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Abstract

This paper examines the effects of the degree of firm heterogeneity on the number of firms and of the difference in this degree between countries on international trade. The change in the mass of firms, trade pattern and welfare effect of trade are examined in a general equilibrium model where firms with different productivity levels in two countries having different degrees of firm heterogeneity in productivity compete in a monopolistically competitive market of a differentiated good. The paper reveals that the number of firms in a country always inversely relates to the degree of firm heterogeneity of its own, both in autarky and under free trade. In contrast, when firms in a country become less (more) heterogeneous, the number of firms in this country's trading partner will decrease (increase). Two countries with different extents of firm heterogeneity will benefit from trade at an equilibrium where the country with less heterogeneous firms has more firms and is the net-exporter in the intra-industry trade of the differentiated good. This paper contributes to the analysis of the effect of asymmetry between countries at firm level on the industrial reallocation and international trade with firm heterogeneity.

JEL classification codes: D21, D43, F12, L11, L13

Key words: firm heterogeneity, international trade, monopolistic competition, intra-industry trade, number of firms.

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I. INTRODUCTION

Most trade theoretical models depict an industry by a representative firm based on the hypothesis of firm homogeneity. However, recent empirical facts supported by increasing availability of firm-level data have shown that firms are heterogeneous in many aspects, even in a narrowly defined industry. Moreover, evidences on different linkages between firm heterogeneity and international trade have also been more spotted out[1]. Since the early 2000s, to be more relevant, many trade theorists have incorporated firm-level parameters and variables into their models in the effort to explain the related facts. Although there are some cases in which it is not different to treat firms as homogenous or as heterogeneous, the relevance of incorporating firm heterogeneity into trade models is recognized in many cases. The impact of trade under firm heterogeneity is different from that in the model of homogenous firms when there exist barriers to trade, endogenously variable elasticity of substitution between varieties, or characteristic asymmetries of trading partner countries etc. Melitz (2003) shows that in the presence of trade costs, trade under firm heterogeneity between identical countries induces inter-firm reallocation within an industry of a country in favor of more efficient firms. The reallocation, in turn, stimulates the improvement in efficiency of the industry. The efficiency gain, together with the welfare gain from love-of-variety effect usually observed in new trade theoretical models with homogenous firms, enhances welfare of trading partners. Besides, some studies such as Bernard, Eaton, Jenson and Kortum (2003), Montagna (2001), Bernard, Redding and Schott (2004), Falvey, Greenaway and Yu (2004) have shown that trade under firm heterogeneity between asymmetric countries triggers not only the inter-country production reallocation but also inter-firm reallocation that jointly affects welfare of the countries involved.

Our paper is among the efforts to clarify the linkage between characteristics of firm heterogeneity and the impact of trade between asymmetric countries. Although firm heterogeneity in our model is treated in somewhat the same way as that in many other models, in the sense that firms in a country are different in marginal productivity, the asymmetry between trading countries is treated differently. In this paper, countries are assumed to be different in the degree of firm heterogeneity. Different firms in each country have different levels of productivity. In addition, the difference between productivity levels of firms in a country is not the same as that in its trading partner. The difference is relevant due to the dissimilarity between countries in the exogenous economic background, such as difference in technology environment, input market, or information condition etc. Under monopolistic competition, the interaction between love-of-variety effect and efficiency effect caused by inter-firm as well as inter-country reallocations is expected. Our results show that the country with lower degree of firm heterogeneity will be the net-exporter in the intra-industry trade, and have more firms and more varieties than its partner country. The country with lower heterogeneity extent has more firms in free trade than in autarky, while the other country observes the opposite. Comparative static analysis in this model hints that the change in the gap between firms' productivity levels in a country will affect the numbers of firms in both countries, with the decrease in the gap in a country leading to an increase in the number of firms and product varieties of its own, but to a decrease in that of the other country. Besides contributing to the efficiency effect, the production reallocations also intensify the love-of-variety effect, and the world enjoys even higher welfare in free trade.

[1] See Tybout (2003) for detailed literature review on the matter.

As many studies have done, we base ourselves on the format of new trade theory where there is the interaction of increasing returns to scale, product differentiation and monopolistic competition. We incorporate firm heterogeneity into the model by assuming that firms are exogenously different in marginal cost. Different from Melitz's (2003), it is assumed in our model that there are no entry cost, no trade costs, no uncertainty about productivity before entry, and no forward-looking behavior, together with constant elasticity of substitution between varieties. With these assumptions, we can concentrate on the analysis of efficiency heterogeneity-induced reallocation in trade between countries with asymmetric degrees of firm heterogeneity. The symmetric-country model of Melitz (2003) shows that trade induces the reallocation within industry of each country in favor of more efficient firms. No inter-country reallocation occurs. The number of firms in each country decreases after trade. In contrast, in our model, the mass of firms in country with lower degree of heterogeneity expands when opening to trade while that in the other country shrinks. This is because, due to the asymmetry of heterogeneity degrees, incumbents and new entrants in the country with lower heterogeneity degree have advantage over its counterparts in the trade partner. Benefits from trade are derived from both inter-firm and inter-country reallocations, together with larger number of varieties.

Some papers have also analyzed trade between asymmetric countries with firm heterogeneity. It is noted in Melitz (2003) that Melitz's model can be easily extended to the case of asymmetric countries in country size. There is no specific impact of this difference on productivity distribution within a country. Bernard, Redding and Schott (2004) generalize Melitz's (2003) model to a model with multiple industries, multiple factors of production and asymmetric countries in terms of relative factor endowments to examine theorems of the Heckscher-Ohlin model under firm heterogeneity. All of the four fundamental theorems continue to hold in this context. However, like in Melitz (2003), two countries are the same as per the productivity distribution and therefore the degree of heterogeneity. Montagna's (2001) model is most close to our model in terms of country asymmetry related to the characteristics of heterogeneity between firms. Her model considers trade between countries having different states of technological advance. The most advanced technology in a country is more advanced than that in its trading partner country so that the most efficient firm in the former has lower marginal cost than the most efficient firm in the latter. The former will be the net-exporter of in the intra-industry trade of the differentiated good with a fall in the average efficiency of the industry when there are more less efficient firms enter the integrated market after trade. The latter faces the opposite. Nevertheless, the degree of heterogeneity in productivity between two firms close to each other in the ranking is the same in the two countries, and there is no comparative static analysis to understand the effects of change in the status of heterogeneity to trade.

Another paper that examines the effects of trade between multiple countries of different states of technology is Bernard, Eaton, Jenson and Kortum (2003). This model is of Ricardian type with firm-specific heterogeneity. Although this paper predicts reallocations induced by trade in the same kind as in some existing literature, its format is very different from that of Melitz's (2003) or that of our model. The difference between countries in their model is the asymmetry in the average productivity of firms in an industry, while the heterogeneity of efficiency is assumed to be the same among countries. Moreover, it relies on the assumptions of fixed total number of varieties, and of firms with variable mark-ups competing in the same variety. This may hinder the analysis of trade-induced reallocations on the number of varieties of the world, which is endogenously determined in our model. Moreover, with their format, the reallocation may be triggered not only by the firm-level efficiency gap as the unique factor as it is in our model due to the assumption of fixed mark-ups, but also via the combination of cross-country firm-level efficiency gap and the nature of competition between firms.

There is also an effort of Falvey, Greenaway and Yu (2004) in studying the effects of trade between countries that are asymmetric both in labor endowment and chances of successful entry. The productivity distribution in each country is of Pareto type, and cross-country difference in firm heterogeneity states is in the maximum marginal cost bound. Firms are uncertain about their productivity before entry. Under this setting, the model implies that countries have different probability of potential successful entry and potential national-wide average marginal cost. Some results derived in their paper are different from those of ours, especially in the effects of trade on the change in the mass of firms and trade patterns under costless trade. It is due to the difference in the setting of the model. Our model is different in the sense that, in order to analyze the pure effect of trade via reallocation under the existence of difference in the degrees of firm heterogeneity between countries, we assume a uniform distribution of productivity with unbound maximum marginal cost in each country, and the certainty firms have about their productivity in advance before entry. The only difference between the two countries is the asymmetry in the ratio of marginal costs of two firms ranked immediately close to each other in terms of productivity. With the unbounded maximum marginal cost and endogenous determination of the number of firms of each country, the advantage of a country over its partner in term of average marginal cost can not be implied beforehand. In addition, firms in the two countries do not face any unequal disadvantage induced by the uncertainty before entry[2].

The paper is organized as follows. We set up the model in section II and analyze in section III. Section IV concludes the analysis.

II. THE MODEL

1. Autarky

1.1 The world economy:

There are two countries, Home (denoted by *h*) and Foreign (by *f*)[3]. In each country, there are two final goods production sectors: a monopolistically competitive sector producing varieties of a horizontally differentiated good; and a perfectly competitive industry producing a homogeneous good. There is only one type of primary production factor, labor, that is homogeneous and assumed perfectly mobile between industries within each country but immobile between countries. Two countries are similar in all aspects except that the relative marginal costs of firms in the differentiated good sector, hereafter called the degree of firm heterogeneity, in a country is different from that in the other country.

