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under  
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**HETEROGENEITY, NETWORKS AND INTERNATIONAL TRADE**  
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**MONOPOLISTIC COMPETITION**

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**Abstract** This paper examines integrated effects of firm heterogeneity and communication network services on international trade. Patterns and effects of trade are to be analyzed in a general equilibrium model where firms with different productivity levels share among them the cost of network services and compete in a monopolistically competitive market for a differentiated good. The paper reveals that the more efficient country in the differentiated good production is not always the net-exporter of the good. The less efficient country also has the chance to expand the industry and then to become the net-exporter in this intra-industry trade due to the combination of the efficiency effect induced by firm heterogeneity and the cost-sharing effect by the existence of the network service industry.

**Key words** international trade, communication networks, monopolistic competition, firm heterogeneity, efficiency gaps

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# **HETEROGENEITY, NETWORKS AND INTERNATIONAL TRADE UNDER MONOPOLISTIC COMPETITION**

## **I. INTRODUCTION:**

Firm homogeneity is the basic assumption in new trade theories in the early stage of their development. In reality, however, firms have actually shown difference in many aspects<sup>1</sup>. Unfortunately, the effect of these differences on international trade has not been dealt with much detail until recently. It was partly due to the difficulty in modeling firm heterogeneity in any satisfactory manner. Only in the early 2000s, some trade theorists have successfully integrated heterogeneous firms into their models and obtained some insights into the effect of differences across firms on trade between countries. Thus, Melitz (2003), Montagna (2001) and Bernard, Redding and Schott (2004) have argued that firms are by nature different in productivity. Firm heterogeneity in these models is assumed to be generated through an endogenous selection process of firms in which firms are assigned their productivity levels randomly. Yeaple (2005) argues that firms are different in technologies they are using or in types of inputs they are employing. Firms are identical when they were born but are free to produce with technologies that differ in their characteristics, and are free to hire workers who are varied in their skills. In Manasse and Turrini (2001), firm heterogeneity is generated by the difference in entrepreneurial ability of firms. It is revealed in these papers that trade induces only the most productive firms to enter the export market and simultaneously forces the least productive firms to exit. Exporting firms are larger, employ more advanced technologies, pay higher wages, have higher entrepreneurial ability and are more productive than firms that do not export. There exists intra-industry trade between countries of different efficiency levels in which the more efficient country is always the net-exporter of the differentiated good, due to the efficiency effect induced by firm heterogeneity, besides the love-of-variety effect usually observed in trade models with goods differentiation and firm homogeneity.

On the other hand, the development of communication networks such as Internet, satellite communication systems, mobile phone networks and so on has been intuitively considered as a trade stimulator, especially in the sense that it helps eliminate unnecessary barriers among countries. However, there has not been much analysis of their role in international trade until recently. By an empirical test Freund and Weinhold (2004) found that the Internet stimulated trade between countries. Also, Fink, Mattoo and Neagu (2002), in an estimation of the impact of communication costs on bilateral trade, observe that international variations in communication costs had a significant influence on trade, especially trade in differentiated products. In international trade theory, impacts of communication networks on

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<sup>1</sup> See discussion in Montagna (2001)

trade between countries have been tackled first by Harris (1995), followed by Toru Kikuchi and his joint efforts with others in the early 2000s<sup>2</sup>. These papers have shown that cost-sharing and congestion effects induced from the characteristics of the communication networks industry are significant in the determination of trade patterns and welfare.

Yet, there has not been any study of the role of communication networks in trade between economies with heterogeneous firms competing in a monopolistically competitive market of a differentiated good. It is worth examining this setting, not only because it is less restrictive in reality but also because that the interaction between love-of-variety, efficiency and cost-sharing effects may give us a hint as to whether and in which case the less efficient country has comparative advantage in the production of the communication-service intensive good. The present paper aims at this analysis.

The model in this paper is formulated by extending Montagna (2001) and Kikuchi and Ichikawa (2002). Both firm heterogeneity and communication networks are integrated into the one-factor, two-good Krugman model to examine the patterns and effects of trade between two countries that are different in terms of efficiency levels. By one-factor, two-good Krugman model, we mean that each country has only one homogenous production factor – labor – and two final good sectors: homogenous good sector using constant-returns-to-scale technology and differentiated goods sector using increasing-returns-to-scale technology. The homogenous good market is perfectly competitive, while the market of the differentiated goods is monopolistically competitive. Preferences are of Dixit-Stiglitz type for both countries. Firms in the differentiated goods industry have different productivity levels and use labor and communication network services as their inputs, while firms in the homogenous good industry use only labor. Communication services are provided by an average-cost-pricing monopolist.

The rest of the paper is organized as follows. The next section, Section II, sets out the model, and Section III provides analyses. The last section concludes the analysis.

## II. THE MODEL

### 1. Autarky

#### 1.1 The world economy:

There are two countries called Home (denoted by  $h$ ) and Foreign (by  $f$ )<sup>3</sup>. In each country, there are two final goods production sectors: (i) an imperfectly competitive sector producing horizontally differentiated goods, whose production process uses labor and communication network services as inputs (hereafter referred to as the network good sector); and (ii) a perfectly competitive industry producing a homogeneous good, using labor only, but not communication network services in its production process (hereafter referred to as the non-network good sector). There is only one type of primary production factor, labor,

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<sup>2</sup> See Kikuchi (2002), Kikuchi and Ichikawa (2002), Kikuchi (2003), Kikuchi and Kobayashi (2003)

<sup>3</sup> All the variables and parameters inherent to them will also be denoted by the subscripts  $h$  and  $f$ , respectively.

that is homogeneous and assumed perfectly mobile between industries within each country but internationally immobile. The communication network services are supplied by one communication network service provider, whose fixed cost is large and marginal cost varies accordingly to the number of firms using the network; the network in each country is country-specific, that is, each country uses its own network.

## **1.2 Consumption:**

The two countries have identical structures of preferences. Each country has a Cobb-Douglas utility over the homogeneous non-network good  $A_j$  and the composite differentiated network good  $D_j$  ( $j = h, f$ ),

the composition of which is expressed by a CES sub-utility over the demanded quantities of  $N_j$  varieties of the differentiated network goods. This preference is known as the Dixit-Stiglitz type<sup>4</sup>. The first stage of utility maximization is to solve the following problem, taking the homogeneous good as the numeraire by setting its price to unity:

$$(1) \quad \max_{D_j, A_j} U_j = A_j^{1-\mu} D_j^\mu, \quad 0 < \mu < 1$$

$$s.t. \quad M_j = A_j + P_j D_j$$

where  $M_j$  is the total income of country  $j$ , and  $P_j$  is the price index of the differentiated network good in country  $j$  ( $j = h, f$ ). The total income  $M_j$  is the sum of country  $j$ 's factor income and total profit of firms,  $\Pi_j$ :

$$(2) \quad M_j = w_j \bar{L}_j + \Pi_j$$

where  $w_j$  is the wage rate,  $\bar{L}_j$  is the total labor endowment in country  $j$  ( $j = h, f$ ).

Solving (1) to obtain demand for the two goods, we have the following demand functions:

$$(3) \quad A_j = (1 - \mu) M_j$$

and

$$(4) \quad D_j = \mu \frac{M_j}{P_j}.$$

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<sup>4</sup> Hence, all the assumptions and the demand derivation procedures are the same as in Dixit-Stiglitz (1977).

Assuming a continuum of varieties, we define the aggregate demand for the differentiated network good as follows:

$$(5) \quad D_j = \left( \int_1^{N_j+1} D_{ji}^{(\sigma-1)/\sigma} di \right)^{\frac{\sigma}{\sigma-1}}$$

where  $D_{ji}$  is the consumed amount of the variety produced by firm  $i \in [1, N_j + 1]$  in country  $j$ , and  $\sigma$  is the constant elasticity of substitution between varieties ( $\sigma > 1$ ).

The second stage of utility maximization is then to maximize (5) subject to  $P_j D_j = \int_1^{N_j+1} P_{ji} D_{ji} di$ .

