Consumption Sustainability in Resource Economies under Price Uncertainty

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I. INTRODUCTION

Environmental sustainability has revived development economists' interests in resource economies. Contrary to the 1970s when optimal resource extraction paths were the main theme of analysis, sustainability arguments ask the possibility of keeping aggregate consumption constant forever in resource economies. An earlier contribution to the question is found in Hartwick (1977) where a sufficient condition for the constancy of consumption (or utility levels) is given for a closed economy subject to a stationary population and technology. An economy that reinvests in reproducible capital the exact amount of competitive rents from its current extraction of exhaustible resources will enjoy constant consumption stream forever. This has been called the Hartwick rule.

The rule has been investigated in much more elaborated fashion afterwards. Long and Hartwick (1996) deal with non-autonomous case in which the current profit is time-dependent. Their analysis includes both exogenous technological change (a la, Kemp and Long (1982), and Weitzman (1995)) and the case of endogenous changes in the terms of trade for open economies (Kemp and Long (1995), Vincent, Panayotou and Hartwick (1997)). Deriving the expressions for economic depreciation, they show that if the rate of interest is time independent, constant consumption paths can be obtained by investing a sum equal to the economic depreciation into a sinking fund.

On the other hand, Svensson (1986) puts the argument in open economies. Intertemporal transactions among countries with lending and borrowing imply that consumption need not equal to the domestic output of the consumption good at each point

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in time. In particular, for a small open economy, her consumption and investment decisions can be separated, with a given interest rate in the world capital market.

When an economy is open to international trade and behaves as a price taker, capital gains (or losses) must be taken into account to discuss the sustainability of consumption. It is not enough to count the return on its domestic resource since the country's net national wealth must be augmented by the return from the past investment abroad. Asheim (1986, 1996) and Hartwick (1995) have suggested modifications to the original Hartwick rule for open economy cases. Taking note of the concern on open economies, Vencent, et.al. (1997) put stress on the exogeneity of world resource prices. It is natural to consider, as they do, that price-taking resource exporting countries have experienced capital gains (or losses) in their exporting activities of natural resources. More importantly, however, prices of internationally traded natural resources are often greatly fluctuating. Thus, volatility and uncertainty are the significant factors for the sustainability of open economies. Indeed, Vincent et. al. (1997) examine a price-taking exporter of non-renewable resources and succeed in deriving a modified sinking fund rule. The modification element is interpretable as the net present value of future terms-of-trade effect.

Although Vincent et. al. (1997) have stressed the importance of volatility of the given world price, the stream of prices in the future is perfectly foreseen in their model. Otherwise, the modification factor in their model could not be calculated for deriving how much the economy should invest at each moment of time. In this sense their analysis is made under perfect foresight and more basic feature of the volatility is assumed away. Departing from perfect foresight, Katayama and Ohta (1999) explicitly examine future price uncertainty. The resource price at present time is certainly known by planners, but it turns uncertain a moment away. They assume, moreover, that the degree of volatility increases as time elapses. We are less certain to predict events in distant future than those on tomorrow. Using a geometric Brownian motion on the price movements, Katayama and Ohta (1999) have shown that there are two ways of modifying Hartwick's rule in the presence of capital gains or losses. They are equivalent in deriving a constant level of expected consumption for the economy. One of them is similar to the formula suggested by Vincent, et.al. (1997). However, it is not appropriate to use it, if the perfect foresight does not hold.

In the present paper we extend the analysis by Katayama and Ohta (1999) to include renewable resources. It may not be surprising to see that the Hartwick rule has so far been examined in the context of exhaustible resources. It seems a lot easier to secure the constancy of consumption when resources are renewable than they are not. With ample opportunities of replenishment in natural resources, countries may have greater degree of intergenerational equity. However, the present paper suggests that under the same rule of investment as under exhaustible resource the average level of national consumption is *decreasing* over time when renewable resource stock is relatively *large* rather than small. Moreover, it is not enough to eliminate the effect of capital gains or losses to arrive at the constant consumption, even if the resource price is the only source of volatility in the model. We maintain that the constancy of consumption may be less rather than more sustainable with renewable resources as far as the same investment rule under exhaustible resource is applied. Indeed, Hartwick (1978) has suggested a modified savings-investment rule for renewable resources to guarantee a constant consumption path. Our task in the present paper is to articulate the role of non-exhaustibility of resources for the validity of the original Hartwick rule.

