International Duopoly and Trade Policies Under Budget Constraint

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1. Introduction

Recently many trade theorists have been studying the international trade theory with imperfect competition[1]. They have constructed various models of international oligopoly and analyzed trade polices. One of the most important results of the analyses is that each country has an incentive of giving export subsidy to the domestic firms and import tariff to foreign firms in order to improve welfare of the country.

When markets of two countries (home and foreign) are segmented, trade polices on the domestic market cannot affects the quantities of foreign market if we assume, as many of the literature do[2], firms' marginal costs are constant. But this assumption is not so realistic, and so the derived results might be unconvincing. As a matter of facts, in the real world trade restrictions, such as tariffs and/or quotas, imposed by one country affects not only the market of the country but also the markets of the rest of the world. For example, when Toyota had to reduce her exports for the U.S. auto market in the cause of "voluntary" export restrains (VER), she shifted the products exceeding the VER quantity to the other markets, such as Japan and EC markets. This kind of phenomenon has not been fully explained by trade theorist except those who has assumed non-linear cost function of an oligopolist[3].

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1) Studies of Brander (1981), Brander and Krugman (1983), Brander and Spencer (1984a, 1984b), Cheng (1988), Dixit (1984, 1988), Eaton and Grossman (1984), Krishna (1989), Krishna and Itoh (1988), Krugman (1984), Uekawa (1993, 1995), Venables (1985) and many other economists have made valuable contribution to this theory.

2) Krugman (1984) and Uekawa (1993) are such exceptions of the above literature in note 1.

3) Krugman (1984) and Uekawa (1993) are such exceptions. But Krugman (1984) has not analyzed the foreign market. Uekawa (1993) is a model of the product differentiation and so different from the model of this paper. In view of the model which will be developed in this paper, models of Okamoto and Yoshida (1991, 1994) and Okuguchi (1990) are the most similar to the present paper. But their purposes of the analyses are different from the current paper.

In this paper, we shall develop a trade model of international duopolists who supply identical product in each others' market, in which demand functions of the markets are non-linear, and whose marginal costs are decreasing. And we shall analyze the two trade policy instruments of home country, production subsidy and import tariff, on each firm's production of the good and consumption of the good in each market, as well as the welfare of home country. Thus, in this paper we shall do the similar analyses which have been done in the famous papers such as Cheng (1988), Dixit (1988), and Uekawa (1993) etc. Though the analysis to be presented is very similar as those of the above papers, there are some important differences between our analysis and theirs. Namely, we take into account the policy maker's budget constraint, and analyze the effects of two trade policies, production subsidy and import tariff.

Although neither of the above mentioned works have explained the source of production subsidy or export subsidy, they have showed that the optimal production subsidy or export subsidy is positive. But this result could be quite obvious when the production subsidy or export subsidy is like 'a gift from Heaven'. So the results which do not specify the source of subsidy may be misleading. It is needed to specify the financial background of the cost of subsidies, if you want to treat welfare aspect of the policies properly. Thus, we introduce the government budget constraint to finance the production subsidy by import tariff.

In this paper we shall establish following results:

(1) Uniqueness of the Cournot equilibrium solution containing both countries' markets is investigated and a set of sufficient conditions for the uniqueness of the solution is presented.

(2) Under the set of conditions for the uniqueness of the solution, the home country production subsidy increases the supply of the imperfectly competitive good in each market and decreases the price of the good if the subsidy is a gift from Heaven.

(3) Under the same set of conditions, an imposition of domestic import tariff reduces the supply of the good to domestic market but it depends on the cost condition whether imposition of tariff increase supply of the good to

foreign market.

(4) Under the same set of conditions, whether the production subsidy increases the supply of each market depends on ϕ', the degree of marginal increase in tariff to finance one unit of production subsidy, if the domestic government's budget constraint is considered. But under the same set of conditions, an increase in production subsidy raises the supply of the home duopolist in each market and reduces that of foreign duopolist when the government budget constraint is binding.

(5) Under the same set of conditions plus one minor condition, an imposition of production subsidy which is financed by import tariff increases national welfare of home country. So the optimal production subsidy and import tariff are definitely positive.

The remainder of this paper is organized as follows. In the next section we shall present the model and the assumptions formally and show that under these assumptions the Cournot equilibrium solution of the model is unique. In section 3, first, we shall examine the effects of a domestic production subsidy and import tariff on consumption as well as production in each country in the case of no government budget constraint. Then, on the base of these analyses, we shall examine the effects of a domestic production subsidy in the presence of government budget constraint. In section 4, we shall turn our attention to the analysis of welfare effects of trade policies when the government budget constraint is imposed. And we shall show that the optimal production subsidy and import tariff are positive. In the last section, we shall give some concluding remarks.

2. The Model

2.1 The framework of the model

There are two countries, home country (country H) and foreign country (country F), in which two good X and Z are produced. Good Z, taken as numeraire, is produced in competitive sector in each country. On the other hand, a homogeneous good X is produced by one firm in each country. Each firm supplies of this good to each market which is segmented and so the two firms are international duopolist of good X.

In this paper we shall analyze two trade polices of home country. The government of home country imposes a specific import tariff on good X at the rate of t and gives a specific production subsidy to the domestic producer of good X at the rate of s in order to maximize national welfare of country H. We assume that domestic government set the two rates s and t first subject to her budget constraint, then each duopolist takes these values into account and decides how much to produce and how much to supply to each country's market which is Cournot competitive. For simplicity of analysis, we assume foreign country will not retaliate for these policies of country H.