The resulting demand for each variety is obtained as follows:

$$(6) \quad D_{ji} = D_j \left( \frac{P_{ji}}{P_j} \right)^{-\sigma}$$

where  $P_{ij}$  is the price of  $i^{\text{th}}$  variety produced in country  $j$  ( $j = h, f$ ), with the price index of the differentiated network good in country  $j$  being measured by

$$(7) \quad P_j = \left( \int_1^{N_j+1} P_{ji}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

### **1.3 Production in the homogenous non-network good industry**

Assume that this industry is characterized by a production technology exhibiting constant returns to scale with unit labor requirement. Then, the market clearing condition is  $A_j^S = A_j$ , where  $A_j^S$  is the supply amount of the non-network good. Denote  $L_{Aj}$  being the amount of labor used in the non-network good sector in country  $j$  ( $j = h, f$ ), we obtain

$$(8a) \quad A_j^S = L_{Aj}.$$

The market for this good is assumed to be perfectly competitive. Hence, its price is equal to the average cost. The zero-profit condition also implies a unit wage rate in country  $j$  ( $j = h, f$ ):

$$(8b) \quad w_j = 1.$$

#### **1.4 Production in the differentiated network good industry**

A firm  $i$  in country  $j$  faces a total cost with the function in the following form:

$$(9) \quad C_{ji} = \alpha + \beta_{ji} D_{ji}^s + \gamma$$

where  $D_{ji}^s$  is the quantity of variety  $i \in [1, N_j + 1]$  supplied by country  $j$  ( $j = h, f$ ). The components of the total cost can be de-structured as follows:

(i)  $\alpha$  is the fixed cost, assumed to be identical for all firms and across countries;

(ii)  $\beta_{ji}$  is the marginal cost of the variety  $i \in [1, N_j + 1]$  in country  $j$  ( $j = h, f$ ), assumed to be firm-specific. Following Montagna (2001), we assume that within each country the first firm is the most efficient one with respect to which all other firms can be ranked. We rank the firms according to efficiency level, by defining a continuous variable  $\rho_j(i)$  such that

$$(10a) \quad \beta_{ji} = \rho_j(i) \text{ with } \rho_j(1) = \phi_j \text{ and } \rho_j'(i) \geq 0 \text{ for all } i \in [1, N_j + 1]$$

where  $\phi_j$  is the marginal cost of the most efficient firm in country  $j$  ( $j = h, f$ ). For the sake of simplicity, we adopt the following specific functional form for firms' marginal cost:

$$(10b) \quad \beta_{ji} = \rho_j(i) = \phi_j i^\delta$$

where  $\delta$  is the degree of technical heterogeneity among firms, assumed to be non-negative and the same in both countries. Firms are homogenous when  $\delta = 0$ . With all the settings, we can see that the difference in efficiency between the two countries is characterized only by  $\phi_h - \phi_f$ . Without loss of

generality, Home is assumed to be more efficient than Foreign, ( $\phi_h - \phi_f < 0$ ). And, finally,

(iii)  $\gamma$  is the cost of network services. We derive this cost in the way as follows:

The total labor cost in providing the network services incurred by the communication network services provider is assumed to have the form:

$$(11a) \quad K(N_j) = F + gN_j^\eta$$

where  $F$  is the fixed cost of network services,  $g$  the marginal cost (with respect to the users of the network), and  $\eta$  the coefficient for congestion ( $\eta \geq 1$ ). Assume that the network services provider

employs average cost pricing<sup>5</sup>. Then, the cost of network services each firm in the differentiated network good industry has to incur is the average cost in the network services sector:

$$(11b) \quad \gamma(N_j) = \frac{K(N_j)}{N_j} = \frac{F}{N_j} + gN_j^{\eta-1}.$$

We can see that although it is a fixed cost to firms, it will vary according to the number of firms who are using the network. This is known as *the quasi-fixed cost*.

Facing this cost structure, firm  $i \in [1, N_j + 1]$  in country  $j$  ( $j = h, f$ ) will choose its optimal price ( $P_{ji}$ ) and the quantity supplied ( $D_{ji}^s$ ) to maximize its profit. As it is well established in the theoretical analysis of monopolistic competition, due to the existence of increasing return to scale (brought forth by fixed-cost effect), each firm will only produce one variety. Specifically, firm  $i$  in country  $j$  will choose the optimal price  $P_{ji}$  to maximize its profit  $\Pi_{ji} = (P_{ji} - \beta_{ji})D_{ji}^s - [\alpha + \gamma(N_j)]$  under the market clearing condition ( $D_{ji}^s = D_{ji}$ ). We assume further that firms do not behave strategically in the sense that each firm considers other firms' prices as given when setting its price (i.e.  $\partial P_{ji} / \partial P_{jk} = 0, i, k = 1, \dots, N_j + 1; i \neq k$ ), and that the influence of an individual price change on the aggregate price index is ignorable (i.e.  $\partial P_{ji} / \partial P_j = 0$ ). Firms also consider the national income level being fixed. Thus, by solving this profit maximization problem, we obtain the optimal price as follows

$$(12) \quad P_{ji} = \omega \beta_{ji}$$

where  $\omega \equiv \frac{\sigma}{\sigma - 1}$ , known as the constant mark-up over the marginal cost. Hence, the profit of firm

$i \in [1, N_j + 1]$  in country  $j$  ( $j = h, f$ ) can be calculated as follows:

$$(13) \quad \Pi_{ji} = \frac{1}{\sigma} \mu M_j \left( \frac{P_{ji}}{P_j} \right)^{1-\sigma} - [\alpha + \gamma(N_j)].$$

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<sup>5</sup> Here, we follow Harris (1995), Kikuchi (2002) or Kikuchi and Ichikawa (2002). This assumption is reasonable in terms of the fact that network service sectors in many countries are heavily subject to government regulation. Since network services are universally demanded goods, when there is a single provider, governments usually urge the sector to employ the average cost pricing to guarantee access to everyone as well as cost recovery to the sector. Besides, this is also reasonable in the context of contestable markets.



### **1.5 The exit-entry process in the market of the differentiated network goods**

In the monopolistically competitive market of the differentiated network goods in each country, a firm will stay in the market while its profit is non-negative and will quit otherwise. A new firm will enter the market when it finds that it is profitable to produce a variety with the marginal cost in producing the variety that it is going to incur. In equilibrium there should be no new entry or exit; hence the marginal firm will break even. Namely, the profit with marginal cost  $\beta_{j(N_j+1)}$  is zero, i.e.,

$\Pi_{j(N_j+1)}(\beta_{j(N_j+1)}) = 0$ , where  $\beta_{j(N_j+1)} = \phi_j(N_j + 1)^\delta$  is the marginal cost of the marginal firm (hereafter referred to as the *efficiency cut-off point* on the marginal cost spectrum in the differentiated network good industry), showing the highest marginal cost that prevails among the existing firms.

Furthermore, given the way of ranking firms, firms whose marginal costs are smaller than  $\beta_{j(N_j+1)}$  will make positive profits ( $\Pi_{ji}(\beta_{ji} | \beta_{ji} < \beta_{j(N_j+1)}) > 0$ ). With (12) and (13), the zero profit condition for the marginal firm in country  $j$  becomes

$$(14) \quad \mu M_j \left( \frac{P_{j(N_j+1)}}{P_j} \right)^{1-\sigma} = \sigma[\alpha + \gamma(N_j)].$$

### **1.6 The labor market**

Given the average cost pricing, there is no profit in the network service industry. Furthermore, the network services are used in the industry of the differentiated network goods as inputs. Therefore, the total cost of the differentiated network goods is the sum of the “pure” labor cost of its own and the total labor cost of the network service industry. Let  $L_{Dj}$  be the total labor required in the differentiated network good industry and network service sector in country  $j$  ( $j = h, f$ ). Since in this paper the cost is

measured in terms of labor, it must hold that  $L_{Dj} w_j = \int_1^{N_j+1} C_{ji} di$ . Deriving this integral by using