1.2 Consumption*:*

The two countries have identical structure of preferences. Country j $(j = h, f)$ has a Cob-Douglas utility, denoted by U_j, over the homogeneous good A_j and the composite differentiated good D_j $(j = h, f)$:

[2] Furthermore, our model makes it easier to do comparative static analysis to search for the change in equilibrium variables due to changes in the heterogeneity condition of each country.

[3] All the variables and parameters inherent to them will also be denoted by the subscripts *h* and *f*, respectively.

$U_j = A_j^{1-\mu} D_j^{\mu}$, ($0 < \mu < 1$). D_j is a CES composition of the demanded quantities of a continuum of N_j varieties of the differentiated good: $D_j = \left(\int_1^{N_j+1} D_{ji}^{(\sigma-1)/\sigma} di \right)^{\frac{\sigma}{\sigma-1}}$, where D_{ji} is the demand for the variety produced by firm $i \in [1, \quad N_j + 1]$ in country *j,* and σ is the constant elasticity of substitution between varieties (assume $\sigma > 1$). This preference is known as the Dixit-Stiglitz type[4].

Take the homogeneous good as the numeraire by setting its price to unity. Denote the total income of country *j* by M_j, and the price index of the differentiated good in country *j* by P_j. The total income M_j of country j is the sum of the country's factor income and total profit of all firms (Π_j), $M_j = w_j \overline{L}_j + \Pi_j$, where w_j is the wage rate, $\overline{L}_j$ is the total labor endowment in country j.

Usual way of demand derivation is employed and we can obtain demand functions for the homogenous good, aggregate differentiated good and each variety i of this differentiated good, respectively, as follows:

(1) $$A_j = (1-\mu) M_j$$

(2) $$D_j = \mu \frac{M_j}{P_j}.$$

and

(3) $$D_{ji} = D_j \left(\frac{P_{ji}}{P_j} \right)^{-\sigma}$$

where P_{ji} is the price of i^{th} variety produced in country j $(j = h, f)$, with the price index of the differentiated good in country *j* being measured by

(4) $$P_j = \left(\int_1^{N_j+1} P_{ji}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}},$$

which is derived by solving $P_j D_j = \int_1^{N_j+1} P_{ji} D_{ji} di$. Note that with this setting, P_j can be considered as the price of the aggregate differentiated good D_j.

1.3 Production in the homogenous good industry

Denote the supply of the homogenous good by A_j^S, and the amount of labor used in this industry of

[4] Hence, all the assumptions and the demand derivation procedures are the same as in Dixit-Stiglitz (1977).

country $j \quad (j = h, f)$ by L_{Aj}. Assume that this industry is characterized by a production technology exhibiting constant returns to scale with unit labor requirement. Thus, $A_j^S = L_{Aj}$. The market clearing condition is $A_j^S = A_j$. The market for this good is assumed to be perfectly competitive. Hence, its price is equal to the average cost. The zero-profit condition also implies a unit wage rate in country j (i.e. $w_j = 1$).

1.4 Production in the differentiated good industry

A firm *i* in country *j* faces a total cost (in labor) with the function in the following form:

$$C_{ji} = \alpha + \beta_{ji} D_{ji}^s$$

where D_{ji}^s is the quantity of variety $i \in [1, \quad N_j + 1]$ supplied by country $j \quad (j = h, f)$; α is the fixed cost, assumed to be identical for all firms and across countries; and β_{ji} is the marginal cost of the variety $i \in [1, \quad N_j + 1]$ in country j, assumed to be firm-specific. Following Montagna (2001), we assume that within each country the first firm is the most efficient one with respect to which all other firms can be ranked. We rank the firms according to efficiency level, by defining a continuous variable $\rho_j(i)$ such that $\beta_{ji} = \rho_j(i)$ with $\rho_j(1) = \phi_j$ and $\rho_j^{'}(i) \geq 0$ for all $i \in [1, \quad N_j + 1]$, where ϕ_j is the marginal cost of the most efficient firm in country $j \quad (j = h, f)$. For the sake of simplicity, we adopt the following specific functional form for firms' marginal cost:

$$\beta_{ji} = \rho_j(i) = \phi_j i^{\delta_j}$$

where δ_j is the degree of technical heterogeneity among firms in country j, assumed to be non-negative. Firms are homogenous when $\delta_j = 0$ and heterogeneous otherwise. However, the profiles of firm heterogeneity are different between Montagna (2001) and ours. Montagna assumes that the productivities of the most efficient firms are different but the degrees of heterogeneity are the same between the two countries. Contrary, our model assumes that the most efficient firms are equally productive in the two countries but the degrees of heterogeneity for other firms are different. Our assumption implies that technological levels may be the same in the two countries but differences in business environment or factor specificity are the source of productivity difference among firms. Countries may have different technology markets, R&D governmental policies, factor markets, legal framework or business environment. That induces the difference between countries in the productivity gap between firms, referred to as degree of firm heterogeneity in this model. Without loss of generality, we assume that $\phi_h = \phi_f$ but $\delta_h < \delta_f$. The difference between the two countries is characterized only by the difference in the degree of technical heterogeneity among firms between the two countries.

Facing this cost structure, firm $i \in [1, \quad N_j + 1]$ in country $j \quad (j = h, f)$ will choose its optimal price (P_{ji}) and the quantity supplied (D^s_{ji}) to maximize its profit. As it is well established in the theoretical analysis of monopolistic competition, due to the existence of increasing return to scale (brought forth by fixed-cost effect), each firm will only produce one variety. Specifically, firm *i* in country *j* will choose the optimal price P_{ji} to maximize its profit $\Pi_{ji} = (P_{ji} - \beta_{ji}) D^S{}_{ji} - \alpha$ under the market clearing condition ($D^s_{ji} = D_{ji}$). We assume further that firms do not behave strategically in the sense that each firm considers other firms' prices as given when setting its price (i.e. $\partial P_{ji} / \partial P_{jk} = 0, i, k = 1, \cdots, N_j + 1; i \neq k$), and that the influence of an individual price change on the aggregate price index is ignorable (i.e. $\partial P_{ji} / \partial P_j = 0$). Firms also consider the national income level being fixed. Thus, by solving this profit maximization problem, we obtain the optimal price as follows

$$P_{ji} = \omega \beta_{ji} \tag{5}$$

where $\omega \equiv \dfrac{\sigma}{\sigma - 1}$, known as the constant mark-up over the marginal cost. Hence, the profit of firm $i \in [1, \quad N_j + 1]$ in country $j \quad (j = h, f)$ can be calculated as follows:

$$\Pi_{ji} = \frac{1}{\sigma} \mu M_j \left(\frac{P_{ji}}{P_j} \right)^{1-\sigma} - \alpha . \tag{6}$$

<u>1. 5 The exit-entry process in the market of the differentiated good</u>

In the monopolistically competitive market of the differentiated goods in each country, a firm will stay in the market while its profit is non-negative and will quit otherwise. A new firm will enter the market when it finds that it is profitable to produce a variety with the marginal cost that it is going to incur. In equilibrium there should be no new entry or exit; hence the marginal firm will break even. That is, the profit of this firm is zero, i.e., $\Pi_{j(N_j+1)}(\beta_{j(N_j+1)}) = 0$, where $\beta_{j(N_j+1)} = \phi_j (N_j + 1)^{\delta_j}$ is the marginal cost of the marginal firm (hereafter referred to as the *efficiency cut-off point* on the marginal cost spectrum in the differentiated good industry), showing the highest marginal cost that prevails among the existing firms. Furthermore, given the way of ranking firms, firms whose marginal costs are smaller than $\beta_{j(N_j+1)}$ will make positive profits ($\Pi_{ji}(\beta_{ji} \mid \beta_{ji} < \beta_{j(N_j+1)}) > 0$). With (5) and (6), the zero profit condition for the marginal firm in country *j* becomes

(7) $$\frac{1}{\sigma}\mu M_j\left(\frac{P_{j(N_j+1)}}{P_j}\right)^{1-\sigma}=\alpha\,.$$

1.6 The labor market

Let L_{Dj} be the total labor required in the differentiated good industry in country j $(j=h,f)$, thus $L_{Dj}=\int_1^{N_j+1}C_{ji}\,di$. Deriving this integral by using (2), (3) and (4) we yield:

(8) $$L_{Dj}=\frac{\mu}{\omega}M_j+\alpha N_j\,.$$

Furthermore, the national labor market equilibrium condition requires:

(9) $$\overline{L}_j=L_{Aj}+L_{Dj}\,.$$

1.7 The autarkic equilibrium

We now try to summarize the equations system governing the equilibrium in each country under autarky. The price of the marginal firm's variety is $P_{j(N_j+1)}=\omega\phi_j(N_j+1)^{\delta_j}$. Using (5) and then (4), we can now express P_j in terms of the number of varieties as follows:

(10) $$P_j=\omega\phi_j\left[\frac{(N_j+1)^{\theta_j}-1}{\theta_j}\right]^{\frac{1}{1-\sigma}}$$

where $\theta_j\equiv\delta_j(1-\sigma)+1\le 1$. One more thing we have to calculate is the economy-wide profit. There will be no profit in the homogeneous good sector thanks to the assumption of perfect competition in the sector. Therefore, the total profit of the economy is the aggregate profit of firms in the differentiated good industry, which is equal to the difference between the revenue and the total labor cost of this sector, keeping in mind that $w_j=1$: $\Pi_j=\int_1^{N_j+1}\Pi_{ji}\,di=P_jD_j-L_{Dj}$. Using (2) and (8), we can get

(11) $$\Pi_j=\frac{1}{\sigma}\mu M_j-\alpha N_j\,.$$

In autarky, the model consists of the following unknown variables: A_j, D_j, D_{ji}, P_j, P_{ji}, M_j, Π_j, Π_{ji}, L_{Aj}, L_{Dj}, and N_j, where $i\in[1,\ N_j+1]$ and $j=h,f$. Taking Walras' Law into consideration, the general equilibrium is characterized by eleven equations *(1)* to *(11)*. By solving this equations system for all the variables, we can capture the characteristics of the autarkic equilibrium.