An economy with single source of income out of its natural resource, which is vulnerable to the world price fluctuation, should be alert to the nature of the resource before establishing a savings-investment rule to stabilize its national consumption.

The paper is organized as follows. Section 2 constructs a model of natural resource dynamics with price uncertainty. It is a standard one for resource economy and covers renewable as well as exhaustible resources. Section 3 reexamines the validity of Hartwick's rule on exhaustible resources and articulates the difference in the conditions for constant consumption between the two types of resources. Section 4 is to reconcile the argument by modifying the investment rule referring to Hartwick (1978) for depleting renewable resources. The final section is for concluding remarks.

Ⅱ. THE MODEL

The model built in the present section is very much of a standard one. The natural resource is augmentative by intrinsic or biological power but reproduction may halt when the resource stock reaches too large. If we denote by $R(t)$ the stock level of the resource at time t, reproduction rate at t is written by $S(R(t))$. Then, it is natural to assume that $S(0) \leqq 0$, $S'(0) > 0$, $S''(R) < 0$ for all $R \geqq 0$, and there exists a stock level $\tilde{R} > 0$ such that $S'(\tilde{R}) = 0$. $\tilde{R}$ is sometime regarded as the golden rule level of stock since it represents the maximum sustainable yield from the natural resource. (See, for example, Dasgupta and Heal (1979).)

Except the above formulation of the resource, the basic structure of the model is similar to Katayama and Ohta (1999). We consider a single small country specializing in extracting and exporting natural resource.[1] The resource price in the trading world is subject to uncertainty. The population of the country is constant and technological progress is absent either for extraction of resources or investment activities in extracted capital. Cost function for the country to extract or harvest an amount of q out of the resource field is denoted by $c(q, R)$. Throughout the paper we use "production" and "harvest" synonymously to express the level of extraction from the resource field in question. It is assumed that $c_q(q, R) > 0$ and $c_R(q, R) \leqq 0$ where subscripts signify partial derivatives of the function by respective arguments. The former inequality implies usual positive marginal cost of production, while the latter shows "stock externality" (see, Smith(1975)) if the inequality is strict. It is often the case in resource production that the more we have, the easier the excavation turns out to be. Resource owners invest abroad part of the net return (resource rent) from the current (say) fish catch under certain constant interest rate ρ.[2] Hence, current consumption is financed out of the remaining resource rents and interest payments from the past investment accumulated abroad.

The resource owner takes a competitive extraction policy and exports the totality of the production under a given world price p. It is assumed here that the price obeys a Wiener process:

$$dp(t) = \alpha p(t)dt + \sigma p(t)dz. \qquad (1)$$

Here dz is the increment of a stochastic process z that obeys the Wiener process.[3] The expected change in p is α, a constant, but there is a disturbance expressed in the second term with a constant $\sigma \geqq 0$. The first term represents the deterministic drift part and the second the diffusion effects. It is assumed that dz independently and normally distributes with mean zero and variance dt : $dz(t) \sim N(0, dt)$, $\forall t$. Notice that with this formula of the price it never become negative.

The resource owner's problem is formulated as

$$\max_{q(t)} E_0 \int_0^{\infty} [p(t)\,q(t) - c(q(t), R(t))] e^{-\rho t} dt \qquad (2)$$

$$\text{s. t.} \qquad dp(t) = \alpha p(t) dt + \sigma p(t) dz\,; p(0) = p_0 > 0 \qquad (3)$$

$$dR(t) = S(R(t)) dt - q(t) dt\,; R(0) = R_0 > 0 \qquad (4)$$

$$q(t) \geqq 0 \text{ and } R(t) \geqq 0.$$

E_t is an expectation operator at time t. The resource owner extracts part of its reserve and exports the harvest under uncertain international market price so as to maximize its expected net cash flow. Following the usual practice, we define the optimal value function for this problem by

$$J(p(t), R(t), t) \equiv \max_{q(t)} E_t \int_t^{\infty} [pq - c(q, R)] e^{-\rho\tau} d\tau \qquad (5)$$

$$= \max_{q(t)} E_t \left\{ \int_t^{t+dt} [pq - c(q, R)] e^{-\rho\tau} d\tau + J(p(t+dt), R(t+dt), t+dt) \right\}.$$

Assuming that J is twice continuously differentiable and by applying Ito's lemma on J, we obtain the following Hamilton-Jacobi-Bellman equation (H-J-B).