Let $p_H(X_H)$ and $p_F(X_F)$ represent the inverse demand function of identical good X in domestic market and foreign market, respectively, where X_H and X_F are the amount of demand in respective markets. We assume that demand curve of each market is negatively sloped, that is $p_H'(X_H)<0$ and $p_F'(X_F)<0$.

The trade policy instruments consist of a domestic import tariff and a domestic production subsidy on imperfectly competitive good X. Let a domestic import tariff rate be denoted by t and a domestic production subsidy rate by s. Then total profits of domestic duopolist π_H and foreign duopolist π_F are represented by

$$(1.1)\quad \pi_H(x_{HH},\ x_{HF};\ x_{FF},\ x_{FH}) = x_{HH}p_H(X_H) + x_{HF}p_F(X_F) - c_H(Q_H) + sQ_H.$$

$$(1.2)\quad \pi_F(x_{FF},\ x_{FH};\ x_{HH},\ x_{HF}) = x_{FF}p_F(X_F) + x_{FH}p_H(X_H) - c_F(Q_F) - tx_{FH}.$$

where x_{Hk} (x_{FK}) denotes the country H (county F) duopolist's supply of good X to the market of country k, thus X_H and X_F can be represented as $X_H = x_{HH} + x_{FH}$ and $X_F = x_{FF} + x_{HF}$, while $Q_H \equiv x_{HH} + x_{HF}$ ($Q_F \equiv x_{FF} + x_{FH}$) represents the amount of production of the country H (county F) duopolist. And $c_H(c_F)$ is the cost function of the country H (country F) duopolist.

2.2 Basic assumptions and equilibrium conditions

In the ensuring analyses we assume the following conditions C.1~C.4 to be satisfied by the inverse demand functions and the cost functions of good X.

C.1: $p_H + x_{HH} p_H' \geqq 0$, $h_{HH} \equiv p_H' + x_{HH} p_H'' < 0$, $h_{HF} \equiv p_F' + x_{HF} p_F'' < 0$;

$p_F + x_{FF} p_F' \geqq 0$, $h_{FH} \equiv p_H' + x_{FH} p_H'' < 0$, $h_{FF} \equiv p_F' + x_{FF} p_F'' < 0$.

C.2: $c_H' > 0$, $c_H'' \leqq 0$; $c_F' > 0$, $c_F'' \leqq 0$.

C.3: $p_H' - 2c_H'' < 0$, $p_H' - 2c_F'' < 0$; $p_F' - 2c_F'' < 0$, $p_F' - 2c_H'' < 0$.

C.4: $h_{HH} - h_{FH} + p_H' - 2c_H'' < 0$, $h_{HF} - h_{FF} + p_F' - 2c_H'' < 0$;

$h_{FH} - h_{HH} + p_H' - 2c_F'' < 0$, $h_{FF} - h_{HF} + p_F' - 2c_F'' < 0$.

First, C.1 requires that each firm's marginal revenue in each market is non-negative and satisfies the Hahn (1962) stability condition with the negative slopedness of each demand function. Second, C.2 requests that each firm's average cost and marginal cost are decreasing and (weakly) convex to the origin. Third, C.3 demands that the inverse demand function of each market is steep and/or each manufacturer's marginal cost curve is flat. Finally, in view of condition C.3 and the definitions of h_{ij}'s (i , j=H, F) , C.4 requires that the absolute values of p_H'' and p_H'' are not too large, i.e. the degree of concavity or convexity to the origin of every inverse demand curve is not too strong.

Under the Cournot assumption on each firm's behavior in each market, the first order conditions for profit maximization are:

(2.1) $p_H(X_H) + x_{HH} p_H'(X_H) - c_H'(Q_H) + s = 0$,

(2.2) $p_F(X_F) + x_{HF} p_F'(X_F) - c_H'(Q_H) + s = 0$,

(2.3) $p_F(X_F) + x_{FF} p_F'(X_F) - c_F'(Q_F) = 0$,

(2.4) $p_H(X_H) + x_{FH} p_H'(X_H) - c_F'(Q_F) - t = 0$.

In the following sections we investigate the nature of the above equation system (2), the Cournot equilibrium and the effects of changes in s and t on the equilibrium. Before proceeding to these, we shall explain the government budget constraint.

2.3 The government budget constraint

We assume that the government of country H collects the tariff from import of good X in order to finance the production subsidy to the domestic firm and the budget constraint is always satisfied. Then the following equation holds:

(3) $sQ_{H}=tx_{FH}$

If we stick to this budget constraint, we can define that the rate of import tariff t as a function of the rate of production subsidy s. Of course, if we do not adhere to this constraint, two rates are simply independent of each other. So let us assume the following expression to treat the government budget constraint flexibly:

(4) $t=\phi(s)$, $\phi'\geqq 0$.

The meaning of this equation is obvious. In the case of $\phi'>0$, the government of home country adhere to the budget constraint and thus in order to raise funds of positive production subsidy s (>0) she has necessary to impose positive import tariff t (>0). In the case of $\phi'=0$, she does not keep the budget constraint and therefore s and t are independently chosen[4), 5)].

3. The Analysis

3.1 The uniqueness of the solution

Now let us consider changes in policy instruments s and t on the Cournot equilibrium. Totally differentiating the equation system (2) yields:

$$(5)\quad \begin{bmatrix} h_{HH}+p_{H}'-c_{H}'' & -c_{H}'' & 0 & h_{HH} \\ -c_{H}'' & h_{HF}+p_{F}'-c_{HF}'' & h_{HF} & 0 \\ 0 & h_{FF} & h_{FF}+p_{F}'-c_{F}'' & -c_{F}'' \\ h_{FH} & 0 & -c_{F}'' & h_{FH}+p_{H}'-c_{F}'' \end{bmatrix} \begin{bmatrix} dx_{HH} \\ dx_{HF} \\ dx_{FF} \\ dx_{FH} \end{bmatrix} = \begin{bmatrix} -ds \\ -ds \\ 0 \\ dt \end{bmatrix}.$$

4) Note that expression (4) allows the following case. Namely, the case that the domestic government wants to keep budget constraint, and so t and s move same direction ($\phi'>0$), but that she cannot adhere to the balanced budget.