2. Free trade

Assume that there are no transport costs and other trade barriers and that consumers do not discriminate amongst goods produced in different countries. All the varieties produced in a country will now also be available to consumers in the other country. Hence, the number of varieties consumers can enjoy under free trade is the total number of varieties of the differentiated goods produced in the two countries[5]:

(12) $$N_t = N_{th} + N_{tf} .$$

where N_{tj} is the total number of varieties, also the number of firms, in country *j* ($j = h, f$) under free trade. The common free trade price index in the differentiated good market is:

$$P_t = \left(\int_1^{N_{th}+1} P_{hi}^{1-\sigma} di + \int_1^{N_{tf}+1} P_{fi}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} .$$

If we denote by $P_{tj} = \left(\int_1^{N_{tj}+1} P_{ji}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ a CES index of the prices of goods produced by country *j*'s firms under free trade, then this price index can be rewritten as

(13) $$P_t = \left(P_{th}^{1-\sigma} + P_{tf}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} .$$

2.1 Consumption

The functions of demand for the two final goods in the world market (denoted by A_t and D_t) under free trade are derived by solving the maximization problem of consumers in the same way as in autarky. They are $A_t = (1-\mu)M_t$ and $D_t = \mu \dfrac{M_t}{P_t}$, where M_t is the total income of the world, being the sum of the income of the two countries under free trade:

(14) $$M_t = M_{th} + M_{tf} .$$

where M_{tj} ($j = h, f$) is the national income in country *j* under free trade.

The demand of country *j* for the homogenous good, A_{tj}, is

(15) $$A_{tj} = (1-\mu)M_{tj} .$$

The world demand for each variety of the differentiated good produced in country *j* , D_{tji} , is

[5] A variable with subscript t implies that it is examined under free trade.

(16) $$D_{tji} = \mu M_t P_t^{\sigma-1} P_{ji}^{-\sigma} ,$$

since the world demand for the aggregate differentiated good is $D_t = \mu \frac{M_t}{P_t}$. The demand by country *j* for the differentiated good produced by the two countries is

(17) $$D_{tj} = \mu \frac{M_{tj}}{P_t} .$$

The world expenditure on the differentiated good produced by country *j* ($j = h, f$) is also the revenue of country j out of selling its differentiated good to the world, denoted by E_{Dtj}. That is $E_{Dtj} = \int_1^{N_{tj}+1} P_{ti} D_{tji} di$. Each country's income is the sum of factor income ($w_{tj}\overline{L}_j$) and total profit of firms in that country under free trade (Π_{tj}):

(18) $$M_{tj} = w_{tj}\overline{L}_j + \Pi_{tj} .$$

where w_{tj} is the wage rate in country *j* under free trade.

2.2 Production

Following the same procedure as in the previous section, we have the supply of the homogenous good by country *j* (A_{tj}^S) as follows, noting that this good is taken as numeraire and the labor wage is equal to 1 ($w_{tj} = 1$):

(19) $$A_{tj}^s = L_{Atj}$$

where L_{Atj} is the amount of labor used in the production of the homogenous good in country *j*, $j = h, f$, under free trade.

In the differentiated good industry, the price of variety $i \in [1, \quad N_{tj} + 1]$ produced in *j* under the condition of market clearance has the same form as in (5):

(20) $$P_{ji} = \omega\beta_{ji} .$$

2.3 The entry-exit process in the market of the differentiated good

The profit of firm $i \in [1, \quad N_{tj} + 1]$ in *j* ($j = h, f$) under free trade is calculated as

$\Pi_{tji} = \left(P_{ji} - \beta_{ji}\right)D_{tji} - \alpha = \frac{1}{\sigma}\mu M_t \left(\frac{P_{ji}}{P_t}\right)^{1-\sigma} - \alpha$, using (16) and (20). The zero profit condition for the marginal firm is given by $\Pi_{tj(N_{tj}+1)}(\beta_{j(N_{tj}+1)}) = 0$, which yields

(21) $$\frac{1}{\sigma}\mu M_t \left(\frac{P_{j(N_{tj}+1)}}{P_t}\right)^{1-\sigma} = \alpha$$

where $P_{j(N_{tj}+1)} = \omega\phi_j (N_{tj}+1)^{\delta_j}$ is the price charged by the marginal firm in country *j* under free trade.

In this model, our assumption of no uncertainty prior to and after entry goes in line with that of no sunk-entry cost. As Melitz (2003) argues, in the model with forward–looking firms facing uncertainty before and after entry and with sunk-entry cost, the total income of the economy is equal to the labor income due to the fact that profits are exhausted by the aggregate investment sunk cost of entrants. However, in our model, due to the absence of uncertainty, in the general equilibrium format, different productivity conditions can imply different market sizes, which in turn determine the population of firms. The elimination of the effect induced by uncertainty helps focus on the analysis of pure effects of the heterogeneity gaps between the two countries.

The price index of all the varieties produced by country j, $j = h, f$ under free trade (P_{tj}) is derived by substituting P_{ji} in (20) into the definition of P_{tj} and then performing the integral. The result is:

(22) $$P_{tj} = \omega\phi_j \left[\frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j}\right]^{\frac{1}{1-\sigma}}$$

We are interested in the total revenue of country *j* from selling its differentiated good under free trade. After substituting D_{tji} in (16) into $E_{Dtj} = \int_1^{N_{tj}+1} P_{ji}D_{tji}di$ and then arranging the integral, we yield, keeping in mind that $\left(P_{tj}\right)^{1-\sigma} = \int_1^{N_{tj}+1} P_{ji}^{1-\sigma}di$

(23) $$E_{Dtj} = \mu M_t \left(\frac{P_{tj}}{P_t}\right)^{1-\sigma}.$$

<u>2.4 The labor market</u>

The labor market clearing condition in country j, $j = h, f$ is

(24) $$\overline{L}_j = L_{Atj} + L_{Dtj}$$

where L_{Atj} and L_{Dtj} are the total labor amounts demanded by the homogeneous good sector and the differentiated good sector, respectively, in country j under free trade. The total labor demand of the differentiated good sector is $L_{Dtj} = \int_1^{N_{tj}+1} C_{ji}\,di$. Calculate this integral after replacing C_{ji} into it, and then combine with (22) and (23), to obtain:

(25) $$L_{Dtj} = \frac{1}{\omega} E_{Dtj} + \alpha N_{tj} .$$

2.5 The final good market clearing conditions

The market clearing condition in the differentiated good market is $D^s_{tji} = D_{tji}$, which is already used to compute the price of each variety and other equations. The market clearing condition in the homogenous good sector is $A^s_{th} + A^s_{tf} = A_{th} + A_{tf}$.

2.6 The general equilibrium equations system

Before summarizing all the equations characterizing the trading equilibrium, the total profit of each country must be computed. As stated before, the total profit of a country is equal to the total profit in the differentiated good industry under free trade. This total profit is the difference between the revenue of this industry and the total cost that this industry has incurred. We can calculate the profit by subtracting L_{Dtj} specified in (25) (after multiplied by the unit wage rate) from E_{Dtj} , and the result is as follows:

(26) $$\Pi_{tj} = \frac{1}{\sigma} E_{Dtj} - \alpha N_{tj} .$$

A_{tj} , A^s_{tj} D_{tji} , D_{tj} , P_t , P_{tj} , P_{ji} , M_t , M_{tj} , E_{Dtj} , Π_{tj} , L_{Atj} , L_{Dtj} , N_t and N_{tj} are unknown variables under free trade. Taking the Walras' Law into account, the general free trade equilibrium is characterized by the equations from (22) to (38).

III. ANALYSIS

1. In autarky:

First, we calculate the income M_j in terms of the number of firms and parameters in the model. Because $A^S_j = L_{Aj}$ and $A^S_j = A_j$, we have $L_{Aj} = A_j$. Replace A_j in (1) into this, we get $L_{Aj} = (1-\mu)M_j$.

