields.
- Axion Monodromy is an interesting exception to the WGC, though there may be other considerations (e.g., backreaction) that limit r.
- Flux flattening can lower r to within current experimental bound and yet detectable in the foreseeable future, e.g., the flux flattened m²φ² family has r ≈ 0.04-0.14.
- We test the WGC from **entropic considerations**.
- Loops of charged particles can lead to unexpected corrections to the classical geometry and entropy of a large extremal BH unless:
 - 3 super-extremal particle for the BH to decay (electric WGC)
 - or, \exists a UV cutoff set by extremal states (magnetic WGC)
- WGC & Multiple Point Principle offer interesting predictions about Higgs and neutrino physics.

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