5) The relation between t and s is just like expression (4) when both rates are in the neighborhood of $t=s=0$. But when t and s are positive large number, it may be doubtful that ϕ' is positive. Thus, in this paper we presupposes t and s are not so quite large.

The 4x4 matrix of equation system (6), which we shall refer to as matrix A hereafter, is the Jacobian matrix of equation system (2). Let us consider the nature of this matrix. From C.1~C.3, all the diagonal elements of A are definitely negative and from C.4 they are the dominant elements in every column[6]. Because, if C.4 holds, then

$$|h_{HH}+p_H'-c_H''| - |-c_H''| - |h_{FH}| = -(h_{HH}-h_{FH}+p_H'-2c_H'')>0,$$
$$|h_{HF}+p_F'-c_H''| - |-c_H''| - |h_{FF}| = -(h_{HF}-h_{FF}+p_F'-2c_H'')>0,$$
$$|h_{FF}+p_F'-c_F''| - |-c_F''| - |h_{HF}| = -(h_{FF}-h_{HF}+p_F'-2c_F'')>0,$$
$$|h_{FH}+p_H'-c_F''| - |-c_F''| - |h_{HH}| = -(h_{FH}-h_{HH}+p_H'-2c_F'')>0.$$

Thus, matrix A has negative dominant diagonals. Consequently it can easily be seen that all principal minors of order two are positive and those of order three negative. Since $\det(A)>0$, we see that A is an N−P matrix, so that the solution to the equation system (2) must be unique[7]. Consequently, we have established the following theorem:

THEOREM 1: *The Jacobian matrix A of the Cournot equilibrium solution system (2) is an N−P matrix and therefore the solution to the system is unique.*

Note that this theorem assures the uniqueness of the Cournot equilibrium solution if there exists a solution to equation system (2) at all. The problem about the existence of a solution in this Cournot competitive intra-industry trade model has been solved by Uekawa and Ohta (1993) in the case of increasing marginal cost. On the other hand, Okuguchi (1990) has shown the existence and the stability of the solution of the model including decreasing marginal cost case.

Let A_{ij} denote the cofactor of the (i, j)-th element in A. Then the solution

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6) An nxn matrix of $A=(a_{ij})$ is said to have dominant diagonals if there exist $d_j>0$ $(j=1, \ldots, n)$ such that

$$d_j|a_{jj}| > \Sigma_{i\neq j} d_i |a_{ji}| \quad \text{for any } j.$$

And a matrix with dominant diagonals is nonsingular. <See McKenzie (1960, p.49) for the proof.>

7) An nxn matrix is said to be an N−P matrix if its principal minors of order r have the sign of $(-1)^r$ $(r=1, \ldots, n)$. And if the Jacobian matrix of equation system is an N−P matrix, the solution of the system is unique. <See Nikaido (1968, p.371) for the proof.>

of equation system (5) can be written as follows:

$$(6)\quad \begin{bmatrix} dx_{HH} \\ dx_{HF} \\ dx_{FF} \\ dx_{FH} \end{bmatrix} = [1/\det(A)] \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{bmatrix} \begin{bmatrix} -ds \\ -ds \\ 0 \\ dt \end{bmatrix}.$$

In the following sections let us consider the nature of this solution.

3.2 The effects of a change in s: independent case

In this sub-section, we shall investigate the effects of a change in s on production and consumption when the domestic government is free from budget constraint and chooses s independently of t.

From (6) we obtain the following equations:

$$(7.1)\quad x_{HHs} \equiv \partial x_{HH}/\partial s = -(A_{11}+A_{21})/\det(A) > 0,$$
$$(7.2)\quad x_{HFs} \equiv \partial x_{HF}/\partial s = -(A_{12}+A_{22})/\det(A) > 0,$$
$$(7.3)\quad x_{FFs} \equiv \partial x_{FF}/\partial s = -(A_{13}+A_{23})/\det(A) < 0,$$
$$(7.4)\quad x_{FHs} \equiv \partial x_{FH}/\partial s = -(A_{14}+A_{44})/\det(A) < 0.$$

(The signs of equations are examined in Appendix A.1. and Appendix A.2.)

From (7.1)～(7.4), we find that an increase in the domestic production subsidy promotes the domestic firm's supply to each market and shrinks the foreign firm's supply to every market. Therefore, we can find that the total production of the domestic duopolist increases and that of foreign counterpart decreases. This establishes the following expressions:

$$(7.5)\quad Q_{Hs} \equiv x_{HHs} + x_{HFs} > 0,$$
$$(7.6)\quad Q_{Fs} \equiv x_{FFs} + x_{FHs} < 0.$$

Now, let us turn to the effects on consumption of this good in each country. We can verify the following expressions:

(7.7) $X_{Hs} \equiv x_{HHs} + x_{FHs} = -(A_{11} + A_{21} + A_{14} + A_{44}) / \det(A)$

$= -(1/2)(h_{HF} + h_{FF} + p_F')[p_H'(p_F' - 2c_F'') + p_F'(p_H' - 2c_F'')] / \det(A) > 0,$

(7.8) $X_{Fs} \equiv x_{FFs} + x_{HFs} = -(A_{12} + A_{22} + A_{13} + A_{23}) / \det(A)$

$= -(1/2)(h_{HH} + h_{FH} + p_H')[p_H'(p_F' - 2c_F'') + p_F'(p_H' - 2c_F'')] / \det(A) > 0.$

(See Appendix A.3 for the derivation of the expressions.) The signs of (7.7)–(7.8) stem from that $(p_F' - 2c_F'') < 0$ and $(p_H' - 2c_F'') < 0$ hold from C.3. Therefore we can claim that an increase in s raises both countries' total consumption of good X. Thus, since the inverse demand functions are negatively sloped, the prices of good X in both markets decrease if s rises.