Replace this L_{Aj} and L_{Dj} in (8) into (9), and then rearrange the terms, to obtain:

(27) $$M_j = \frac{\sigma}{\sigma - \mu}(\overline{L}_j - \alpha N_j)$$

Let us define $Y_{aj} \equiv \mu M_j$ and $S_{aji} \equiv \left(\frac{P_{ji}}{P_j}\right)^{1-\sigma}$. Y_{aj} is the total expenditure of country j

($j = h, f$) on the differentiated good in autarky. In other words, Y_{aj} is the size of the market for the differentiated good in autarky. S_{aji} is the market share a firm i ($i \in [1, \quad N_j + 1]$) can acquire. $Y_{aj}S_{aji}$ is the revenue of firm i, $\frac{1}{\sigma}Y_{aj}S_{aji}$ is the variable profit (the difference between the revenue and variable cost), and $\frac{1}{\sigma}Y_{aj}S_{aji} - \alpha$ is its profit. With (27), we have the market size of the differentiated good as

(28)
$$Y_{aj} = \frac{\sigma\mu}{\sigma - \mu}(\bar{L}_j - \alpha N_j)$$

The income, and then the market size, depends inversely on the number of firms in the differentiated good industry. Let $S_{ajm} \equiv \left(\frac{P_{j(N_j+1)}}{P_j}\right)^{1-\sigma}$ denote the market share of the marginal firm. Calculating this, using (10), we get

(29)
$$S_{ajm} = \theta_j \frac{(N_j + 1)^{\theta_j - 1}}{(N_j + 1)^{\theta_j} - 1},$$

The marginal firm's zero-profit condition (7) can be expressed as:

$$\frac{1}{\sigma}Y_{aj}S_{ajm} - \alpha = 0$$

Let $ZP_{aj} \equiv \frac{1}{\sigma}Y_{aj}S_{ajm} - \alpha$, then the condition is equivalent to $ZP_{aj} = 0$. Substitute Y_{aj} and S_{ajm} in (28) and (29) into this condition to obtain:

(30)
$$ZP_{aj} = \frac{\sigma\mu}{\sigma - \mu}\theta_j(\bar{L}_j - \alpha N_j)\frac{(N_j + 1)^{\theta_j - 1}}{(N_j + 1)^{\theta_j} - 1} - \alpha = 0$$

There is only one unknown variable N_j in equation (30). Therefore we can solve for N_j in terms of all the parameters appeared in this equation. Since it is quite intricate, we will not solve that but instead analyze some characteristics of the number of firms in autarkic equilibrium based on this equation.

We can immediately see from this condition that the number of firms in autarkic equilibrium does not depend on ϕ_j. In other words, it does not depend on the productivity of the most efficient firm in the economy. However, it does depend on the degree of heterogeneity in productivity of firms, in addition to the level of labor endowment.

We now examine the relationship between the number of firm and the degree of heterogeneity by analyzing the effect of the change in the degree of firm heterogeneity to the change in the number of firms in equilibrium. As shown in Appendix (a), $\frac{dN_j}{d\delta_j} < 0$. This implies that the more heterogeneous the firms are, the smaller number of firms in the differentiated good sector is. This is because the more efficient firm has higher market power in comparison to the less efficient one in the case when firms are more different than the case when firms are more similar. Therefore, the market share of the marginal firm will be smaller when firms are more heterogeneous, ($\frac{dS_{ajm}}{d\delta_j} < 0$). That deters firms in the margin from the market. When firms get less heterogeneous, firms have more equal market power, and then more firms are expected to exist.

Besides, we have $\frac{dP_j}{d\delta_j} > 0$ [proof in Appendix (b)]. When firms are more heterogeneous, consumers will face a higher price of the differentiated good.

Furthermore, $\frac{dN_j}{d\delta_j} < 0$ and $\frac{dP_j}{d\delta_j} > 0$ also imply that the country with higher degree of firm heterogeneity (Foreign) will have less firms and face higher price of the differentiated good than the country with firms being less heterogeneous (Home).

All the analysis done above can be summarized in the following proposition.

Proposition 1: *In autarky, the number of firms in the differentiated good sector does not depend on the efficiency level of the most productive firm, but does depend on the heterogeneity in productivity between firms and labor endowment. When firms are less heterogeneous, the number of firms will increase. Between two countries that have the same levels of labor endowment, the one that has lower degree of firm heterogeneity will have more firms in the differentiated good sector. This country also enjoys a lower aggregate price of the differentiated good.*

2. In free trade:

We are to derive the characteristics of the free trade equilibrium. First, we examine the numbers of firms of the two countries under free trade, relying on the zero-profit condition in free trade.

Let define $Y_t \equiv \mu M_t$, the total expenditure of the world on all the varieties of the differentiated good produced by the two countries in free trade. Y_t is also understood as the size of the common market for all the firms in the differentiated good industry of the two countries. Let $S_{tji} \equiv \left(\frac{P_{ji}}{P_t}\right)^{1-\sigma}$ being the market share in the world market that a firm i ($i \in [1, \quad N_{tj} + 1]$) in country j ($j = h, f$) can acquire under free trade. $Y_t S_{tji}$ is the revenue of firm i, $\frac{1}{\sigma} Y_t S_{tji}$ is variable profit, and $\frac{1}{\sigma} Y_t S_{tji} - \alpha$ is net profit. Let S_{tjm} be the market share of the marginal firm in country j. Then, the zero-profit condition (21) can be expressed as follows

$$ZP_{tj} \equiv \frac{1}{\sigma} Y_t S_{tjm} - \alpha = 0$$

In order to derive N_{th} and N_{tf}, we must solve for M_t and P_t, then Y_t and S_{tjm} in terms of the parameters in the model as well as N_{th} and N_{tf}. From (13) and (23), we have

(31) $$\sum_{j=h,f} E_{Dtj} = \mu M_t$$

From (14), (18), (26) and (31), we calculate the world income in terms of the numbers of firms as follows

(32) $$M_t = \frac{\sigma}{\sigma - \mu}\left(\sum_{j=h,f} \overline{L}_j - \alpha \sum_{j=h,f} N_{tj} \right).$$

With (32), we have the market size of the differentiated good as

(33) $$Y_t = \frac{\sigma\mu}{\sigma - \mu}\left[\sum_{j=h,f} \overline{L}_j - \alpha \sum_{j=h,f} N_{tj} \right]$$

The world income depends negatively on the world number of firms in the differentiated good industry. And so does the world market size.

On the other hand, replacing P_{tj} in (13) by the same variable in (22), we have

(34) $$P_t = \omega\left(\sum_{j=h,f} \phi_j^{1-\sigma} \frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j} \right)^{\frac{1}{1-\sigma}}$$

Using this to calculate the market share of the marginal firm in each country, we obtain

$$S_{tj} = \frac{\phi_j^{1-\sigma}(N_{tj}+1)^{\theta_j - 1}}{\sum_{j=h,f} \phi_j^{1-\sigma} \frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j}}, \quad j = h, f$$

Then, the marginal firm's zero-profit condition in country j, $j = h, f$ will be:

$$\frac{\mu}{\sigma - \mu}\left[\sum_{j=h,f} \overline{L}_j - \alpha \sum_{j=h,f} N_{tj} \right] \frac{\phi_j^{1-\sigma}(N_{tj}+1)^{\theta_j - 1}}{\sum_{j=h,f} \phi_j^{1-\sigma} \frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j}} - \alpha = 0$$

We can see that if $\phi_f \neq \phi_h$, the difference in the number of firms between the two countries do depend on these levels of efficiency. However, we just examine here the case where $\phi_f = \phi_h$ but $\delta_f > \delta_h$ to analyze the effect of difference in the degree of firm heterogeneity. Thus, the zero-profit condition becomes

(35) $$\frac{\mu}{\sigma-\mu}\left[\sum_{j=h,f}\overline{L}_j-\alpha\sum_{j=h,f}N_{tj}\right]\frac{(N_{tj}+1)^{\theta_j-1}}{\sum_{j=h,f}\frac{(N_{tj}+1)^{\theta_j}-1}{\theta_j}}-\alpha=0$$

In the case of $\phi_f=\phi_h$, the numbers of firms in free trade are invariant to ϕ_h and ϕ_f. If we plot the zero-profit condition of a country on $N_{th}-N_{tf}$ plane, this curve illustrates the number of firms in a country corresponding to different levels of that in the other country. This curve is known as allocation curve of the country. Suppose that all the parameters are relevant for the system to have solutions.

We can also immediately see that $S_{thm}=S_{tfm}$. This also implies $P_{h(N_{th}+1)}=P_{f(N_{tf}+1)}$ and $\beta_{h(N_{th}+1)}=\beta_{f(N_{tf}+1)}$. That is, in free trade, the marginal firms of the two countries have the same marginal cost, and charge the same price for their varieties. Furthermore, $\beta_{h(N_{th}+1)}=\beta_{f(N_{tf}+1)}$ means $\phi_h(N_{th}+1)^{\delta_h}=\phi_f(N_{tf}+1)^{\delta_f}$. With $\phi_f=\phi_h$ *and* $\delta_f>\delta_h$, we have $N_{tf}<N_{th}$. The country with lower degree of firm heterogeneity will have more firms in the differentiated good sector under free trade.

We can prove that this equilibrium is unique and stable. As shown in Appendix (d), we have that along the allocation curve of a country, the number of firms of a country relates negatively and monotonically to the number of firms in the other country. Therefore, if the system has solution and $\phi_f=\phi_h$, this solution is unique. We illustrate this in Figure 1. In this figure, A and E are the autarkic equilibrium and the free trade equilibrium, respectively.