(4), (6), (7) and (8b), we yield:

$$(15) \quad L_{Dj} = \frac{\mu}{\omega} M_j + [\alpha + \gamma(N_j)] N_j.$$

Further, the national labor market equilibrium condition requires:

$$(16) \quad \bar{L}_j = L_{Aj} + L_{Dj}.$$

### 1.7 The autarkic equilibrium

Equipped with all the above preparations, we try to derive the equation governing the equilibrium number of firms in the network goods industry in each country under autarky. Replacing the marginal cost of the marginal firm into (12), we have the price of this firm's variety is

$$(17) \quad P_{j(N_j+1)} = \omega \phi_j (N_j + 1)^\delta .$$

Using (10b), (12) and then (7), we can now express  $P_j$  in terms of the number of varieties as follows:

$$(18) \quad P_j = \omega \phi_j \left[ \frac{(N_j + 1)^\theta - 1}{\theta} \right]^{\frac{1}{1-\sigma}}$$

where  $\theta \equiv \delta(1 - \sigma) + 1$ . One more thing we have to calculate is the economy-wide profit. There will be no profit in the homogeneous good sector thanks to the assumption of perfect competition in the sector. Furthermore, no profit in network services sector either. Therefore, the total profit of the economy is the aggregate profits in the network good sector, which is equal to the difference between the

revenue and the total labor cost of this sector:  $\Pi_j = \int_1^{N_j+1} \Pi_{ji} di = P_j D_j - L_{Dj}$ . Using (4) and (15),

we can get

$$(19) \quad \Pi_j = \frac{\mu}{\sigma} M_j - [\alpha + \gamma(N_j)] N_j .$$

In autarky, the model consists of the following unknown variables:  $A_j, D_j, D_{ji}, P_j, P_{ji},$

$M_j, \Pi_{ji}, \Pi_j, L_{Aj}, L_{Dj},$  and  $N_j$ , where  $i \in [1, N_j + 1]$  and  $j = h, f$ . Taking Walras'

Law into consideration, the general equilibrium is characterized by the following eleven equations: (2)-(4),(6),(12)-(16),(18) and (19). By solving this equations system for all the variables, we can capture the characteristics of the autarkic equilibrium. However, the system is so intricate that it is difficult to find specific analytical solutions of the system. Consequently, numerical solutions and simulations are partly employed in the analysis of this paper, as done in Kikuchi and Ichikawa (2002) and Montagna (2001).

Before we fall back on numerical solutions, we now come to analyze some of the main characteristics of the autarkic equilibrium. Solving for  $L_{Aj}$  by putting  $A_j$  in (3) into (8a), and then substituting the result, together with  $L_{Dj}$  specified in (15), into (16), we yield the following expression

after changing terms:

$$(20) \quad M_j = \sigma(\sigma - \mu)^{-1} \{ \bar{L}_j - [\alpha + \gamma(N_j)]N_j \}.$$

Replace  $P_{j(N_j+1)}$  in (17) and  $P_j$  in (18) with (14), we obtain

$$(21a) \quad \mu M_j (N_j + 1)^{\theta-1} \left( \frac{(N_j + 1)^\theta - 1}{\theta} \right)^{-1} = \sigma[\alpha + \gamma(N_j)].$$

We re-write (21a) after substituting  $M_j$ 's from (20) into them to obtain

$$(21b) \quad \frac{\bar{L}_j - [\alpha + \gamma(N_j)]N_j}{\frac{1}{\mu} - \frac{1}{\sigma}} \left[ \frac{(N_j + 1)^{\theta-1}}{\frac{(N_j + 1)^\theta - 1}{\theta}} \right] = \sigma[\alpha + \gamma(N_j)], \quad (j = h, f).$$

From these two equations, we can solve for  $N_h$  and  $N_f$  under autarkic equilibria. Notice that equations

(21b) do not include parameters  $\phi_j$  ( $j = h, f$ ). It implies that  $N_h = N_f$  if  $\bar{L}_h = \bar{L}_f$ . Then, given

the efficiency gap,  $\phi_h < \phi_f$ , we have  $\beta_{h(N_h+1)} < \beta_{f(N_f+1)}$  due to  $\beta_{ji} = \phi_j i^\delta$ . Therefore, we have the

following proposition.

**Proposition 1.** [Autarkic Equilibrium] *The number of firms in autarkic equilibrium is invariant to  $\phi_j$  and the numbers of firms of the two countries are the same if they have the same labor endowments. Also, the more efficient Home country is characterized by a higher minimum efficiency and higher efficiency throughout all the operating firms than the Foreign country at autarkic equilibrium.*

## 2. Free trade

Now, we move on to free trade. Assume that there are no transport costs and other trade barriers and consumers do not discriminate amongst goods produced in different countries. All the varieties produced in a country will now also be available to consumers in the other country. Hence, the number of varieties each consumer can enjoy in free trade is the total number of varieties of the differentiated goods produced in the two countries<sup>6</sup>:

$$(22) \quad N_t = N_{th} + N_{fh}.$$

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<sup>6</sup> Variables with subscript  $t$  implies that they are examined in free trade.

where  $N_{tj}$  is the total number of varieties, also the number of firms, in country  $j$  ( $j = h, f$ ) under free trade. The common free trade price index in the differentiated network goods market is:

$$(23a) \quad P_t = \left( \int_1^{N_{th}+1} P_{hi}^{1-\sigma} di + \int_1^{N_{tf}+1} P_{fi}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

If we denote by  $P_{tj} = \left( \int_1^{N_{tj}+1} P_{ji}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$  a CES index of the prices of goods produced by country  $j$ 's firms under free trade, then (23a) can be rewritten as

$$(23b) \quad P_t = \left( P_{th}^{1-\sigma} + P_{tf}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

## 2.1 Consumption

The functions of demand for the two final goods in the world market (denoted by  $A_t$  and  $D_t$ ) under free trade are derived by solving the maximization problem of consumers in the same way as in autarky.

They are  $A_t = (1 - \mu)M_t$  and  $D_t = \mu \frac{M_t}{P_t}$ , where  $M_t$  is the total income of the world, being

the sum of the income of the two countries under free trade:

$$(24) \quad M_t = M_{th} + M_{tf}.$$

where  $M_{tj}$  ( $j = h, f$ ) is the national income in country  $j$  under free trade.

The demand of country  $j$  for the non-network good,  $A_{tj}$ , is characteristically

$$(25) \quad A_{tj} = (1 - \mu)M_{tj}.$$

The world demand for each variety of the differentiated network goods produced in country  $j$ ,  $D_{tji}$ , is

$$(26) \quad D_{tji} = \mu M_t P_t^{\sigma-1} P_{ji}^{-\sigma},$$

since the world demand for the network goods is expressed by  $D_t = \mu \frac{M_t}{P_t}$  as shown earlier.

The demand by country  $j$  for the differentiated network goods produced by the two countries is

$$(27) \quad D_{ij} = \mu \frac{M_{ij}}{P_t},$$

since the national income of country  $j$  under free trade is  $M_{ij}$  and the free trade price index for the network goods is  $P_t$ .

The world expenditure on the differentiated network goods produced by country  $j$  ( $j = h, f$ ) is also the revenue (denoted by  $E_{Dij}$ ) of country  $j$  out of selling its differentiated network goods to the world.

That is  $E_{Dij} = \int_1^{N_j+1} P_{ti} D_{tji} di$ . Each country's income is the sum of factor income ( $w_{ij} \bar{L}_j$ ) and the total profit of that country under free trade ( $\Pi_{ij}$ ):

$$(28) \quad M_{ij} = w_{ij} \bar{L}_j + \Pi_{ij}.$$

where  $w_{ij}$  is the wage rate held in country  $j$  under free trade.

## **2.2 Production**

Following the same procedure as in autarkic equilibrium, we have the supply of non-network good by country  $j$  ( $A_{ij}^s$ ) as follows, noting that this good is taken as numeraire and the labor wage is equal to 1

( $w_{ij} = 1$ ):

$$(29) \quad A_{ij}^s = L_{Aij}$$

where  $L_{Aij}$  is the amount of labor allocated to the non-network good production in country  $j$  under free trade.

In the differentiated network good industry, the price of variety  $i \in [1, N_j + 1]$  produced in  $j$  ( $j = h, f$ ) under the condition of market clearance has the same form as in (12):

$$(30) \quad P_{ji} = \omega \beta_{ji}.$$

### **2.3 The entry-exit process in the market of the differentiated network good**

Now, with the aid of (26) and (30), the profit of firm  $i \in [1, N_{ij} + 1]$  in  $j$  ( $j = h, f$ ) under free trade

is calculated as  $\Pi_{tji} = (P_{ji} - \beta_{ji})D_{tji} - [\alpha + \gamma(N_{ij})] = \frac{\omega^{-\sigma}}{\sigma - 1} \mu M_t P_t^{\sigma-1} \beta_{ji}^{1-\sigma} - [\alpha + \gamma(N_{ij})]$ .