Consequently, we have established the following theorem:

THEOREM 2: *When the production subsidy is determined independently by the domestic government, an increase in the production subsidy raises domestic production, domestic consumption and foreign consumption, while it curtails foreign production. And it reduces the prices of good X in both markets.*

3.3 The effects of a change in t: independent case

In this sub-section, we examine the effects of a domestic import tariff on production and consumption of good X in each country in the case that the domestic government is free from budget constraint and chooses t independently of s.

From (6) we obtain the following equations:

(8.1) $x_{HHt} \equiv \partial x_{HH} / \partial t = A_{41} / \det(A) > 0,$

(8.2) $x_{HFt} \equiv \partial x_{HF} / \partial t = A_{42} / \det(A) \geqq 0,$

(8.3) $x_{FFt} \equiv \partial x_{FF} / \partial t = A_{43} / \det(A) \leqq 0,$

(8.4) $x_{FHt} \equiv \partial x_{FH} / \partial t = A_{44} / \det(A) < 0,$

(The signs of equations are examined in Appendix A.1.)

From (8.1)~(8.4) we find that an increase in the import tariff promotes the domestic firm's supply to each market and shrinks the foreign firm's supply to every market. Therefore, we can find that the total production of the domestic duopolist increases and that of foreign counterpart decreases. This establishes the following expressions:

(8.5) $Q_{Ht} \equiv x_{HHt} + x_{HFt} > 0$,

(8.6) $Q_{Ft} \equiv x_{FFt} + x_{FHt} < 0$.

Note that in equations (8.2) and (8.3) the equality holds only if $c_H" = c_F" = 0$, that is every firm's marginal cost is constant.

Now let us consider the effects of a change in the import tariff on consumption in each country. Considering (8.1)~(8.4), we can verify the following expressions:

(8.7) $X_{Ht} \equiv x_{HHt} + x_{FHt} = (A_{41} + A_{44}) / \det(A)$
$= (1/2)\{(h_{FF} + p_F' - c_F")[p_H'(p_F' - 2c_H") + p_F'(p_H' - 2c_H")]$
$+ h_{HF}[p_H'(p_F' - 2c_F") + p_F'(p_H' - 2c_H")]\} / \det(A) < 0$,

(8.8) $X_{Ft} \equiv x_{FFt} + x_{HFt} = (A_{42} + A_{43}) / \det(A)$
$= \{h_{HH}(p_F' - c_F")(c_F" - c_H") + (1/2)c_F"$
$[p_H'(p_F' - 2c_H") + p_F'(p_H' - 2c_H")]\} / \det(A)$.

Thus, from (8.7) we see that an increase in the domestic import tariff reduces consumption of good X in the home country. This means that price of good X increases in the domestic market. But whether it increases consumption in the foreign market or not depends on changes in cost conditions of the duopolists. From (8.8), we see that if $c_F" - c_H" \leqq 0$, i.e. the degree of marginal cost reduction in the domestic firm is not so large compared with the foreign firm, the total supply of good X in the foreign market decreases. On the other hand, if $c_F" - c_H" \geqq 0$, the degree of marginal cost reduction in the domestic firm is large enough, it is possible that the total supply of this good in the foreign market increases.

Consequently, we have established the following:

THEOREM 3: *When the import tariff is determined independently by the domestic government, an increase in the import tariff raises domestic production while it curtails domestic consumption and foreign production. And it increases the price of good X in domestic market. On the other hand, whether it increases consumption in the foreign market or not depends on the cost conditions. But if* $c_F'' - c_H'' \leqq 0$*, an increase in the import tariff decreases foreign consumption.*

3.4 The effects of a change in s: binding budget constraint case

In this sub-section we shall investigate the effects of changes in s on production and consumption of good X in each country in the case of binding budget constraint. From (3), (7.1)~(7.4), and (8.1)~(8.4), we can derive the following results:

(9.1) $dx_{HH}/ds = x_{HHs} + x_{HHt}(dt/ds) = x_{HHs} + \phi' x_{HHt} > 0,$

(9.2) $dx_{HF}/ds = x_{HFs} + x_{HFt}(dt/ds) = x_{HFs} + \phi' x_{HFt} > 0,$

(9.3) $dx_{FF}/ds = x_{FFs} + x_{FFt}(dt/ds) = x_{FFs} + \phi' x_{FFt} < 0,$

(9.4) $dx_{FH}/ds = x_{FHs} + x_{FHt}(dt/ds) = x_{FHs} + \phi' x_{FHt} < 0.$

From the above equations we find that a rise in the domestic production subsidy financed by a corresponding rise in the import tariff increases the domestic firm's supply to each market and reduces the foreign firm's supply to every market. Thus it promotes the production of domestic duopolist and shrinks that of foreign duopolist. Therefore, we have the following equation:

(9.5) $dQ_H/ds = dx_{HH}/ds + dx_{HF}/ds > 0,$

(9.6) $dQ_F/ds = dx_{FF}/ds + dx_{FH}/ds < 0.$

Let us consider the effects of a change in s with a accompanying change in t on consumption in each country. We can derive the following expressions:

(9.7) $dX_H/ds = dx_{HH}/ds + dx_{FH}/ds = X_{Hs} + \phi' X_{Ht},$

(9.8) $dX_F / ds = dx_{FF} / ds + dx_{HF} / ds = X_{Fs} + \phi' X_{Ft}$.