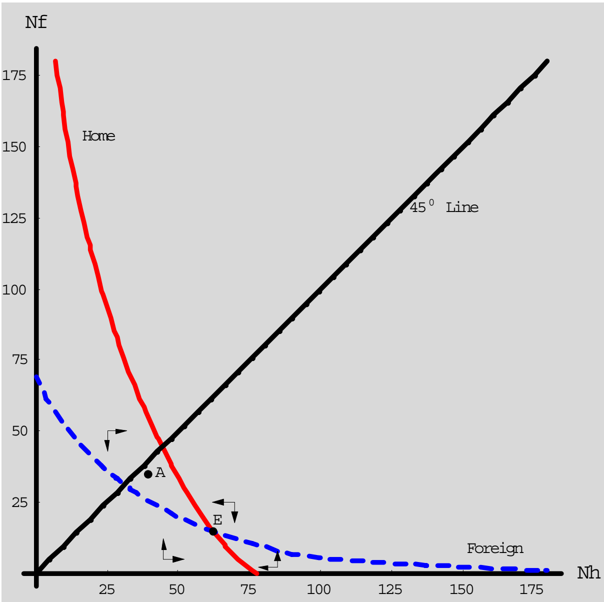

Figure 1: The equilibrium is unique and stable.

$(\overline{L}_h=\overline{L}_f=400;\ \alpha=1,\phi_h=\phi_h=1;\delta_h=0.2;\delta_f=0.3\ \mu=0.25;\sigma=2.1)$

The stability of this equilibrium can be examined by analyzing the change in the demand for the variety of the marginal firm located in a country when the population of firms in the other country changes. Market demand for the variety produced by the marginal firm in country j $(j = h, f)$ is:

$$D_{tj(N_{tj}+1)} = \mu M_t \left(\frac{P_{j(N_{tj}+1)}}{P_t} \right)^{1-\sigma} \left(P_{j(N_{tj}+1)} \right)^{-1} = Y_t S_{tjm} \left(P_{j(N_{tj}+1)} \right)^{-1}$$

Differentiate this demand by the number of firms in the other country, country $\bar{j} = h, f$ and $\bar{j} \neq j$ to have:

$$\frac{\partial D_{tj(N_{tj}+1)}}{\partial N_{t\bar{j}}} = \left[Y_t \frac{\partial S_{tjm}}{\partial N_{t\bar{j}}} + S_{tjm} \frac{\partial Y_t}{\partial N_{t\bar{j}}} \right] \left(P_{j(N_{tj}+1)} \right)^{-1}$$

As shown in Appendix (e), $\frac{\partial D_{tj(N_{tj}+1)}}{\partial N_{t\bar{j}}} < 0$. The demand for the variety produced by the marginal firm in a country will decrease (increase) when the number of firms in the other country increases (decreases). This implies that, in a country, at any point below its allocation curve, the marginal firm makes positive profit. Thus, more firms will enter and the number of firms in this country will increase. The reverse holds for any point above the allocation curve. These motions in two countries are illustrated by arrows in Figure 1. With the characteristics of these motions, the free trade equilibrium is stable. We now come to the following proposition.

Proposition 2: *The equilibrium in free trade is unique and stable. At this equilibrium, the country with lower degree of heterogeneity in firm productivity will have more firms in the differentiated good sector.*

We turn to examine the effect of changes in the degree of firm heterogeneity in a country on the numbers of firms in both countries in free trade. In Appendix (c), we have proved that $\frac{dN_{tj}}{d\delta_{\bar{j}}} > 0$ and $\frac{dN_{t\bar{j}}}{d\delta_{\bar{j}}} < 0$, $j, \bar{j} = h, f; j \neq \bar{j}$. These results imply that when the degree of firm heterogeneity in one country decreases, the number of firms in the differentiated good sector in its own economy will increase while that of its partner decreases. We have the following proposition.

Proposition 3: *Under free trade, if firms in the differentiated good sector of a country are less heterogeneous, then the number of firms in this country will increase, while that of its trading partner decreases.*

Intuitively, when the degree of firm heterogeneity in a country is high, the gap between the prices charged by any two firms operating in the monopolistically competitive market is large. Thus, the gap of the market power is large. This hinders new entrants from entering the market. The aggregate price of the differentiated good will be high, and the comparative advantage (disadvantage) of the country as compared to its trading partner is low (high). When firms are less heterogeneous, the "monopoly" of each firm is pulled down, and the market gets closer to the "perfectly competitive" side. Thus, more firms can join to serve the market and intra-industry, inter-firm production reallocation occurs. More firms in the market also make the aggregate price of the differentiated good go down. This, in turn, implies that under free trade, a decrease in firm heterogeneity intensifies (mitigates) the comparative advantage (disadvantage) of that country over its trading partner. The inter-country production relocation is induced in favor of the country with the improved degree of heterogeneity. The number of firms in its trading partner will therefore decrease.

We are now interested in trade pattern and welfare effects of trade. We have proved in Appendix (f) that $P_{tf} > P_{th}$. In words, the aggregate price of the differentiated good produced by Home is lower than that of the differentiated good produced by Foreign. Furthermore, from (23), we have $\frac{E_{Dth}}{E_{Dtf}} = \left[\frac{P_{th}}{P_{tf}}\right]^{1-\sigma}$. Because $P_{tf} > P_{th}$ and $\sigma > 1$, $E_{Dth} > E_{Dtf}$: *A* larger share of the market income is spent on the differentiated good produced in Home, the country with lower degree of firm heterogeneity.

Which country will have a larger share of its labor endowment employed in the differentiated good sector? It is Home. We can prove this by calculating the difference in the labor force used in the differentiated good industry between the two countries. From (25), we have $L_{Dth} - L_{Dtf} = \omega^{-1}\left(E_{Dth} - E_{Dtf}\right) + \alpha\left(N_{th} - N_{tf}\right)$. We immediately see that $L_{Dth} > L_{Dtf}$ because $E_{Dth} > E_{Dtf}$ and $N_{th} > N_{tf}$. Home employs more labor in the differentiated good than Foreign does. This also implies that labor used in the homogeneous good sector in Home is less than that in Foreign $L_{Ath} < L_{Atf}$.

As far as the incomes of the two countries are concerned, the country with higher total profit will have higher income, because the two countries have the same labor income due to having the same labor endowments and wage rates. As shown in Appendix (g), $\Pi_{th} > \Pi_{tf}$. Therefore, $M_{th} > M_{tf}$. The country having lower degree of heterogeneity will earn more profit and have higher income under free trade.

We now compare the supply of and the demand for the homogenous good of the two countries. Because $L_{Ath} < L_{Atf}$, we have $A_{th}^S < A_{tf}^S$. Moreover, we can derive from (25) that $\frac{A_{th}}{A_{tf}} = \frac{M_{th}}{M_{tf}}$. $M_{th} > M_{tf}$ implies $A_{th} > A_{tf}$. From the homogenous good market clearance condition, we have $A_{th}^S - A_{th} = -\left(A_{tf}^S - A_{tf}\right)$. Hence, $A_{th}^S < A_{th}$ and $A_{tf}^S > A_{tf}$. This means that Home produces the homogenous good less than it needs. Home is the importer of the homogeneous good. According to Walras' Law, it is the net-exporter of the differentiated good. We summarize this into a proposition as follows.

Proposition 4: *Under free trade, the country with lower degree of firm heterogeneity is the net-exporter in the intra-industry trade of the differentiated good.*

We can say that the country with lower degree of firm heterogeneity have the comparative advantage in producing the differentiated good, because it produces the good at lower price than its partner ($P_{th} < P_{tf}$) .

Come to the analysis of welfare effect of trade. We do this by comparing the welfare of a country under free trade to that in autarky. Denote by V_j and V_{tj} the indirect utility of country j , $(j = h, f)$ in autarky and under free trade, respectively. Using the demand functions we have derived to replace into the utility of each country in autarky and free trade, we obtain $V_j = \nu M_j (P_j)^{-\mu}$ and $V_{tj} = \nu M_{tj} (P_t)^{-\mu}$, respectively, where $\nu \equiv (1-\mu)^{(1-\mu)} \mu^{\mu}$. Taking the ratio of the two and we get $\frac{V_j}{V_{tj}} = \frac{M_j}{M_{tj}} \left(\frac{P_j}{P_t} \right)^{-\mu}$. We can see from formulas of the indirect utility that the change in welfare when moving from autarky to free trade depends on changes in income and price index. In other words, the change in welfare is induced by the income effect and price effect.

We first examine the price effect. In order to do so, we observe the change in the number of firms with the transition to free trade. Because the equilibrium equation system is rather complicated, we rely on some simulations. The simulation result (as shown in Appendix (h) shows that $N_{th} > N_h$ and $N_{tf} < N_f$ as well as $M_{th} > M_h$, $M_{tf} < M_f$ and $M_t < M_h + M_f$. When moving from autarky, Home sees its number of firms in the differentiated good sector and total income increase while Foreign observes the opposite. Because $N_{th} > N_h$ and $N_{tf} < N_f$, then $P_{th} < P_h$ and $P_{tf} > P_f$. Moreover, we can immediately see from $P_t = \left(P_{th}^{1-\sigma} + P_{tf}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ that $P_t < P_{th} < P_{tf}$. Thus, $P_t < P_{th} < P_h < P_f < P_{tf}$, or $\left(\frac{P_h}{P_t} \right)^{-\mu} < 1$ and $\left(\frac{P_f}{P_t} \right)^{-\mu} < 1$.