The zero profit condition for the marginal firm is given by  $\Pi_{tj(N_{ij}+1)}(\beta_{j(N_{ij}+1)}) = 0$ , which yields

$$(31) \quad \mu M_t \left( \frac{P_{j(N_{ij}+1)}}{P_t} \right)^{1-\sigma} = \sigma [\alpha + \gamma(N_{ij})]$$

where  $P_{j(N_{ij}+1)}$  is the price charged by the marginal firm in country  $j$  in free trade and

$$(32) \quad P_{j(N_{ij}+1)} = \omega \phi_j (N_{ij} + 1)^\delta.$$

The price index of all the varieties produced by country  $j$ ,  $j = h, f$  under free trade ( $P_{ij}$ ) is

derived by substituting  $P_{ji}$  in (30) into the definition of  $P_{ij}$  and then performing the integral. The result is:

$$(33) \quad P_{ij} = \omega \phi_j \left[ \frac{(N_{ij} + 1)^\theta - 1}{\theta} \right]^{\frac{1}{1-\sigma}}$$

where  $\theta = \delta(1 - \sigma) + 1$  as defined earlier. After substituting  $D_{tji}$  in (26) into  $E_{Dij} = \int_1^{N_{ij}+1} P_{ji} D_{tji} di$

and then arranging the integral, we yield, keeping in mind that  $(P_{ij})^{1-\sigma} = \int_1^{N_{ij}+1} P_{ji}^{1-\sigma} di$ , that the total

expenditure of country  $j$  under free trade is

$$(34) \quad E_{Dij} = \mu M_t \left( \frac{P_{ij}}{P_t} \right)^{1-\sigma}.$$

### **2.4 The labor market**

The labor market clearing condition in country  $j$ ,  $j = h, f$  is

$$(35) \quad \bar{L}_j = L_{Aij} + L_{Dij}$$

where  $L_{A_{tj}}$  and  $L_{D_{tj}}$  are the total labor amounts demanded by the homogeneous non-network good sector and the differentiated network good sector, respectively, in free trade in country  $j$ ,  $j = h, f$ .

The total labor demand of the differentiated network good sector is  $L_{D_{tj}} = \frac{1}{w_j} \int_1^{N_{tj}+1} C_{ji} di$ . Calculate

this integral after replacing into it  $C_{ji}$  from (9), and then combine with (33) and (34), we obtain:

$$(36) \quad L_{D_{tj}} = \frac{1}{\omega} E_{D_{tj}} + [\alpha + \gamma(N_{tj})]N_{tj}.$$

## **2.5 The final good market clearing conditions**

The market clearing condition in the differentiated network good market is  $D_{tji}^s = D_{tji}$ , which has been used to compute the price of each variety and other equations. The market clearing condition in the non-network good sector is:

$$(37) \quad A_{th}^s + A_{tf}^s = A_{th} + A_{tf}.$$

## **2.6 The general equilibrium equations system**

Before summarizing all the equations characterizing the trading equilibrium, the total profit of each country must be computed. As stated before, the non-network good sector and network services sector earn zero profits. The total profit of a country is therefore equal to the total profit in the differentiated network good industry under free trade. This total profit is the difference between the revenue of the industry and the total cost that this industry has incurred. We can calculate the profit by subtracting  $L_{D_{tj}}$  specified in (36) (after multiplied by the unit wage rate) from  $E_{D_{tj}}$ , and the result is as follows:

$$(38) \quad \Pi_{tj} = \frac{1}{\sigma} E_{D_{tj}} - [\alpha + \gamma(N_{tj})]N_{tj}.$$

Thus, altogether, we have the following unknown variables:  $A_{tj}, A_{tj}^s, D_{tji}, D_{tj}, P_t, P_{tj}, P_{ji}, M_t, M_{tj}, E_{D_{tj}}, \Pi_{tj}, L_{A_{tj}}, L_{D_{tj}}, N_t$  and  $N_{tj}$ . The general equilibrium is characterized by the equations (22),(23b),(24)-(31),(33)-(36), and (38). We do not include (37) because it is dependent on the equations system listed above, due to the Walras' Law.

### III. ANALYSIS

#### 1. Equilibria in free trade

We are now ready to derive the number of firms in each country under free trade ( $N_{th}$  and  $N_{tf}$ ) in terms of the parameters in the model. In order to do so, first we must solve for  $M_t$ ,  $P_t$  and  $P_{j(N_{ij}+1)}$  ( $j = h, f$ ) in terms of all the parameters as well as of  $N_{th}$  and  $N_{tf}$  to be replaced into (31). From (24), (28) and (38), we can obtain, by noting that the labor wage rate in each country is the numeraire,

$$(39a) \quad M_t = \sum_{j=h,f} \bar{L}_j + \frac{1}{\sigma} \sum_{j=h,f} E_{Dij} - \sum_{j=h,f} [\alpha + \gamma(N_{ij})] N_{ij}.$$

However, from (23b) and (34), we have

$$(39b) \quad \sum_{j=h,f} E_{Dij} = \mu M_t.$$

By substituting  $M_t$  from (39a) into (39b) and then by changing terms, we obtain

$$(40) \quad M_t = \frac{\sum_{j=h,f} \bar{L}_j - \sum_{j=h,f} [\alpha + \gamma(N_{ij})] N_{ij}}{1 - \frac{\mu}{\sigma}}.$$

On the other hand, replacing  $P_{ij}$  in (23b) by (33) gives us

$$(41) \quad P_t = \omega \left( \sum_{j=h,f} \phi_j^{(1-\sigma)} \left[ \frac{(N_{ij} + 1)^\theta - 1}{\theta} \right] \right)^{\frac{1}{1-\sigma}}.$$

Substitute  $M_t$ ,  $P_t$  and  $P_{j(N_{ij}+1)}$  from (40), (41) and (32), respectively, into (31) and then obtain

$$(42) \quad \frac{\sum_{j=h,f} \bar{L}_j - \sum_{j=h,f} [\alpha + \gamma(N_{ij})] N_{ij}}{\frac{1}{\mu} - \frac{1}{\sigma}} \frac{\phi_j^{1-\sigma} (N_{ij} + 1)^{\theta-1}}{\sum_{j=h,f} \phi_j^{1-\sigma} \left[ \frac{(N_{ij} + 1)^\theta - 1}{\theta} \right]} = \sigma [\alpha + \gamma(N_{ij})], \quad j = h, f.$$

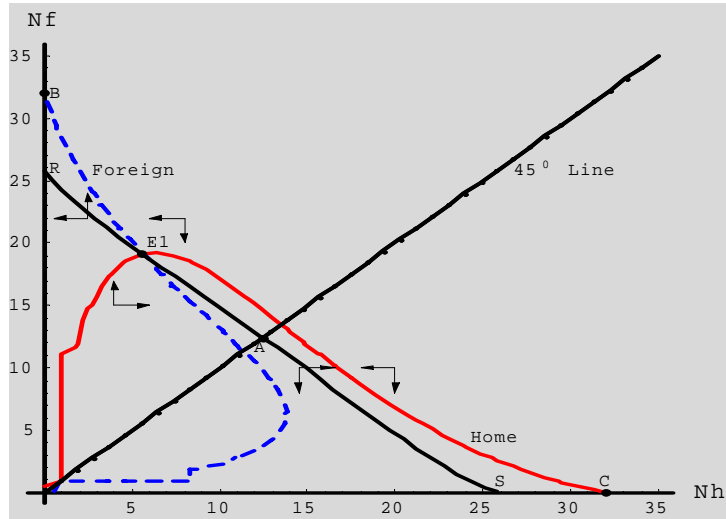
Solving the equations system in (42) will give us the free trade equilibrium  $N_{th}$  and  $N_{tf}$  with respect to the parameters of the model. Notable characteristics we can observe straightforward from (42) are that the efficiency parameters  $\phi_j$ 's do present themselves in each of the equation, while the total labor endowments of the two countries appear in both equations. Thus, we establish the following proposition.



**Proposition 2.** [Free Trade Equilibrium] *Under free trade, the difference in the numbers of operating firms between the two countries does not depend on the difference in the labor endowments of the two countries, but does depend on the efficiency levels of each country<sup>7</sup>.*

This proposition shows a sharp contrast to the result in the autarkic equilibrium derived in the previous section.