In this case, whether consumption of good X increases or not in each market mainly depends on the magnitude of $\phi' \equiv dt / ds$, which represents the marginal rate of tariff to finance a unit of production subsidy rate added. ϕ' depends on the relative magnitude of the volume of import and the volume of domestic production. From equations (7.7), (7.8), (8.7), and (8.8) we can estimate that relatively small value of ϕ' assures positive values of dX_H / ds and dX_F / ds. As a matter of fact, in the case of constant marginal costs case, we can derive the following equation:

(9. 9) $dX_H / ds = X_{Hs} + \phi' X_{Ht} = -(1-\phi')[p_H' p_F' (h_{HF} + h_{FF} + p_F')] / \det(A)$
if $c_H'' = c_F'' = 0$.

(9.10) $dX_F / ds = X_{Fs} + \phi' X_{Ft} = -p_H' p_F' (h_{HH} + h_{FH} + p_H') / \det(A) > 0$
if $c_H'' = c_F'' = 0$.

From (9.9) we see that consumption in country H increases if $\phi' < 1$ in the case of $c_H'' = c_F'' = 0$. On the other hand, from (9.10) consumption in country F increases without any additional conditions when the marginal costs are constant. This is because the effects of production subsidy transmits to the foreign market but that of import tariff does not pass through to the market of country F when $c_H'' = c_F'' = 0$[8].

Thus we have established the following theorem:

THEOREM 4: *In the case of binding budget constraint, an increase in the production subsidy promotes the supply of domestic duopolist and curtails that of foreign duopolist. But whether the consumption in each market increases or not depends on the value of* $\phi' \equiv dt / ds$, *the marginal rate of tariff to finance a unit of production subsidy rate added.*

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8) The reader can easily check this result from (7.7), (7.8), (8.7), and (8.8) by substituting the condition $c_H'' = c_F'' = 0$.

4. The Optimal Trade Policy

Now, let us proceed to optimal trade policy discussions. First we shall assume that the social utility function takes the semi-linear form:

$$U \equiv u(X_H) + Z,$$

where Z represents the domestic demand for competitively produced good Z, which is the numeraire in both country, and so the relative price of good X is $p_H(X_H)$ and $p_F(X_F)$ in country H and F, respectively. In this case welfare effects of trade policy can be expressed by standard surplus measures[9]. Let W denote the national welfare of the home country, which consists of the consumers' surplus, the rent of the domestic duopolist, and the domestic government budget surplus.

$$\begin{aligned}(10.1)\quad W(s) \equiv & [u(X_H) - X_H p_H(X_H)] + [x_{HH} p_H(X_H) + x_{HF} p_F(X_F) - c_H(Q_H) + sQ_H] \\ & + [t x_{FH} - sQ_H] \\ & = u(X_H) - x_{HH} p_H(X_H) + x_{HF} p_F(X_F) - c_H(Q_H) + t x_{FH}.\end{aligned}$$

Differentiating $W(s)$ with respect to s yields[10]:

$$\begin{aligned}(10.2)\quad W'(s) = & (p_H - x_{FH} p_H' - c_H') dx_{HH}/ds + (t - x_{FH} p_H') dx_{FH}/ds + (p_F + x_{HF} p_F' - c_H') \\ & dx_{HF}/ds + x_{HF} p_F' dx_{FF}/ds + x_{FH} \phi' \\ & = (-x_{HH} p_H' - s - x_{FH} p_H') dx_{HH}/ds + (t - x_{FH} p_H') dx_{FH}/ds - s dx_{HF}/ds \\ & + x_{HF} p_F' dx_{FF}/ds + x_{FH} \phi'.\end{aligned}$$

The optimal rate of production subsidy s^{op} and the corresponding optimal im-

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9) Assuming that (1) labor is the only production factor, (2) labor is fully employed, (3) international payments are always balanced by numeraire good Z, and (4) one unit of numeraire good is produced by one unit of labor, we can express the social utility level of country H as follows:

$$U = u(X_H) - X_H p_H + \pi_H + t x_{FH} + L_H$$

where L_H represents the endowment of labor in country H. The first two terms of the RHS express consumers' surplus of country H. Thus, if we assume the constancy of labor endowment, a change in the social utility level in country H is fully described by the social welfare function W.

Note also that this model can be regarded as a fully general equilibrium model with imperfectly competitive good, once we assume there is only one factor of production in this model.

10) The last equality is derived from using the equilibrium conditions (2.1)~(2.4).

port tariff t^{op} are defined by

(11) $W'(s^{op})=0$ and $t^{op}=\phi(s^{op})$.

In order to establish whether the optimal rate of production subsidy is positive or not we must investigate the sign of $W'(0)$. If $W'(0)>0$ then the optimal rate of production subsidy should be positive. If $W'(0)<0$ then it should be negative. If $W'(0)=0$ then free trade is the best policy. Let us examine the sign of $W'(0)$.