In Home, due to $\frac{M_{th}}{M_h} > 1$ and $\left(\frac{P_h}{P_t} \right)^{-\mu} < 1$, thus $\frac{V_h}{V_{th}} < 1$ or $V_{th} > V_h$. Home enjoys an increase in welfare when opening to trade. The increase in welfare in Home is due to both the increase in the income and the decline in aggregate price the consumers in Home are facing.

In Foreign, consumers can also enjoy the decline in the aggregate price of the composite differentiated good as shown above. However, the income of this country decreases. To the total effect, our simulation shows that welfare in Foreign also increases after trade ($V_{tf} > V_f$, Appendix (h)). This implies that the price effect dominates the income effect in Foreign, making this country better off after trade.

Furthermore, $M_t < M_h + M_f$ shows that the income of the world, in terms of the homogeneous good, declines after trade. However, this also implies $N_t = N_{th} + N_{tf} > N_h + N_f$, based on (27) and (32). The world has more firms and more varieties of differentiated good in free trade than in autarky. That the two countries are better off after opening to trade while the total world income declines assures that the price effect dominates the income effect at the world level in this model. Therefore, it is worth going into more detail to elaborate these effects. Because the two countries have different levels of firm heterogeneity, when the world goes into trade, there occurs the reallocation of production within the differentiated good sector in each country and between countries. We have derived that the marginal cost of the marginal firm (cut-off point efficiency) of Home is the same as that of Foreign ($\beta_{h(N_{th}+1)} = \beta_{f(N_{tf}+1)}$) under free trade. In other words, trade has leveled off the cut-off point efficiency levels between the two countries. With $N_{th} > N_h$ and $N_{tf} < N_f$, we have $\beta_{h(N_{th}+1)} > \beta_{h(N_h+1)}$ and $\beta_{f(N_{tf}+1)} < \beta_{f(N_f+1)}$. Note that between two firms having the same rank, the firm in Home is more efficient than its counterpart in Foreign. These imply that trade induces more efficient firms in Home to enter and less efficient firms in Foreign to exit. Inter-country reallocation as well as within-country inter-firm reallocation both occur. Efficiency in the world level is enhanced, reflected in the decline of the aggregate price of the differentiated good. This effect does not prevail in the case of identical technology between two countries. Besides, in free trade, consumers can enjoy more varieties than in autarky. The number of varieties available to consumers is greater than the total number of firms of the two countries in autarky. This is also due to the efficiency effect via reallocations. Reallocations intensify the love-of-variety effect. With more varieties, consumers having love-of-variety preference will benefit from trade. In summary, the decline in the aggregate price of the differentiated good is due to the efficiency effect on the one hand and love-of-variety effect on the other hand. The love-of-variety effect in turn is induced by the increase in the mass of firm brought forth by not only that the consumers can now consume foreign varieties but also that the reallocations of production have given chance for more firms to enter. The reallocations can only occur in the existence of firm heterogeneity and difference of degree of firm heterogeneity.

We now summarize the analysis so far as a proposition as follows.

Proposition 5*: Both countries benefit from trade. Free trade world has more firms and enjoys more varieties of differentiated good than a closed one.*

IV. Conclusion

Patterns and effects of trade between asymmetric countries have been studied at the very beginning of the birth of economic science. However, as more firm-level data become available, trading countries are not considered as "black boxes" in trade theories anymore. Interaction between firms within the "black box" is also taken into account in modeling trade theories.

The primary purpose of our paper is to examine trade between asymmetric countries in terms of degree of firm heterogeneity and the effects of change in this degree on the evolution of industrial structure of trading countries, especially in the number of firms. The asymmetry between countries is treated characteristically in our model where countries are different in the relative productivity levels of firms. The asymmetry is relevant due to the difference in the exogenous economic background. Under monopolistic competition, the interaction between love-of-variety effect and efficiency effect induced by inter-firm as well as inter-country reallocations is shown to prevail. Our model is set up in such a manner to isolate the pure effect of degree of firm heterogeneity. The main results of our paper show that the country with lower degree of firm heterogeneity will be the net-exporter in the intra-industry trade, and have more firms and more varieties than its partner country. The country with lower heterogeneity extent also has more firms in free trade than in autarky, while the other country observes the opposite. The change in the heterogeneity degree of firms' productivity levels in a country will affect the numbers of firms in both countries. A decrease in the degree in a country leads to an increase in the number of firms and product varieties of its own, but to a decrease in those of the other country. Besides contributing to the efficiency effect, the production reallocations also intensify the love-of-variety effect, and the world enjoys even higher welfare in free trade. This paper contributes to the analysis of the effect of asymmetry between countries at firm level on the industrial reallocation and international trade with firm heterogeneity.

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APPENDIX

(a) Determination of the sign of $\frac{dN_j}{d\delta_j}$:

From the zero-profit condition, we have

$$\frac{\partial ZP_{aj}}{\partial N_j} dN_j + \frac{\partial ZP_{aj}}{\partial \theta_j} d\theta_j = 0 \quad \text{or} \quad \frac{dN_j}{d\theta_j} = -\frac{\partial ZP_{aj}}{\partial N_j} / \frac{\partial ZP_{aj}}{\partial \theta_j}.$$

We calculate

$$\frac{\partial ZP_{aj}}{\partial N_j} = \frac{1}{\sigma}\left[S_{ajm}\frac{\partial Y_{aj}}{\partial N_j} + Y_{aj}\frac{\partial S_{ajm}}{\partial N_j}\right] \text{ and } \frac{\partial ZP_{aj}}{\partial \theta_j} = \frac{1}{\sigma}\left[S_{ajm}\frac{\partial Y_{aj}}{\partial \theta_j} + Y_{aj}\frac{\partial S_{ajm}}{\partial \theta_j}\right].$$

Partially differentiate S_{ajm} and Y_{aj} with respect to N_j and θ_j, we obtain

$$\frac{\partial S_{ajm}}{\partial N_j} = \theta_j \frac{[(\theta_j - 1)(N_j+1)^{\theta_j - 2}[(N_j+1)^{\theta_j} - 1] - \theta_j (N_j+1)^{\theta_j - 1}(N_j+1)^{\theta_j - 1}}{[(N_j+1)^{\theta_j} - 1]^2}$$

$$= \frac{\theta_j - 1}{(N_j+1)} S_{ajm} - S_{ajm}^2 < 0 \text{ due to } \theta_j \le 1 \text{ and } S_{ajm} \ge 0;$$

$$\frac{\partial Y_{aj}}{\partial N_j} = -\frac{\sigma\mu\alpha}{\sigma - \mu} < 0 \text{ due to } \sigma > 1, 0 < \mu < 0; \text{and } \alpha > 0;$$

$$\frac{\partial S_{ajm}}{\partial \theta_j} = \frac{(N_j+1)^{\theta_j - 1}[(N_j+1)^{\theta_j} - \theta_j \ln(N_j+1) - 1)]}{[(N_j+1)^{\theta_j} - 1]^2} > 0$$

due to $[(N_j+1)^{\theta_j} - \theta_j \ln(N_j+1) - 1)] \ge 0 \; \forall N_j \ge 0, \theta_j \le 1; (N_j+1)^{\theta_j - 1} \ge 1$ and $[(N_j+1)^{\theta_j} - 1]^2 \ge 0$

and

$$\frac{\partial Y_{aj}}{\partial \theta_j} = 0.$$

Therefore, we have

$$\frac{\partial ZP_{aj}}{\partial \theta_j} = \frac{1}{\sigma}\left[S_{ajm}\frac{\partial Y_{aj}}{\partial \theta_j} + Y_{aj}\frac{\partial S_{ajm}}{\partial \theta_j}\right] > 0$$

$$\frac{\partial ZP_{aj}}{\partial N_j} = \frac{1}{\sigma}\left[S_{ajm}\frac{\partial Y_{aj}}{\partial N_j} + Y_{aj}\frac{\partial S_{ajm}}{\partial N_j}\right] < 0 \quad .$$

Thus, we obtain

$$\frac{dN_j}{d\theta_j} = -\frac{\partial ZP_{aj}}{\partial N_j} / \frac{\partial ZP_{aj}}{\partial \theta_j} > 0 .$$

From $\theta_j \equiv \delta_j(1-\sigma)+1$, we have $\dfrac{d\theta_j}{d\delta_j} = (1-\sigma) < 0$.

Therefore, $\dfrac{dN_j}{d\theta_j} > 0$ implies $\dfrac{dN_j}{d\delta_j} < 0$.

(b) Determination of the sign of $\dfrac{dP_j}{d\delta_j}$:

From (10) , we have

$$\frac{\partial P_j}{\partial N_j} = \omega\phi_j \frac{1}{1-\sigma}(N_j+1)^{\theta_j-1}\left[\frac{(N_j+1)^{\theta_j}-1}{\theta_j}\right]^{\frac{1}{1-\sigma}-1} < 0 ,$$

and $$\frac{\partial P_j}{\partial \delta_j} = \omega\phi_j \left[\frac{\theta_j(N_j+1)^{\theta_j}\ln(N_j+1)-(N_j+1)^{\theta_j}+1}{\theta_j}\right]\left[\frac{(N_j+1)^{\theta_j}-1}{\theta_j}\right]^{\frac{1}{1-\sigma}-1} \geq 0$$

due to $\dfrac{\theta_j(N_j+1)^{\theta_j}\ln(N_j+1)-(N_j+1)^{\theta_j}+1}{\theta_j} \geq 0 \ \forall N_j \geq 0; \theta_j \leq 1; \theta_j \neq 0$.