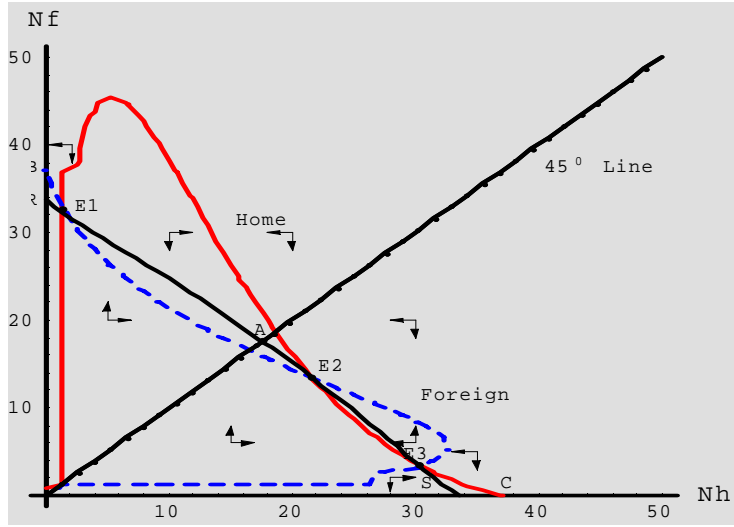
For further analyses, given the complexity of the algebra involved, numerical examples are to be utilized to solve the model. Each equation in (42) specifies the number of firms in a country given a specific firm number in the other. The curve which is plotted in the  $N_{th} - N_{tf}$  space based on the equation of country  $j$  ( $j = h, f$ ) is called country  $j$ 's *allocation curve*, following Kikuchi (2002). The intersections between the two curves are free trade equilibria specifying the equilibrium numbers of firms in the differentiated network good sectors in the two countries. By plotting allocation curves of the Home and Foreign with different values of parameters, we are able to consider several possible trading equilibria. As far as intra-industry trade is concerned in our scope, *it is revealed that the model may have a unique equilibrium or multiple equilibria*, such as illustrated in Figure 1 and Figure 2, respectively.



**Figure 1:** The case of unique intra-industry trading equilibrium

$$(\bar{L}_h + \bar{L}_f = 800, F = 17, g = 0.005, \alpha = 1, \phi_h = 1; \phi_f = 1.1, \delta = 0.5, \mu = 0.25, \eta = 2, \sigma = 2.1)$$

<sup>7</sup> This is a new result as compared to Kikuchi (2002) because this study deals with efficiency gap while Kikuchi does not.



**Figure 2:** The case of three intra-industry trading equilibria

$$(\bar{L}_h + \bar{L}_f = 800, F = 10, g = 0.005, \alpha = 1, \phi_h = 1; \phi_f = 1.1, \delta = 0.5, \mu = 0.25, \eta = 2, \sigma = 2.1)$$

In these figures, Home's allocation curve (labeled as *Home*) and Foreign's (labeled as *Foreign*) are plotted with the number of operating firms in Home on the horizontal axis. We also plot the autarkic equilibria curve in each figure (the curve RS) by using the equations (21b). The autarkic equilibrium curve plots all the pair of the numbers of firms in the two countries at autarkic equilibrium with respect to the ratio  $\bar{L}_h / \bar{L}_f$  while keeping  $\bar{L}_h + \bar{L}_f$  constant. Note that autarkic equilibrium is not affected by the firms' efficiency levels. Therefore, the autarkic equilibria curves of the two are identical and overlapped. We can see that this curve always passes through all the intra-industry trade equilibria. If the two countries have the same labor endowment, the autarkic equilibrium will be at **A**, the intersection point between the RS curve and the  $45^0$  line.

Observe that there are equilibria on both sides of the  $45^0$  line. Especially, the only one intra-industry trade equilibrium in Figure 1 lies on the left of the line. This implies that there is an equilibrium characterized by a set of operating firms in the less efficient country's industry of the differentiated goods being larger than that in the more efficient country. In other words, if the two countries get to this trading equilibrium, the less efficient country will produce more varieties of the differentiated goods than the more efficient one does. This result is different from what is specified in the analysis of Montagna (2001), in which there are always more firms in the differentiated network goods sector in the more efficient country than in the less efficient one. This must be brought forth by the effects induced by the existence of the communication network industry in the economies. Therefore, the chance of the less efficient country to become the net-exporter of the differentiated goods is certainly be expected.

Before going into the examination of trade patterns and effects of trade to specify the chance, we conduct dynamic analyses to check the stability of these trading equilibria. To do so, we first examine

the dynamics of the entry-exit process in the differentiated network good sector in the two countries by analyzing changes in the quantity demanded at any pair of  $(N_{th}, N_{tf})$  during the process to the equilibrium. Substitute  $M_t$  and  $P_t$  from (40) and (41), respectively, into the aggregate demand for each variety (26), we obtain:

$$(43) \quad D_{tji} = \frac{\omega^{1-\sigma}}{\frac{1}{\mu} - \frac{1}{\sigma}} \frac{\sum_{j=h,f} \bar{L}_j - \sum_{j=h,f} [\alpha + \gamma(N_{tj})] N_{tj}}{\sum_{j=h,f} \phi_j^{1-\sigma} \left[ \frac{(N_{tj} + 1)^\theta - 1}{\theta} \right]} P_{ji}^{-\sigma}$$

From (43), we obtain the quantity demanded of the good produced by the marginal firm in country  $j$  ( $j = h, f$ ) as follows:

$$(44) \quad D_{tj(N_{tj+1})} = \frac{\omega^{1-\sigma}}{\frac{1}{\mu} - \frac{1}{\sigma}} \frac{\sum_{j=h,f} \bar{L}_j - \sum_{j=h,f} [\alpha + \gamma(N_{tj})] N_{tj}}{\sum_{j=h,f} \phi_j^{1-\sigma} \left[ \frac{(N_{tj} + 1)^\theta - 1}{\theta} \right]} P_{j(N_{tj+1})}^{-\sigma}$$

Partially  
differentiate

$$\frac{\partial D_{tj(N_{tj+1})}}{\partial N_{tj}} = \frac{\omega^{1-\sigma}}{\frac{1}{\mu} - \frac{1}{\sigma}} \frac{-(\eta g N_{tj}^{\eta-1} + \alpha) \sum_{j=h,f} \phi_j^{1-\sigma} \left[ \frac{(N_{tj} + 1)^\theta - 1}{\theta} \right] - \left( \sum_{j=h,f} \bar{L}_j - \sum_{j=h,f} [\alpha + \gamma(N_{tj})] N_{tj} \right) \phi_j^{1-\sigma} (N_{tj} + 1)^{\theta-1}}{\left( \sum_{j=h,f} \phi_j^{1-\sigma} \left[ \frac{(N_{tj} + 1)^\theta - 1}{\theta} \right] \right)^2} P_{j(N_{tj+1})}^{-\sigma} < 0$$

where  $\bar{j} \neq j$  ( $j, \bar{j} = h, f$ ). Relying on (45), we can determine the directions of movement in the dynamic process. For example, consider the movement in Home. We begin with any point on the home allocation curve, such as point  $E_1$  in Figure 1. At this point, the marginal firm in Home earns zero profit according to the characteristics of the curve. Now consider any point vertically below this point. The number of firms in Home is the same as that at point  $E_1$ , but the number of firms in Foreign is less than that at point  $E_1$ . Therefore, (45) implies that the quantity demanded for the good produced by the marginal firm in Home will be larger than that at point  $E_1$ . Note that the price of the good charged by the marginal firm depends only on the marginal cost of the firm, hence does not change by this movement. This, in turn, implies that the marginal firm can now make profit. This positive profit induces other firms to enter. The number of firms in home will increase. Similarly, consider any point vertically above point  $E_1$ . At this point, the number of firms in Foreign is less than that at point  $E_1$ . Relying on (45), this implies that the quantity demanded of the good produced by the marginal firm in Home is less than that at point  $E_1$ . The firm incurs a loss and therefore withdraws from the market. Other firms who earn negative profit also exit. The number of firms in Home will decrease. We can argue in the same way

with the case of Foreign country. In summary, *firms will enter when the economy is somewhere below the allocation curve and exit somewhere above the allocation curve*. These movement directions are shown by arrows on the graove-mentioned arguments, we can see that  $\mathbf{E}_1$  in Figure 1 is a saddle-point stable intra-industry trade equilibrium. There is no stable equilibrium in this case. If the two countries have the same labor endowment levels and depart to trade from the autarkic equilibrium (point  $\mathbf{A}$ ), the equilibrium reached in the dynamic process will be point  $\mathbf{C}$ , where Home is the only country who produces the differentiated network goods. In Figure 2, among the intra-industry trade equilibria,  $\mathbf{E}_2$  is stable, while  $\mathbf{E}_1$  and  $\mathbf{E}_3$  are saddle-point stable ones. If the world economy moves from an autarkic equilibrium on the segment  $\mathbf{E}_1\mathbf{E}_3$  of RAS, it will move to the stable equilibrium  $\mathbf{E}_2$ , where both countries produce the differentiated network goods and the number of firms of Home is larger than that of Foreign's. Any autarkic equilibrium on the segment  $\mathbf{RE}_1$  ( $\mathbf{SE}_3$ ) will lead to equilibrium at  $\mathbf{B}$  ( $\mathbf{C}$ ), where Foreign (Home) will specialize completely in the production of the differentiated network goods.