Substituting $s=t=0$ into (10.2) yields:

(12) $$W'(0)=-x_{HH}p_H'dx_{HH}/ds-x_{FH}p_H'[dx_{HH}/ds+dx_{FH}/ds-\phi'/p_H'] +x_{HF}p_F'dx_{FF}/ds.$$

In the above equation, the derivatives are evaluated at $s=t=0$. The first and the third terms of the RHS of equation (12) are positive. Therefore, if the second term of the RHS of (12) is not negative, $W'(0)$ is positive. The key problem is the sign of [ ]. If [ ] of (12) is positive, then the second term of the RHS of (12) is also positive. By using conditions of C.1~C.4, we can prove that it is surely positive.

LEMMA 5: *Under Conditions C.1~C.4, if* $h_{FH}-2c_F''\leqq 0$ *holds, then*

$$dx_{HH}/ds+dx_{FH}/ds-\phi'/p_H'>0.$$

Proof: Using (9.1)~(9.7) we can derive the following equation:

(13.1) $$dx_{HH}/ds+dx_{FH}/ds-\phi'/p_H'=x_{HHs}+x_{FHs}+\phi'(x_{HHt}+x_{FHt}-1/p_H').$$

From (7.7) $x_{HHs}+x_{FHs}=X_{Hs}$ is positive. $x_{HHt}+x_{FHt}-1/p_H'$ can be represented as follows:

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11) Note that $\det(A)=h_{FH}A_{41}+(-c_F'')A_{43}+(h_{FH}+p_H'-c_F'')A_{44}$.

(13.2) $x_{HHt}+x_{FHt}-1/p_H'=[1/p_H'\det(A)][p_H'A_{41}+p_H'A_{44}-\det(A)]$.

If we expand the det(A) by the last row, and rearrange the above expression we obtain the following expression[11]:

(13.3) $$p_H'A_{41}+p_H'A_{44}-\det(A)=(p_H'-h_{FH})A_{41}+c_F''A_{43}-(h_{FH}-c_F'')A_{44}$$
$$=-(h_{FH}-2c_F'')(A_{41}+A_{44})$$
$$+(p_H'-2c_F'')A_{41}-c_F''(A_{44}-A_{43}).$$

From (8.7) $A_{41}+A_{44}<0$, and from Appendix A.1 and Appendix A.4 $A_{41}>0$ and $A_{44}-A_{43}<0$. We can conclude that the LHS of (13.3) is negative if the following inequalities holds:

(14) $h_{FH}-2c_F''=p_H'+x_{FH}p_H''-2c_F''\leqq 0$.

Then from (13.2) we can easily check that the LHS of (13.2) is positive. Therefore, the LHS of (13.1) is positive if (14) is satisfied. (Q.E.D.)

Since $p_H'-2c_F''<0$ from C.3, if p_H'' is not greater than some sufficiently small positive number (i.e. $p_H''\leqq-[p_H'-2c_F'']/x_{FH}$), the above condition is satisfied. Thus, the condition (14) demands that the inverse demand function of country H should not be too concave to the origin.

Now we have established the effects of an increase in the production subsidy from 0, which is financed by the corresponding increase in the import tariff on the national welfare of the domestic country. It improves national welfare.

THEOREM 6: *In the case of binding budget constraint, if one additional condition (14) is satisfied, an increase in the production subsidy from 0 to some positive number, which is financed by the corresponding increase in the import tariff rate, improves the national welfare of the domestic country. Thus the optimal production subsidy rate and the associated import tariff rate are definitely positive, which are defined in equation (11).*

5. Concluding Remarks

In this paper we have examined the characteristics of the international duopolist model with intra-industry trade of identical product. The model we have constructed is the most general and realistic one in which each firm has decreasing marginal cost curve and each market has non-linear inverse demand curve, and the policy maker should be subject to budget constraint. We have investigate that a change in a production subsidy and an import tariff on production and consumption in each country especially in the case that the two policy instruments are combined with budget constraint. By these analyses we have extended the Krugman (1984) analyses which have examined only the effects of an import tariff on each firm's supply to home market alone. As we have mentioned above, the non-linearity of cost function means that domestic and foreign markets cannot be separated, and so we must examine both markets to investigate the trade policy effects fully and properly.

We also have examined the welfare effects of trade policies which are combined with budget constraint. We have shown that a small positive production subsidy which is financed by corresponding positive import tariff improves the national welfare of the domestic country if the inverse demand function of domestic market is not too concave to the origin. This result is quite new one. Of course this result is more general than that of Cheng (1988) and Dixit (1988) which have assumed the constant marginal cost curves. And this is also more general than that of Okamoto and Yoshida (1991, 1994) which have assumed that the marginal cost curves are non-constant but the inverse demand functions are linear.

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Appendix

In this Appendix we shall show (1) the expressions and the sign of A_{ij}'s, (2) the proof of equation (7.4), (3) the proof of equations (7.7) and (7.8), (4) the derivation of equations (8.7) and (8.8), and (5) the proof of $A_{44}-A_{43}<0$.