From Appendix (a), we have $\dfrac{dN_j}{d\delta_j} < 0$.

Thus, $\dfrac{dP_j}{d\delta_j} = \dfrac{\partial P_j}{\partial N_j}\dfrac{dN_j}{d\delta_j} + \dfrac{\partial P_j}{\partial \delta_j} > 0$.

(c) Determination of the signs of $\dfrac{dN_{tj}}{d\delta_{\bar{j}}}$ and $\dfrac{dN_{t\bar{j}}}{d\delta_{\bar{j}}}$, keeping other factors, including δ_j unchanged:

Applying the Implicit Function Theorem to the zero-profit condition, we have

$$\sum_{k=j,\bar{j}} \frac{\partial ZP_{tj}}{\partial N_{tk}} dN_{tk} + \frac{\partial ZP_{tj}}{\partial \theta_{\bar{j}}} d\theta_{\bar{j}} = 0 \quad , \quad j,\bar{j} = h,f; j \neq \bar{j} \quad \text{with } d\theta_{\bar{j}} = (1-\sigma)d\delta_{\bar{j}} < 0 .$$

This can be rewritten in the matrix form as $AX = B$
where

$$A \equiv \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \equiv \begin{pmatrix} \dfrac{\partial ZP_{tj}}{\partial N_{tj}} & \dfrac{\partial ZP_{tj}}{\partial N_{t\bar{j}}} \\ \dfrac{\partial ZP_{t\bar{j}}}{\partial N_{tj}} & \dfrac{\partial ZP_{t\bar{j}}}{\partial N_{t\bar{j}}} \end{pmatrix};\ \mathrm{X} \equiv \begin{pmatrix} \dfrac{dN_{tj}}{d\theta_{\bar{j}}} \\ \dfrac{dN_{t\bar{j}}}{d\theta_{\bar{j}}} \end{pmatrix};\ \mathrm{B} \equiv \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \equiv \begin{pmatrix} -\dfrac{\partial ZP_{tj}}{\partial \theta_{\bar{j}}} \\ -\dfrac{\partial ZP_{t\bar{j}}}{\partial \theta_{\bar{j}}} \end{pmatrix}.$$

$$\frac{dN_{tj}}{d\theta_{\bar{j}}} = \frac{1}{|A|}\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11}a_{22} - a_{21}a_{12}}$$

$$\frac{dN_{t\bar{j}}}{d\theta_{\bar{j}}} = \frac{1}{|A|}\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = \frac{b_2 a_{11} - b_1 a_{21}}{a_{11}a_{22} - a_{21}a_{12}}.$$

We calculate all the derivatives as follows.

$$\frac{\partial Y_t}{\partial \theta_{\bar{j}}} = 0, \quad \frac{\partial Y_t}{\partial N_{tj}} = -\frac{\mu\alpha}{\sigma - \mu} < 0; \quad \frac{\partial Y_t}{\partial N_{t\bar{j}}} = -\frac{\sigma\mu\alpha}{\sigma - \mu} < 0;$$

$$\frac{\partial S_{tjm}}{\partial N_{tj}} = \frac{\theta_j - 1}{(N_{tj}+1)} S_{tjm} - S_{tjm}^2 \leq 0;$$

$$\frac{\partial S_{tjm}}{\partial N_{t\bar{j}}} = -S_{tjm} S_{t\bar{j}m} \leq 0;$$

$$\frac{\partial S_{tjm}}{\partial \theta_{\bar{j}}} = -S_{tjm} \frac{\left(\partial \sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right) / \partial \theta_{\bar{j}}}{\left(\sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right)} \leq 0$$

due to

$$\frac{\left(\partial \sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right)}{\partial \theta_{\bar{j}}} = \frac{\theta_{\bar{j}} (N_{t\bar{j}}+1)^{\theta_{\bar{j}}} \ln(N_{t\bar{j}}+1) - [(N_{t\bar{j}}+1)^{\theta_{\bar{j}}} - 1]}{\theta_{\bar{j}}^2} \geq 0 \; \forall N_{tj} \geq 0; \theta_{\bar{j}} \leq 1; \theta_{\bar{j}} \neq 0;$$

$$\frac{\partial S_{t\bar{j}m}}{\partial N_{tj}} = -S_{t\bar{j}m} S_{tjm} \leq 0;$$

$$\frac{\partial S_{t\bar{j}m}}{\partial N_{t\bar{j}}} = \frac{\theta_{\bar{j}} - 1}{(N_{t\bar{j}}+1)} S_{t\bar{j}m} - S_{t\bar{j}m}^2 \leq 0;$$

$$\frac{\partial S_{t\bar{j}m}}{\partial \theta_{\bar{j}}} = S_{t\bar{j}m} \ln(N_{t\bar{j}}+1) - S_{t\bar{j}m} \frac{\left(\partial \sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right) / \partial \theta_{\bar{j}}}{\left(\sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right)}$$

$$= S_{t\bar{j}m} \left(\frac{\ln(N_{t\bar{j}}+1)\left(\sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right) - \frac{\theta_{\bar{j}} (N_{t\bar{j}}+1)^{\theta_{\bar{j}}} \ln(N_{t\bar{j}}+1) - [(N_{t\bar{j}}+1)^{\theta_{\bar{j}}} - 1]}{\theta_{\bar{j}}^2}}{\left(\sum_{k=j,\bar{j}} \frac{(N_{tk}+1)^{\theta_k} - 1}{\theta_k}\right)} \right)$$

$$= S_{t\bar{j}m}\left(\frac{\frac{(N_{tj}+1)^{\theta_j}-1}{\theta_j}\ln(N_{t\bar{j}}+1)+\frac{(N_{t\bar{j}}+1)^{\theta_{\bar{j}}}-\theta_{\bar{j}}\ln(N_{t\bar{j}}+1)-1}{\theta_{\bar{j}}^2}}{\left(\sum_{k=j,\bar{j}}\frac{(N_{tk}+1)^{\theta_k}-1}{\theta_k}\right)}\right)$$

Since $\frac{(N_{t\bar{j}}+1)^{\theta_{\bar{j}}}-\theta_{\bar{j}}\ln(N_{t\bar{j}}+1)-1}{\theta_{\bar{j}}^2}\geq 0\ \ \forall \mathrm{N}_{t\bar{j}}\geq 0,\theta_{\bar{j}}\leq 1$, we can immediately prove that

$$\frac{\partial S_{t\bar{j}m}}{\partial\theta_{\bar{j}}}\geq 0\,;$$

Therefore, we have the following sign patterns.

$$a_{11}=Y_t\frac{\partial S_{tjm}}{\partial N_{tj}}+S_{tjm}\frac{\partial Y_t}{\partial N_{tj}}<0\,;\ \ a_{21}=Y_t\frac{\partial S_{t\bar{j}m}}{\partial N_{tj}}+S_{t\bar{j}m}\frac{\partial Y_t}{\partial N_{tj}}<0\,;$$

$$a_{12}=Y_t\frac{\partial S_{tjm}}{\partial N_{t\bar{j}}}+S_{tjm}\frac{\partial Y_t}{\partial N_{t\bar{j}}}<0\,;\ \ a_{22}=Y_t\frac{\partial S_{t\bar{j}m}}{\partial N_{t\bar{j}}}+S_{t\bar{j}m}\frac{\partial Y_t}{\partial N_{t\bar{j}}}<0\,;$$

$$b_1=-\left(Y_t\frac{\partial S_{tjm}}{\partial\theta_{\bar{j}}}+S_{tjm}\frac{\partial Y_t}{\partial\theta_{\bar{j}}}\right)=-Y_t\frac{\partial S_{tjm}}{\partial\theta_{\bar{j}}}\geq 0\,;\ \ b_2=-\left(Y_t\frac{\partial S_{t\bar{j}m}}{\partial\theta_{\bar{j}}}+S_{t\bar{j}m}\frac{\partial Y_t}{\partial\theta_{\bar{j}}}\right)=-Y_t\frac{\partial S_{t\bar{j}m}}{\partial\theta_{\bar{j}}}\leq 0\,;$$

Because $S_{tjm}=S_{t\bar{j}m}$, we have

$$a_{11}\leq a_{21}<0\ \text{ and }\ a_{22}\leq a_{12}<0\,.$$

Then

$$|A|=a_{11}a_{22}-a_{21}a_{12}>0$$

$$b_1a_{22}-b_2a_{12}<0$$

and

$$b_2a_{11}-b_1a_{21}>0$$

with an assumption that $|A|\neq 0$ for the equation system to have solutions.