## 2. Effects and patterns of trade

Let us consider the effects of international trade. To simplify the analysis, we examine only the situation where the two countries have the same labor endowments ( $\bar{L}_h = \bar{L}_f$ ). The difference between the two countries is the only difference in the efficiency level. In this case, the numbers of firms of the two countries at autarkic equilibrium are the same and at point  $\mathbf{A}$  in all the figures. We choose three specific cases to analyze the effect of trade, patterns of specialization and patterns of trade: (a) point  $\mathbf{E}_1$  in Figure 1, (b) point  $\mathbf{E}_2$  in Figure 2, and (c) point  $\mathbf{C}$  in Figure 1.

### a. Case 1:

Our numerical simulations suggest that the case illustrated by Figure 1 is likely to occur when the existing fixed cost of the network services industry ( $F$ ) is large relative to other parameters, especially to the total labor endowment of the world economy and the marginal cost in this network services sector ( $g$ ); or when the marginal cost of the network services production is small relative to other parameters. As we have specified, the network services sector in reality has the two of above-mentioned characteristics (namely, a large fixed cost and small marginal cost). In addition, this case may also exist when the efficiency gap between the two countries ( $\phi_f - \phi_h$ ) is large; the difference in the marginal costs of the two firms that locate close to each other in the marginal cost spectrum is large (i.e., large  $\delta$ ); the congestion coefficient ( $\eta$ ) is small; and the elasticity of substitution ( $\sigma$ ) is large.

#### (a-1) The patterns of trade:

As we can see in Figure 1, at  $\mathbf{E}_1$  there is the intra-industry trade between the two countries. Besides,  $\mathbf{E}_1$  lies on the left of the  $45^\circ$  line, this implies  $N_{th} < N_{tf}$ . The less efficient country (Foreign)

has more number of firms in the differentiated network goods industry than the more efficient country (Home) does. This case does not exist in the model having no network services sector, examined in Montagna (2001). Our simulations have shown that *the average price, measured by the aggregate price index, of the differentiated network goods produced by the more efficient country is higher than that of the differentiated network goods produced by the less efficient country* ( $P_{th} > P_{tf}$ , Figure A-1), and *Home's income is less than Foreign's* ( $M_{th} < M_{tf}$ , Figure A-2).

We now examine the market share held by each country in the integrated market of the differentiated network goods. From (34), we yield

$$(46) \quad \frac{E_{Dth}}{E_{Dtf}} = \left( \frac{P_{th}}{P_{tf}} \right)^{1-\sigma}.$$

Because  $P_{th} > P_{tf}$ , we have  $E_{Dth} < E_{Dtf}$ . In words, *a larger share of the market income is spent on the differentiated network goods produced in Foreign, the less efficient country, while the more efficient country holds a smaller share in the world expenditure.*

We also wish to know which country has the larger share of its labor endowment employed in the differentiated network goods sector and in the non-network good sector. By using (36), we yield the difference in the labor force used in the differentiated network goods industry between the two countries as follows:

$$(47) \quad L_{Dth} - L_{Dtf} = \frac{1}{\omega} (E_{Dth} - E_{Dtf}) + \alpha (N_{th} - N_{tf}) + g (N_{th}^{\eta} - N_{tf}^{\eta}).$$

We immediately see that  $L_{Dth} - L_{Dtf} < 0$ . *Home employs a smaller share in the world of its labor endowment in producing differentiated network goods than Foreign does.* Also we have

$$(48) \quad L_{Ath} - L_{Atf} = (\bar{L}_h - L_{Dth}) - (\bar{L}_f - L_{Dtf}) = -(L_{Dth} - L_{Dtf}) > 0,$$

since we assume that  $\bar{L}_h = \bar{L}_f$  in the present case. This implies that labor used in the homogeneous non-network good industry in Home is larger than that in Foreign. This, together with (29), gives that  $A_{th}^S > A_{tf}^S$ . *The quantity of the non-network good supplied by Home is larger than that supplied by*

*Foreign.* Besides, we have from (25) that  $\frac{A_{th}}{A_{tf}} = \frac{M_{th}}{M_{tf}}$ . Since  $M_{th} < M_{tf}$ , it is known that

$A_{th} < A_{tf}$ . In words, *the quantity of non-network good demanded by Home is smaller than that demanded by Foreign*. Now, we re-state the market clearing condition in the homogeneous good sector specified in (37) as  $A_{th}^S - A_{th} = -(A_{tf}^S - A_{tf})$ . Since  $A_{th}^S > A_{tf}^S$  and  $A_{th} < A_{tf}$ , we have  $A_{th}^S > A_{th}$  and  $A_{tf}^S < A_{tf}$ . This implies that *the more efficient country will export the homogeneous non-network good to the less efficient country*. According to Walras' Law, this also implies that the total value of the differentiated network goods imported by Home from Foreign will exceed its total value exported to Foreign. *The more efficient country will have a net import of the differentiated network goods from the less efficient country in the intra-industry trade*.

These results proved above can be summarized by the following proposition, which does not hold when the economy does not consist of communication network service sector.

**Proposition 3.** [Patterns of Trade] *Suppose that the existing fixed cost of the network services industry (F) is large, the marginal cost of the network services production is small, the efficiency gap between the two countries ( $\phi_f - \phi_h$ ) is large, the degree of firm heterogeneity is large, the congestion coefficient ( $\eta$ ) is small and the elasticity of variety substitution ( $\sigma$ ) is large. Then, there is an equilibrium at point  $E_1$ , where the less efficient country will be the net-exporter in the intra-industry trade of the differentiated network goods, and the scale of the differentiated network goods industry in the less efficient country is larger than that in the more efficient country, while the more efficient country will be the net-exporter of the non-network good.*

**(a-2) Welfare effects of trade**

Next, we analyze the effects of trade by comparing two states of the world economy: the economy at autarkic equilibrium (point A) and the economy at free trade equilibrium of point  $E_1$ . In order to observe the overall welfare effect of trade to the two countries, we now examine the indirect utility functions of consumers in the two countries. Using (1) and the demand functions for the network and non-network goods in autarky and those under free trade, we calculate the indirect utilities (denoted by  $V$ ) at autarkic equilibrium (A) and free trade equilibrium ( $E_1$ ) in country  $j$  ( $j = h, f$ ) as  $V_j = \nu M_j (P_j)^{-\mu}$  and

$V_{ij} = \nu M_{ij} (P_t)^{-\mu}$ , respectively, where  $\nu \equiv (1 - \mu)^{(1-\mu)} \mu^\mu$ . Take the ratio of the two and obtain

$$(49) \quad \frac{V_j}{V_{ij}} = \frac{M_j}{M_{ij}} \left( \frac{P_j}{P_t} \right)^{-\mu}, j = h, f.$$

Our simulations, reported in Appendix, shown that  $M_{th} < M_h = M_f < M_{tf}$  (Figure A-2),  $P_h < P_{th}$  (Figure A-3) and  $P_f > P_{tf}$  (Figure A-4). Hence, in Foreign, the intra-industry trade has a positive effect on the overall welfare ( $\frac{V_f}{V_{tf}} < 1$ ), since trade has both positive effects, through the income ( $\frac{M_f}{M_{tf}} < 1$ ) and through the price index ( $\frac{P_f}{P_t} > 1$ ). *The less efficient country benefits from trade.*

However, in Home, trade has a positive welfare effect through the price index ( $\frac{P_h}{P_t} > 1$ ) but a negative effect through the income ( $\frac{M_h}{M_{th}} > 1$ ). Therefore, the overall welfare effect cannot be determined straight away through analytical calculations, unlike what have been done in the case of Foreign. However, with our numerical simulations (see Figure A-5), we can see that  $V_h < V_{th}$ . The overall welfare effect of trade in Home is also positive. In other words, Home, the more efficient country, can also benefit from trade liberalization when trade induces the world economy to equilibrium at  $E_1$ . Thus, we have

**Proposition 4.** [Welfare Effects of Trade] *The both countries can be better off, if the world economy reaches  $E_1$  as the result of intra-industry trade.*

As the remark to the proposition, let us elaborate the welfare effects in detail. Welfare of consumers is affected by trade through changes in income and price indices induced by trade. As known in the model of monopolistic competition with firm heterogeneity (Montagna (2001)), changes in incomes and price indices are brought forth through two main channels: the number of varieties of the differentiated network goods and the efficiency structure of the industry. With love of variety, change in the number of varieties makes the price index vary, which in turn induces changes in utility level. Besides, the change in efficiency structure in each country, together with trade pattern, affects the income of each country as well as the price index of goods.