$$0 = \max_{q(t)} \left\{ [pq - c(q, R)] e^{-pt} + J_t + \alpha p J_p + \frac{1}{2} \sigma^2 p^2 J_{pp} + [S(R(\mathrm{t})) - q] J_R \right\} \qquad \text{(H-J-B)}$$

The first order condition for the optimization is

$$[p - c_q(q, R)] e^{-\rho t} = J_R. \qquad (6)$$

The left-hand side is the present value of the incremental profit that can be obtained by exporting an additional unit of the resource. It should be equal to the shadow price of the resource expressed by the right-hand side. Equation (6) implies that the optimal level of extraction, q^*, is a function of the resource price p and the existing resource stock level R at each moment of time:

$$q^*(t) = q^*(p(t), R(\mathrm{t}), t). \qquad (7)$$

Substituting (6) back into (H-J-B) yields a partial differential equation for $J(p, R, t)$.

To obtain explicitly the optimal extraction trajectory $q(t)^*$ we need to solve the resulting partial differential equation of second-order on J. Usually, however, it is difficult to solve it. Instead, in the following we examine the movements of variables in their expected levels. In this sense we depart from existing literature on Hartwick rules which has concentrated on non stochastic world.

From (H-J-B) and the first order condition (see, Appendix 1 below), we can readily show that

$$\frac{1}{dt}E_t dJ_R = \left\{c_R - S'(R)(p-c_q)\right\}e^{-\rho t} \tag{8}$$

where $\frac{1}{dt}E_t d(\quad)$ is Ito's differential operator.

From the first order condition (6) it is also derived by differentiation that

$$-\rho[p-c_q(q,R)]e^{-\rho t}+\frac{1}{dt}E_t[d(p-c_q)]e^{-\rho t}=\frac{1}{dt}E_t dJ_R. \tag{9}$$

Combining (8) and (9), we have

$$\frac{1}{dt}E_t[d(p-c_q)]=(\rho-S'(R))(p-c_q)+c_R. \tag{10}$$

Equation (10) can be regarded as the generalized Hotelling rule for the optimal production of renewable resources with stock externality. It reduces to the original Hotelling r-percent (or rather, in the present paper, ρ -percent) rule when the resource is non-renewable and production cost is independent from the resource stock.

$$\frac{\frac{1}{dt}E_t[d(p-c_q)]}{p-c_q}=\rho-S'(R)+\frac{c_R}{p-c_q} \tag{11}$$

Since $p-c_q>0$ under positive resource rent and $c_R<0$ if the stock externality presents, the difference between resource price and marginal cost changes less rapidly in the present model than without stock externality. The rate of change even becomes negative if the own rate of return of the resource, $S'(R)$, is larger than the discount rate of the economy, ρ. When the resource is scarce, the resource rent decreases and it is not wise to exploit it further.

Now we expand dc_q to obtain the expression for the optimal level of harvest.

$$dc_q=c_{qq}dq+\frac{1}{2}c_{qqq}(dq)^2+c_{qR}dR+c_{qqR}dRdq \tag{12}$$

Since q is a function of p, R, and t, and p obeys a stochastic process, we have by Ito's lemma that

$$dq = q_p dp + \frac{1}{2} q_{pp} (dq)^2 + q_R dR + q_t dt. \tag{13}$$

Using (1) and (4),

$$E_t (dq)^2 = \sigma^2 p^2 q_p^2 dt + o(dt) \tag{14}$$

where $o(dt)$ represents terms that vanish as dt tends to zero.
From (12)-(14) we have

$$\frac{1}{dt} E_t dc_q = c_{qq} \frac{1}{dt} E_t dq + \frac{1}{2} \sigma^2 p^2 q_p^2 c_{qqq} + (S-q) c_{qR}. \tag{15}$$

The dynamics of production is now determined with the aid of (15) and the generalized Hotelling rule, (10).

$$\frac{1}{dt} E_t dq = \left\{ \alpha p - (\rho - S'(R))(p - c_q) - c_R - \frac{1}{2} \sigma^2 p^2 q_p^2 c_{qqq} - (S-q) c_{qR} \right\} \Big/ c_{qq} \tag{16}$$

It is difficult to determine the sign of the right-hand side in general. However, one important property of the expected level of optimal production is that uncertainty affects only when the cost function is higher order than quadratic in production level. It is because that σ appears only in the forth term in the numerator of the right-hand side in (16). Our conclusion here is that the optimal level of harvest is neither monotone increasing nor decreasing even in its expected sense. The possibility of stationary policy of $(1/dt)E_t dq = 0$ is even further remote.