A.1 The signs of A_{ij}'s

In this sub-section we shall show the expressions and signs of the cofactor of matrix A. Since matrix A is an N−P matrix, we can immediately obtain the following results:

$\det(A)>0, \quad A_{jj}<0 \quad (j=1, \ldots, 4).$

Using the technique of dominant diagonal matrix, we can determine the signs of every other cofactors of A as follows:

$$A_{21}=\begin{vmatrix} c_H''h_{FF}+p_F'-c_F'' & -c_F'' \\ -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix}+c_F''h_{HH}h_{FF}\leqq 0,$$

$$A_{41}=-c_H''c_F''h_{HF}-h_{HH}\begin{vmatrix} h_{HF}+p_F'-c_F'' & h_{HF} \\ h_{FF} & h_{FF}+p_F'-c_F'' \end{vmatrix}>0,$$

$$A_{12}=c_H''\begin{vmatrix} h_{FF}+p_F'-c_F'' & -c_F'' \\ -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix}+c_F''h_{FH}h_{HF}\leqq 0,$$

$$A_{42}=-c_F''h_{HF}(h_{HH}+p_H'-c_H'')-c_H''h_{HH}(h_{FF}+p_F-c_F'')\geqq 0,$$

$$A_{13}=-c_H''h_{FF}(h_{FH}+p_H'-c_F'')-c_F''h_{FH}(h_{HF}+p_F'-c_H'')\geqq 0,$$

$$A_{23}=-h_{FF}\begin{vmatrix} h_{HH}+p_H'-c_H'' & h_{HH} \\ h_{FH} & h_{FH}+p_H-c_F'' \end{vmatrix}-c_H''c_F''h_{FH}>0,$$

$$A_{43}=c_H''h_{HH}h_{FF}+c_F''\begin{vmatrix} h_{HH}+p_H'-c_H'' & -c_H'' \\ -c_H'' & h_{HF}+p_F'-c_H'' \end{vmatrix}\leqq 0,$$

$$A_{14}=-c_H''c_F''h_{FF}-h_{FH}\begin{vmatrix} h_{HF}+p_F'-c_F'' & h_{HF} \\ h_{FF} & h_{FF}+p_F'-c_F'' \end{vmatrix}>0,$$

$$A_{24}=-c_F''h_{FF}(h_{HH}+p_H'-c_H'')-c_H''h_{FH}(h_{FF}+p_F'-c_F'')\geqq 0,$$

A.2 The proof of equation (7.4)

By using the elementary properties of determinant, we can derive the following equation:

$$A_{14}+A_{44}=$$

$$\begin{vmatrix} p_H'-c_H'' & -c_H'' & 0 \\ -c_H'' & h_{HF}+p_F'-c_H'' & h_{HF} \\ c_F'' & h_{FF} & h_{FF}+p_F'-c_F'' \end{vmatrix}=\begin{vmatrix} p_H'-c_H'' & -c_H'' & 0 \\ -c_H'' & p_F'-c_H'' & h_{HF} \\ c_F'' & -p_F'+c_F'' & h_{FF}+p_F'-c_F'' \end{vmatrix}$$

$$=(h_{FF}+p_F'-c_F'')[(p_H'-c_H'')(p_F'-c_H'')-(c_H'')^2]$$

$$+h_{HF}[(p_H'-c_H'')(p_F'-c_F'')-c_H''c_F'']$$

$$=(h_{FF}+p_F'-c_F'')(1/2)[p_H'(p_F'-2c_H'')+p_F'(p_H'-2c_H'')]$$

$$+h_{HF}(1/2)[p_H'(p_F'-2c_F'')+p_F'(p_H'-2c_H'')].$$

Thus we obtain equation (7.4). (Q.E.D.)

A.3 The proof of equations (7.7) and (7.8)

First, we derive the expression for $(A_{11}+A_{21})+(A_{14}+A_{24})\equiv\alpha$. By the definition of cofactor, we obtain the following equation:

$$\alpha=\begin{vmatrix} h_{HF}+p_F' & h_{HF} & -h_{HH} \\ h_{FF} & h_{FF}+p_F'-c_F'' & -c_F'' \\ 0 & -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix}$$

$$+\begin{vmatrix} h_{HH}+p_H' & -(h_{HF}+p_F') & -h_{HF} \\ 0 & h_{FF} & h_{FF}+p_F'-c_F'' \\ h_{FH} & 0 & -c_F'' \end{vmatrix}$$

$$=\begin{vmatrix} h_{HF}+p_F' & h_{HF} & -h_{HH} \\ h_{FF} & h_{FF}+p_F'-c_F'' & -c_F'' \\ 0 & -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix}-\begin{vmatrix} h_{HF}+p_F' & h_{HF} & -h_{HH}-p_H' \\ h_{FF} & h_{FF}+p_F'-c_F'' & 0 \\ 0 & -c_F'' & h_{FH} \end{vmatrix}$$

$$=\begin{vmatrix} h_{HF}+p_F' & h_{HF} & p_H' \\ h_{FF} & h_{FF}+p_F'-c_F'' & -c_F'' \\ 0 & -c_F'' & p_H'-c_F'' \end{vmatrix}=\begin{vmatrix} h_{HF}+p_F' & h_{HF} & -h_{HF}+p_H' \\ h_{FF} & h_{FF}+p_F' & -h_{FF}-p_F'-p_H' \\ 0 & -c_F'' & p_H' \end{vmatrix}$$

$$=p_H'[(h_{HF}+p_F')(h_{FF}+p_F')-h_{HF}h_{FF}]+c_F''\begin{vmatrix} h_{HF}+p_F' & p_H'+p_F' \\ h_{FF} & -p_F'-p_H' \end{vmatrix}$$

$$=p_H'p_F'(h_{HF}+h_{FF}+p_F')-c_F''(p_H'+p_F')(h_{HF}+h_{FF}+p_F')$$

$$=(h_{HF}+h_{FF}+p_F')[p_H'p_F'-c_F''(p_H'+p_F')]$$

$$=(h_{HF}+h_{FF}+p_F')(1/2)[p_H'(p_F'-2c_F'')+p_F'(p_H'-2c_F'')].$$

In the above expression, the first equality is derived from calculating $(A_{11}+A_{21})+(A_{14}+A_{24})$ by using definition of cofactor. The ensuing equalities are derived from elementary properties of determinant.