Since  $N_{th} < N_{tf}$ , we have  $\beta_{h(N_{th}+1)} < \beta_{f(N_{tf}+1)}$ , and hence  $P_{h(N_{th}+1)} < P_{f(N_{tf}+1)}$  (due to the fact that  $P_{ji} = \omega\beta_{ji}$ ), that is, that the price of the variety produced by the marginal firm in Home is lower than the price of the variety produced by the marginal firm in Foreign. By substituting  $P_j$  and  $P_{ij}$  ( $j = h, f$ ) from (18) and (33), respectively, into  $P_h < P_{th}$  and  $P_f > P_{tf}$ , and then comparing the obtained value of the numbers of firms in autarky and in free trade in each country, we have  $N_h > N_{th}$

and  $N_f < N_{ff}$ . Therefore we now have  $N_{th} < N_h = N_f < N_{ff}$ . In words, *the number of firms in free trade in Home, the more efficient country, is less than that in autarky, and the number of firms in free trade in Foreign, the less efficient country, is larger than that in autarky.* With this, we have  $N_t = N_{th} + N_{ff} > N_j$ ,  $j = h, f$ : there are more varieties of the differentiated network goods which consumers can enjoy at the trading equilibrium point  $\mathbf{E}_1$  than at autarkic equilibrium. The increase in the number of varieties lowers the price faced by consumers in the two countries.

However, the price index is also affected by the change in the efficiency structure. With  $N_{th} < N_h = N_f < N_{ff}$ , we have  $\beta_{h(N_{th}+1)} < \beta_{h(N_h+1)} < \beta_{f(N_f+1)} < \beta_{f(N_{ff}+1)}$ , due to  $\phi_h < \phi_f$  and  $\beta_{ji} = \phi_j i^\delta$ ,  $j = h, f$  and  $i \in [1, N_j + 1]$  (in autarky) or  $i \in [1, N_{ij} + 1]$  (in free trade). This implies that the average efficiency level in Home at the trading equilibrium is lower than that in autarky, while Foreign, the country with lower efficiency level, has the opposite. The reason for this result is that under free trade the least efficient firms in Home are displaced by some firms in Foreign, even though the new firms in Foreign are less efficient than those have disappeared in Home. This is explained by the fact that, in the world economy with the presence of communication network services, those less efficient firms in the less efficient country can earn non-negative profits and survive the competition due to the domination of the network cost sharing effect over its disadvantage in production cost efficiency during the dynamic process. With the absence of network services, as analyzed in Montagna (2001), this case does not exist. The inequality  $\beta_{h(N_{th}+1)} < \beta_{h(N_h+1)}$  shows that free trade induces a decrease in the price index via the efficiency effect in Home, while  $\beta_{f(N_f+1)} < \beta_{f(N_{ff}+1)}$  implies that the reverse holds in Foreign. However, with our numerical simulations, we have  $P_t < P_h < P_f$  (Figure A-6). This implies that the total effect of the change in the number of firms and the change in efficiency structures to the price index is positive, that is, that *the two countries' consumers benefits from trade through the fall in the overall price index.*

As for the effect of trade to welfare via income change, we can see that trade brings more income to Foreign than in autarky ( $M_f < M_{ff}$ ). At the trading equilibrium (point  $\mathbf{E}_1$ ), the less efficient country enjoys both positive effects through the income and price index as stated before. Therefore, it is unambiguous that *Foreign will gain from trade due to the increase in the number of varieties, the decrease in the price index of the network goods and the increase in national income.* On the other hand, Home



enjoys the positive effect of trade only under the domination of the positive effect through the price index over the negative effect through the income. The cost-sharing effect induced by the presence of the network services sector in the economy in combination with love-of-variety effect and efficiency effect in the economy with product differentiation and firm heterogeneity accounts for this fact.

**b. Case 2:**

We are now to examine the pattern and effects of trade if the world economy reaches the trading equilibrium at point  $E_2$ . Be noted that this trading equilibrium is stable. If the case in Figure 2 prevails,  $E_2$  is more likely to be reached, especially if the two economies open themselves to trade from autarkic equilibrium. With our numerical simulations, we can see the case illustrated in Figure 2 is likely to occur when the existing fixed cost of the network services industry ( $F$ ) is small relative to other parameters; the marginal cost of the network services production ( $g$ ) is relatively large; the efficiency gap between the two countries ( $\phi_f - \phi_h$ ) is small; the difference in the marginal costs of the two network differentiated firms that locate close to each other in the marginal cost spectrum is small (namely, for small  $\delta$ ); the congestion coefficient ( $\eta$ ) is large; or the elasticity of substitution ( $\sigma$ ) is small.

Proceeding in the same way as in Case 1, we have obtained that at point  $E_2$ , there is intra-industry trade between the two countries in which the more efficient country (Home) is the net-exporter of the differentiated good, and it also imports the homogeneous good from Foreign. The number of varieties that Home produces is larger than that of Foreign. The more efficient country benefits from both positive effects through the income and price index. Although Foreign faces a loss in income after trade, it also benefits from trade due to the domination of the positive effect through price index over the negative income effect. This case is the same result as in the analysis by Montagna (2001).

**Proposition 5.** [Montagna (2001)] *There is a stable equilibrium where the both countries can be better off under free trade, provided that the fixed cost ( $F$ ) is small, the marginal cost ( $g$ ) is large, the efficiency gap between the two countries ( $\phi_f - \phi_h$ ) is small, the degree of firms' heterogeneity ( $\delta$ ) is small, the congestion coefficient ( $\eta$ ) is large and the elasticity of the variety substitution ( $\sigma$ ) is small.*

Note that the proposition 5 articulates the conditions for the result of Montagna (2001) to survive the introduction of network services industry.

**c. Case 3:**

Finally, we are interested in a case of complete specialization. If the case in Figure 1 prevails and the two countries open themselves to trade from the autarkic equilibrium, the world economy will reach a trading equilibrium at point C where Home is the only one who produces and exports the differentiated goods. Foreign will completely specialize in producing the homogeneous not-network good and export it to Home. Our analysis can show that the two countries enjoy higher levels of welfare than in the case where they

were at the autarkic equilibrium. More specifically, the more efficient country benefits from both positive income and price index effects, while the less efficient country enjoys the welfare increase due to the domination of the positive effect through price index over the negative income effect.

**Proposition 6.** [Complete specialization] *The world trading equilibrium with complete specialization is possible, under the same situation stipulated in Proposition 3.*

#### IV. CONCLUSION

The purpose of this study is to examine the patterns and effects of trade in the world economy with the presence of both communication networks and heterogeneity in firms. To do so, country-specific communication networks and heterogeneous firms have been integrated into a one-factor, two-good model of trade between two countries that are similar in all aspects except the cost efficiency level of the most efficient firm in the differentiated goods sector. There exist firms that can earn positive profits. Therefore, the total income of each country depends not only on the labor income but also on the total profit of all existing firms. This makes the endogenous process to determine the number of operating firms examined in this model more complex than that in the simple model of firm homogeneity. Moreover, those existing firms will share the cost of the communication network. This implies that a firm's network cost and, hence, total fixed cost in turn is also endogenously determined.