Ⅲ. HARTWICK'S RULE AND ITS MODIFICATION

Suppose that the economy applies Hartwick's rule for exhaustible resources. It should invest the margin of the resource price over marginal cost of extraction for each unit of the resource dug out. In other words the resource rent $H = (p - c_q)q$ is invested abroad and the economy will receive in turn its earnings for consumption.

$$H(p(t), R(t), t) \equiv (p(t) - c_q(q, R)) q(t) \tag{17}$$

Since the economy has no other source of income except the export revenue and the investment return, the current aggregate consumption $G(t)$ is described by

$$G(t)=p(t)q(t)-c(q,R)-H+\rho\int_0^t H(\tau)d\tau. \tag{18}$$

The last term in the right-hand side indicates the earnings from the past investment, assuming the interest rate is constant and equal to the rate of discount.

Applying Ito's lemma on the investment rule (17) we have

$$dH=qdp+\frac{1}{2}q_p(dp)^2-qc_{qq}dq-\frac{1}{2}qc_{qqq}(dq)^2-\frac{1}{2}c_{qq}(dq)^2-qc_{qR}dR$$
$$+\left\{p-c_q(q,R)\right\}dq-\frac{1}{2}c_{qq}(dq)^2 \tag{19}$$

Therefore, the current increment in the consumption becomes

$$dG(t)=qdp+\frac{1}{2}q_p(dp)^2+(p-c_q)dq-\frac{1}{2}c_{qq}(dq)^2-c_RdR-dH+\rho Hdt$$
$$=-c_RdR+qc_{qq}dq+\frac{1}{2}qc_{qqq}(dq)^2+\frac{1}{2}c_{qq}(dq)^2+qc_{qR}dR+\rho(p-c_q)qdt. \tag{20}$$

Taking expectation, dividing by dt and using (16) derived in the previous section, we have

$$\frac{1}{dt}E_tdG=\alpha pq+\frac{1}{2}c_{qq}\sigma^2p^2q_p^2-Sc_R+S'(p-c_q)q. \tag{21}$$

The first term in the right-hand side represents capital gains (if $\alpha>0$) or losses ($\alpha<0$). The effect of uncertainty appears in the second term. Notice that the uncertainty does not matter if the cost function is linear in production level. The rest of the right-hand side results from the fact that the resource is renewable in the present model rather than purely exhaustible. At this stage there is no possibility to secure the constancy of national consumption.

To eliminate the first term, let us apply the same formula as in Katayama and Ohta (1999). We add a second term to (17) in order to adjust price changes in the Hartwick rule.[4]

$$\hat{H}(p(t),R(t),t)=(p(t)-c_q(q,R))q(t)+\int_0^t q(p(\tau),R(\tau),\tau)e^{-\rho(\tau-t)}dp(\tau) \tag{22}$$

The second term in the right-hand side captures the discounted sum of the terms-of-trade changes. It is the net present value of all the past terms-of-trade shifts up to the present point in time.

Accordingly, the current consumption possibility is modified as

$$\hat{G}(t)=p(t)q(t)-c(q(t),R(t))-\hat{H}+\rho\int_0^t\hat{H}d\tau. \tag{23}$$

Apply Ito's lemma, take expectation, divide by dt, use equation (17), and find that

$$\frac{1}{dt}E_t d\hat{G}=\frac{1}{2}c_{qq}\sigma^2p^2q_p^2-Sc_R+S'(p-c_q)q. \tag{24}$$

Appendix 2 shows the derivation in detail.

Now, equation (24) represents major findings of the paper. First of all, as we have already noticed, the price uncertainty matters only when the cost function is nonlinear. Recall that in the previous section the uncertainty affects the optimal path of the resource harvest only if the cost is of higher order than quadratic. Therefore, with quadratic cost the average consumption is affected by price uncertainty even if the average production is not. Consumption is more vulnerable to price volatility than production under the Hartwick investment rule. Secondly, if the economy has nonlinear cost, the more volatile the price is, the more adverse effect appears against the constancy of consumption. In either case of $c_{qq}>0$ or $c_{qq}<0$, the first term in (24) becomes larger in absolute value under the same level of resource production when σ increases. Since the purpose of Hartwick's rule is to guarantee the economy a constant level of consumption, we can conclude that there is no "gains from uncertainty" in the resource economy. Thirdly, suppose a linear cost (or absence of uncertainty), and thus (24) becomes