Therefore, we determine as

$$\frac{dN_{tj}}{d\theta_{\bar{j}}}<0\ \text{ and }\ \frac{dN_{t\bar{j}}}{d\theta_{\bar{j}}}>0$$

or

$$\frac{dN_{tj}}{d\delta_{\bar{j}}}>0\ \text{ and }\ \frac{dN_{t\bar{j}}}{d\delta_{\bar{j}}}<0\,.$$

(d) The movement along allocation curves:

In country $j,(j=h,f)$: $\dfrac{dN_{tj}}{dN_{t\bar{j}}} = -\dfrac{\partial ZP_{tj}}{\partial N_{t\bar{j}}} / \dfrac{\partial ZP_{tj}}{\partial N_{tj}} = -\dfrac{a_{12}}{a_{11}} < 0$, where $\bar{j}=h,f;\bar{j}\neq j$. This means that, the numbers of firms of the two countries negatively and monotonically relate to each other along the allocation curve of a country.

(e) Determination of the sign of $\dfrac{\partial D_{tj(N_{tj}+1)}}{\partial N_{t\bar{j}}}$:

We have $\dfrac{\partial D_{tj(N_{tj}+1)}}{\partial N_{t\bar{j}}} = \left[Y_t \dfrac{\partial S_{tjm}}{\partial N_{t\bar{j}}} + S_{tjm} \dfrac{\partial Y_t}{\partial N_{t\bar{j}}} \right] \left(P_{j(N_{tj}+1)} \right)^{-1} < 0$

due to $\dfrac{\partial S_{tjm}}{\partial N_{t\bar{j}}} = -S_{tjm} S_{t\bar{j}m} \le 0$ and $\dfrac{\partial Y_t}{\partial N_{t\bar{j}}} = -\dfrac{\sigma\mu\alpha}{\sigma-\mu} < 0$.

(f) Proof of $P_{tf} > P_{th}$:

$$P_{tj} = \omega\phi_j \left[\frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j} \right]^{1/(1-\sigma)}$$

$$\frac{\partial P_{tj}}{\partial N_{tj}} = \omega\phi_j \frac{1}{1-\sigma} (N_{tj}+1)^{\theta_j - 1} \left[\frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j} \right]^{\sigma/(1-\sigma)} < 0$$

$$\frac{\partial P_{tj}}{\partial \theta_j} = \omega\phi_j \frac{1}{1-\sigma} \frac{\theta_j (N_{tj}+1)^{\theta_j} \ln(N_{tj}+1) - [(N_{tj}+1)^{\theta_j} - 1]}{\theta_j^2} \left[\frac{(N_{tj}+1)^{\theta_j} - 1}{\theta_j} \right]^{\sigma/(1-\sigma)} < 0 \quad \text{or} \quad \frac{\partial P_{tj}}{\partial \sigma_j} > 0.$$

Due to $\phi_f = \phi_h, \delta_f > \delta_h$ and $N_{tf} < N_{th}$, we have $P_{tf} > P_{th}$.

(g) Proof of $\Pi_{th} > \Pi_{tf}$:

Due to $N_{th} > N_{tf}$, we have $\Pi_{th} = \sum_1^{N_{tf}+1} \Pi_{thi} + \sum_{N_{tf}+1}^{N_{th}+1} \Pi_{thi}$ and $\Pi_{tf} = \sum_1^{N_{tf}+1} \Pi_{tfi}$.

It is known that $\sum_1^{N_{tf}+1} \Pi_{thi} > \sum_1^{N_{tf}+1} \Pi_{tfi}$ because of the fact that the firm in Home that has the same ranking as the firm in Foreign has lower marginal cost, therefore has higher profit when they compete in the monopolistically competitive world integrated market of the differentiated good. Furthermore, $\sum_{N_{tf}+1}^{N_{th}+1} \Pi_{thi} > 0$. Therefore, $\Pi_{th} > \Pi_{tf}$.

(h) Simulations have been taken with several possible sets of parameters. The results are the same as those in the following numeric solutions. ($\overline{L}_h = \overline{L}_f = 400$; $\alpha = 1, \phi_h = \phi_h = 1; \delta_h = 0.2; \mu = 0.25; \sigma = 2.1$)

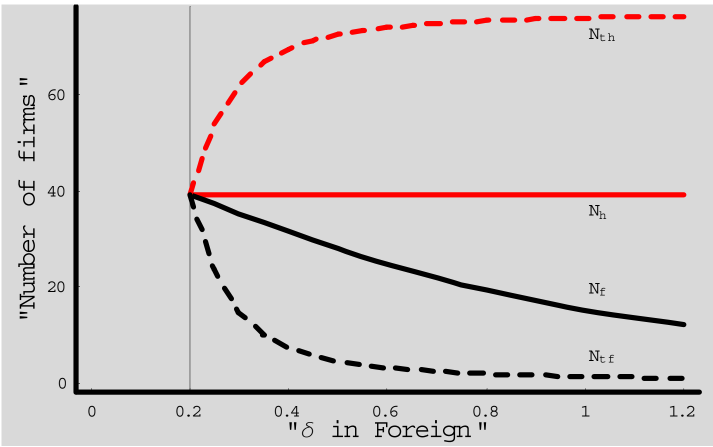

Figure 2: $N_{th} > N_h$ and $N_{tf} < N_f$.

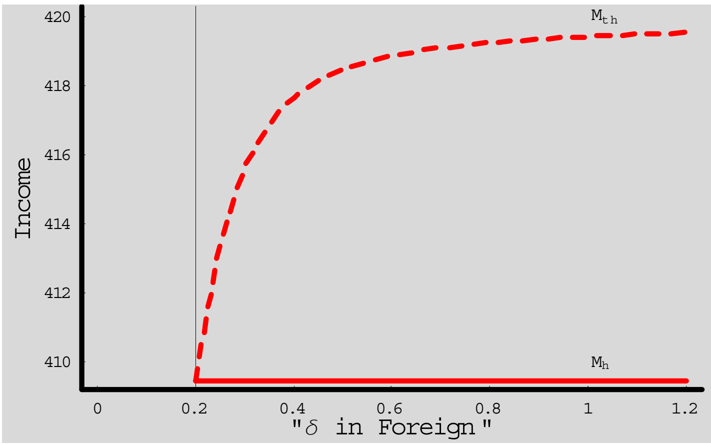

Figure 3: $M_{th} > M_h$

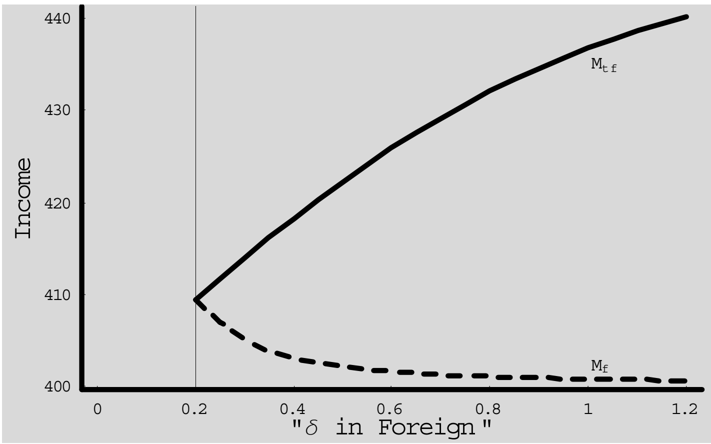

Figure 4: $M_{tf} < M_f$

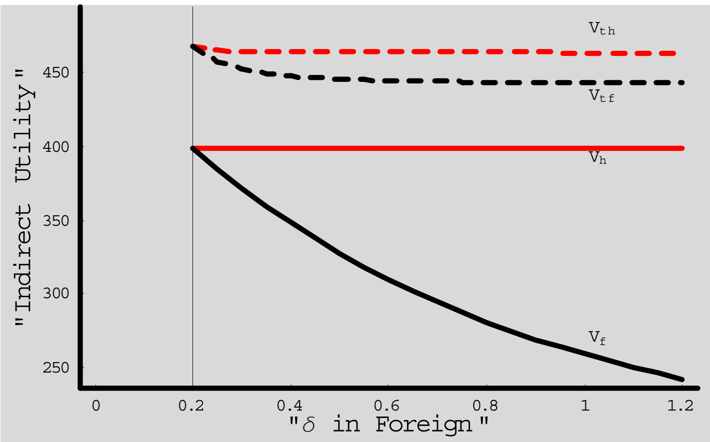

Figure 5: $V_h < V_{th}$ and $V_f < V_{tf}$

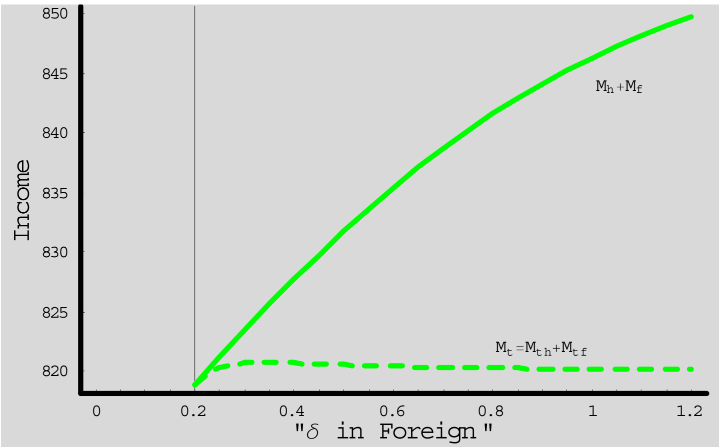

Figure 6: $M_t < M_h + M_f$