It is revealed in the analysis of this paper that, under free trade with no trade costs where all the varieties of the differentiated network goods produced in a country are available to the two countries' consumers, the world economy may have a unique or multiple intra-industry trading equilibria, and the characteristics of these equilibria are different from one another, due to the presence of communication networks and heterogeneous firms. In addition to the case that is consistent to the established literature, where the more efficient country is the net-exporter of the differentiated goods, there is also a case that is different in that the less efficient country is the net-exporter of the differentiated goods at trading equilibrium. In this case, there are more firms in the differentiated goods industry in the less efficient country than in the more efficient country. The new firms of the less efficient country are even less productive than the ones in the more efficient country that have withdrawn out of the market. These firms can earn non-negative profits and survive the competition due to the fact that during the dynamic process, the effect of network cost sharing in the less efficient country overweighs its disadvantage in cost efficiency. Although the total welfare of each country increases, the more efficient country, not the less efficient one, faces a reduction in total profit (and hence in income) after trade, due to the contraction of its differentiated goods industry. Its gain in the total welfare is due to the domination over this negative income effect of the positive effect through price index which is brought forth by the greater availability of differentiated good varieties. This is because of the interaction among the efficiency effect induced from firm heterogeneity, the love-of-variety effect from product differentiation and the cost-sharing effect from

communication networks. Therefore, if the less efficient country can open itself to trade at an appropriate time, it may have a good chance to expand its differentiated goods industry by fostering its communication network services sector.

Appendix

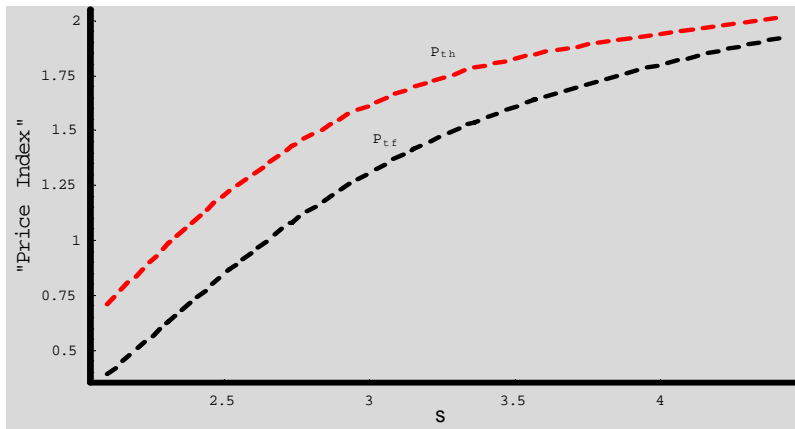


Figure A-1: Free trade price indices of differentiated good produced by Home and Foreign

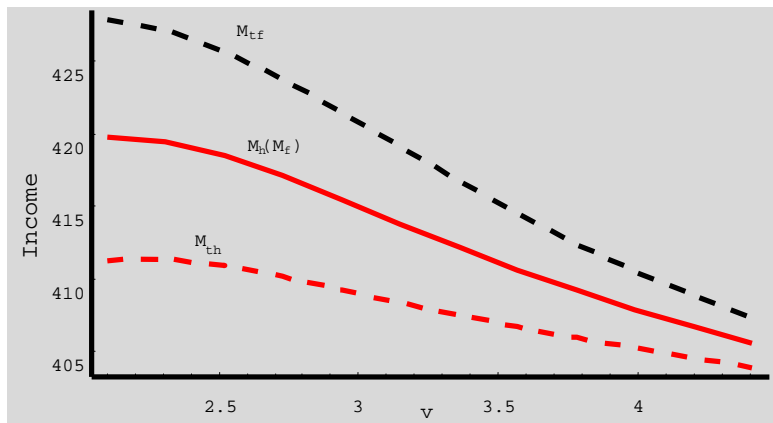


Figure A-2: Autarkic and free trade incomes

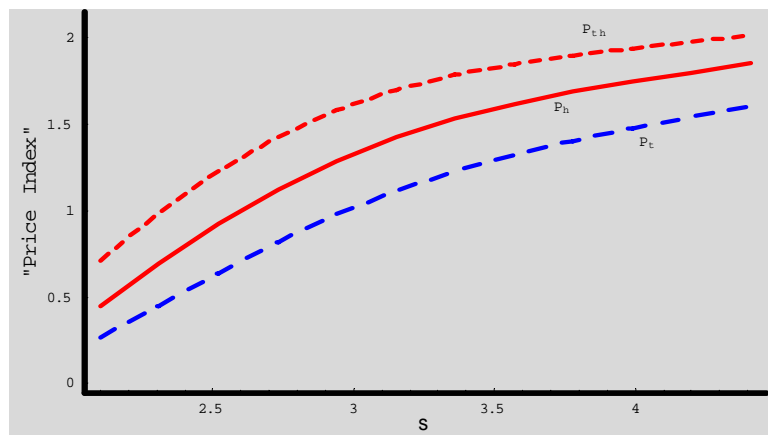


Figure A-3: Differentiated good price indices in autarky and free trade in Home

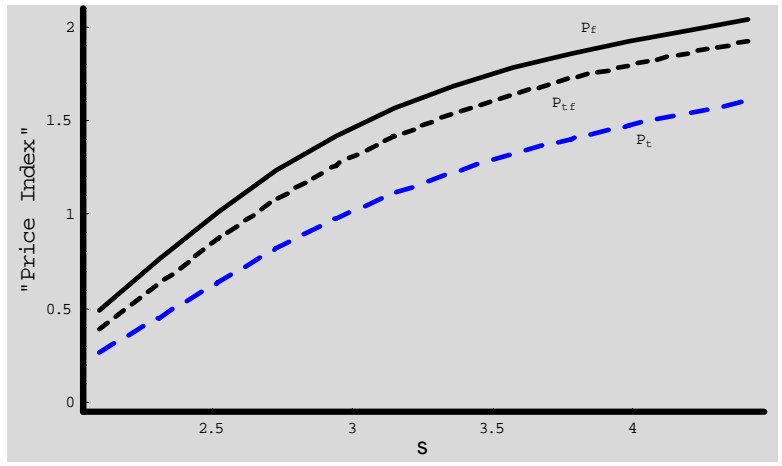


Figure A-4: Differentiated good price indices in autarky and free trade in Foreign

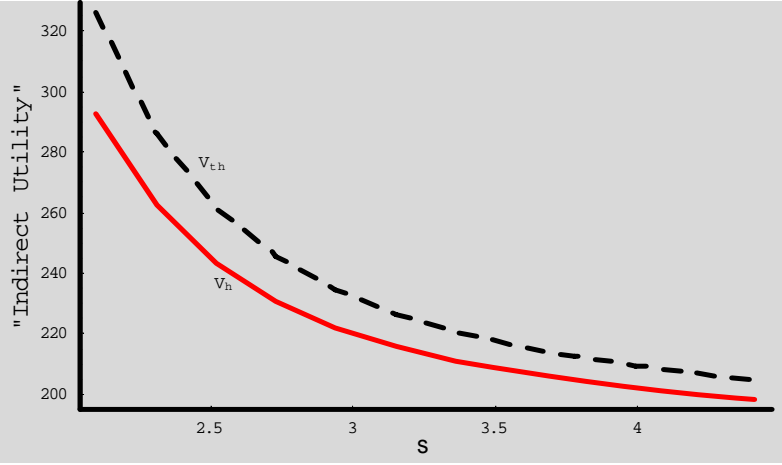


Figure A-5: Autarkic and free trade indirect utility levels of Home

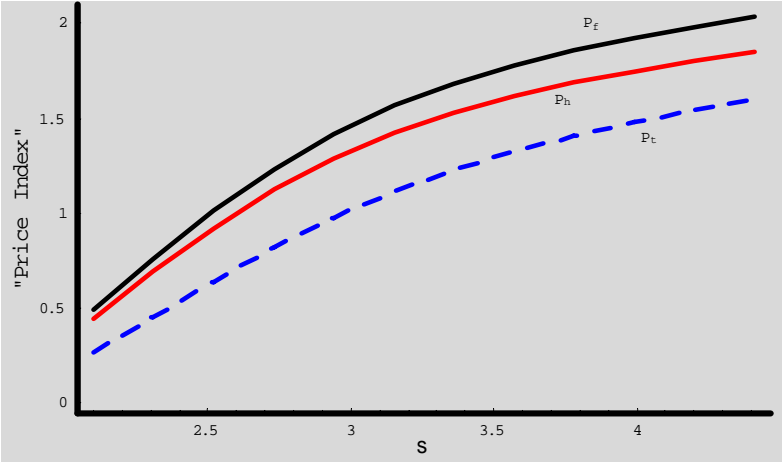


Figure A-6: The free trade world price index and autarkic price indices

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