$$\frac{1}{dt}E_t d\hat{G}=-Sc_R+S'(p-c_q), \tag{25}$$

the right-hand side of which is positive if $R\leq\tilde{R}$, where $\tilde{R}$ is such that $S'(\tilde{R})=0$. It implies that the Hartwick rule in its original sense tends to give an *increasing* average consumption when the stock level of renewable resource is relatively *small.* This somewhat counter-intuitive result may be interpreted as follows. Since we assume $c_R<0$, a smaller stock means a smaller stock externality in production cost. In other words, the production becomes costlier and the resource rent to invest becomes smaller in turn. Hartwick's rule, then, underestimates the investment amount that is necessary to secure a constant national consumption. As the result, the consumption tends to increase over time. Therefore, we can conclude that the Hartwick rule works properly only when the economy with stock externality has sufficiently large stock of renewable resources. Finally, if the stock externality is assumed away, the first term in (25) disappears. The right hand side becomes zero, the target level of the rule, when $R=\tilde{R}$ It is the golden rule stock level in the sense that it is feasible and maximum amount of harvest out of the resource field. However, there is no guarantee that the optimal production level derived in the previous section is governed by a stationary policy $q^*=S(\tilde{R})$ so that $dR=0$ and the resource stock remains at $\tilde{R}$ forever. Indeed, it is easy to check that the conditions for (16) to become zero are different from those for (25) even when the stock externality is absent.

VI. INVESTMENT RULE FOR RENEWABLE RESOURCES

The savings-investment rule applied in the previous section is the one that is originally proposed for finding constancy of consumption in an economy with exhaustible resources. However, if planners take full account of the renewable nature of resources for deciding the amount of foreign investment, they may conceive other rules than (22). Hartwick (1978), indeed, has suggested one of them. Assuming a single consumption good and two natural resource stocks, one exhaustible and one renewable, he proves the following. If current returns from the current net decline in the stock of the renewable resource are invested in reproducible capital, per capita consumption will remain constant along dynamically efficient paths. Since the resource stock depreciates by q, the amount of extraction, at each moment of time but is replenished by S, the amount of reproduction, at the same time, the net decline in the stock becomes $q-S$. The purpose of the present section is to examine if the rule contributes to the constancy of expected consumption in our model.

Taking account of Hartwick's (1978) suggestion, we replace (17) by

$$H=(p-c_q)(q-S) \tag{26}$$

The current aggregate consumption of the economy continues to be expressed by (18) in Section 3. Without indulging ourselves in repeating the same procedure on the derivation of dH as in the previous sections, let us show directly the result on dG under assumption (26):

$$\begin{aligned} dG=&\frac{1}{2}c_{qq}\,q_p^2\,\sigma^2 p^2 dt+c_R\,(q-S)dt+S\alpha p dt+(q-S)c_{qq}dq \\ &+\frac{1}{2}(q-S)c_{qqq}\,q_p^2\,\sigma^2 p^2 dt+c_{qR}\,(q-S)dR \end{aligned}$$

$$-(q-c_q)S'(q-S)dt+\rho(p-c_q)(q-S). \tag{27}$$

Accordingly we have the expression of expected rate of change in consumption as

$$\begin{aligned}\frac{1}{dt}E_t dG=&\frac{1}{2}c_{qq}\,q_p^2\sigma^2p^2+(q-S)c_R+\alpha pS\\&+(q-S)c_{qq}\frac{1}{dt}E_t dq+\frac{1}{2}(q-S)c_{qqq}\,q_p^2\sigma^2p^2\\&-c_{qR}(q-S)^2-(p-c_q)S'(q-S)+\rho(p-c_q)(q-S)\\=&\alpha pq+\frac{1}{2}c_{qq}\,q_p^2\sigma^2p^2\end{aligned} \tag{28}$$

Notice that use has been made of (16) to obtain the last equality.

Now, terms on reproduction rate, S, entirely disappear from the last equation in (28). Residual terms are related to uncertainty in resource prices. It is proved in Section 3 that an adjustment for capital gains or losses due to the price fluctuation eliminates the αpq part. Finally, if the extraction cost is linear, the right-hand side becomes zero, and thus a constancy of consumption is made possible at lease in the expected sense. Therefore, Hartwick's (1978) rule for renewable resources survives an introduction of resource price uncertainty, although it cannot eliminate the effect on consumption path of the price uncertainty itself.

V. CONCLUDING REMARKS

We have analyzed the impact of price uncertainty on Hartwick's rule. It is well known, by the rule, that the consumption level for a resource exporting small open economy is kept constant over time as far as the economy extracts the resource competitively under given world price and invests abroad the Hotelling rent of extraction to receive investment earnings. However, once the price ceases to be stable, the rule is to be modified. This is because of the effect of terms-of-trade shifts on the consumption possibility. It is already proved elsewhere, assuming the resource being exhaustible, that two candidates for the modification term are plausible, but one of them loses ground under future uncertainty. The present paper extends the scope of analysis to include renewable resource cases. Our main concern is to know what happens to the expected rate of change in consumption if the

Hartwick type investment rule for exhaustible resources is applied to renewable resource cases. It is shown that the stock level of resource affects the validity of the rule. When the stock is small, the rule by no means holds in its original sense of constant consumption in resource economies. The analysis sheds lights on the nature of resource that affects the applicability of saving-investment rules for consumption sustainability.

Appendix 1

Substituting (7) into (H-J-B), we have the following identity.

$$0=[pq^*-c(q^*,R)]e^{-\rho t}+J_t+\alpha pJ_p+\frac{1}{2}\sigma^2p^2J_{pp}-q^*J_R$$

Partially differentiate by R and obtain with the aid of Ito's lemma on J_R that

$$0=[pq_R^*-c_q(q^*,R)q_R^*-c_R]e^{-\rho t}+(S'(R)-q_R^*)J_R+J_{tR}+\alpha pJ_{pR}+\frac{1}{2}\sigma^2p^2J_{ppR}+(S-q^*)J_{RR}$$

$$=-c_Re^{-\rho t}+S'(R)J_R+\frac{1}{dt}E_t dJ_R.$$

Thus, from the first order condition (6), we prove that (8) is true.

Appendix 2

Here we assume (22). Then,

$$d\hat{H}=qdp+\frac{1}{2}q_p(dp)^2-qc_{qq}dq-\frac{1}{2}qc_{qqq}(dq)^2-\frac{1}{2}c_{qq}(dq)^2$$

$$+(p-c_q)dq-\frac{1}{2}c_{qq}(dq)^2-qc_{qR}dR+qdp+\rho\int_0^t qe^{-\rho(\tau-t)}dp(\tau)dt.$$

Therefore, we have that

$$d\hat{G}=-c_RdR+qc_{qq}dq+\frac{1}{2}qc_{qqq}(dq)^2+\frac{1}{2}c_{qq}(dq)^2+qc_{qR}dR-qdp+\rho(p-c_q)qdt$$

and in turn that

$$\frac{1}{dt}E_td\hat{G}=-c_R(S-q)+qc_{qq}\frac{1}{dt}E_tdq+\frac{1}{2}(qc_{qqq}+c_{qq})\frac{1}{dt}E_t(dq)^2$$

$$+(S-q)qc_{qR}-q\frac{1}{dt}E_tdq+\rho(p-c_q)q$$

$$=-c_R(S-q)+\alpha pq-q(\rho-S')(p-c_q)-qc_R-\frac{1}{2}q\sigma^2p^2q_p^2c_{qqq}$$

$$-q(S-q)c_{qR}+\frac{1}{2}q\,\alpha^2p^2q_p^2c_{qqq}+\frac{1}{2}c_{qq}\sigma^2p^2q_p^2+qc_{qR}(S-q)-\alpha pq+\rho(p-c_q)q$$
$$=\frac{1}{2}c_{qq}\sigma^2p^2q_p^2-Sc_R+S'(p-c_q)q\,.$$

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NOTES

1 The small-country assumption is not essential in the present analysis. If the country exercises a monopoly power over the resource, the analysis proceeds with the average movement of the marginal revenue for the monopolist rather than the resource price itself. This does not cause any analytical difficulty. In the present paper, however, we just follow the original Hartwick's setting of competitive extraction.

2 If resource owners are accessible to a well-developed reproducible capital market, they need not invest abroad the part of the resource return. The same amount of interest payments may be raised from the market as from overseas.

3 For an earlier application of Wiener process or Brownian motion to the analysis of resource economy, see Pindyck (1980).

4 We do not duplicate here the discussion on two different ways of modification to eliminate the capital gains/losses term proposed by Katayama and Ohta (1999). The one in (22) is good enough for both exhaustible and renewable resources.

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