Next, we derive the expression for $(A_{12}+A_{22})+(A_{13}+A_{23})\equiv\beta$. Using the same technique in the above, we have the following:

$$\beta=\begin{vmatrix} h_{HH}+p_H' & -h_{HF} & h_{HH} \\ 0 & h_{FF}+p_F'-c_F'' & -c_F'' \\ h_{FH} & -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix}$$

$$-\begin{vmatrix} h_{HH}+p_H' & -(h_{HF}+p_F') & h_{HH} \\ 0 & h_{FF} & -c_F'' \\ h_{FH} & 0 & h_{FH}+p_H'-c_F'' \end{vmatrix}$$

$$= \begin{vmatrix} h_{HH}+p_H' & p_F' & h_{HH} \\ 0 & p_F'-c_F'' & -c_F'' \\ h_{FH} & -c_F'' & h_{FH}+p_H'-c_F'' \end{vmatrix} = \begin{vmatrix} h_{HH}+p_H' & p_F' & -p_H' \\ 0 & p_F'-c_F'' & -c_F'' \\ h_{FH} & -c_F'' & p_H'-c_F'' \end{vmatrix}$$

$$=(h_{HH}+p_H')[(p_F'-c_F'')(p_H'-c_F'')-(c_F'')^2]+h_{FH}[p_H'(p_F'-c_F'')-p_F'c_F'']$$

$$=(h_{HH}+h_{FH}+p_H')(1/2)[p_H'(p_F'-2c_F'')+p_F'(p_H'-2c_F'')].$$

A.4 The proof of equations (8.7) and (8.8)

First, we derive the expression for $A_{41}+A_{44}$. By the definition of cofactor, we obtain the following equation:

$$A_{41}+A_{44}=-\begin{vmatrix} c_H'' & 0 & h_{HH} \\ h_{HF}+p_F'-c_H'' & h_{HF} & 0 \\ h_{FF} & h_{FF}+p_F'-c_F'' & -c_F'' \end{vmatrix} + \begin{vmatrix} h_{HH}+p_H'-c_F'' & -c_H'' & 0 \\ -c_H'' & h_{HF}+p_F'-c_H'' & h_{FH} \\ 0 & h_{FF} & h_{FF}+p_F'-c_F'' \end{vmatrix}$$

$$= \begin{vmatrix} p_F' & -c_H'' & 0 \\ -c_H'' & h_{HF}+p_F'-c_H'' & h_{HF} \\ c_F'' & h_{FF} & h_{FF}+p_F'-c_F'' \end{vmatrix} = \begin{vmatrix} p_F'-c_H'' & -c_H'' & 0 \\ -c_H'' & p_F'-c_H'' & h_{HF} \\ c_F'' & -p_F'-c_F'' & h_{FF}+p_F'-c_F'' \end{vmatrix}$$

$$=(h_{FF}+p_F'c_F''{}_F+)[(p_H'-c_H'')(p_F'-c_H'')-(c_H'')^2]$$
$$+h_{HF}[(p_H'-c_H'')(p_F'-c_F'')-c_H''c_F''].$$

Next, we derive the expression for $A_{42}+A_{43}$. Using the same technique in the above, we have the following:

$$A_{42}+A_{43}=\begin{vmatrix} h_{HH}+p_H'-c_H'' & 0 & h_{HH} \\ -c_H'' & h_{HF} & 0 \\ 0 & h_{FF}+p_F'-c_F'' & -c_F'' \end{vmatrix} - \begin{vmatrix} h_{HH}+p_H'-c_H'' & -c_F'' & p_F' \\ -c_H'' & h_{HF}+p_F'-c_H'' & 0 \\ 0 & h_{FF} & -c_F'' \end{vmatrix}$$

$$= \begin{vmatrix} h_{HH}+p_H'-c_H'' & -c_H'' & h_{HH} \\ -c_H'' & -p_F'-c_H'' & 0 \\ 0 & p_F'-c_F'' & -c_F'' \end{vmatrix} = \begin{vmatrix} p_H'-c_H'' & -c_H'' & h_{HH} \\ -c_H'' & -p_F'-c_H'' & 0 \\ -c_F'' & p_F'-c_F'' & -c_F'' \end{vmatrix}$$

$= | h_{HH}(p_F' - c_F'')(c_F'' - c_H'') + c_F''[(p_F' - c_H'')(p_F' - c_F'') - c_H'' c_F''].$

A.5 The proof of $A_{44} - A_{43} < 0$

Let us define $\det(D) \equiv A_{44} - A_{43}$. Then from the definition of cofactors, we can derive the following expression:

$$\det(D) \equiv A_{44} - A_{43} = \begin{vmatrix} h_{HH} + p_H' - c_H'' & -c_H'' & h_{HH} \\ -c_H'' & h_{HF} + p_F' - c_H'' & h_{HF} \\ 0 & h_{FF} & h_{FF} + p_F' - 2c_F'' \end{vmatrix}.$$

From above equation, it is obvious that all the diagonal elements of matrix D are negative. Furthermore they are the dominant elements in every row. Because following inequalities hold:

$$| h_{HH} + p_H' - c_H'' | - | -c_H'' | - | h_{HH} | = -(p_H' - 2c_H'') > 0,$$
$$| h_{HF} + p_F' - c_H'' | - | -c_H'' | - | h_{HF} | = -(p_F' - 2c_H'') > 0,$$
$$| h_{FF} + p_F' - 2c_F'' | - | h_{FF} | = -(p_F' - 2c_F'') > 0.$$

Thus matrix D has negatively dominant diagonals and so it is a N−P matrix. Therefore $\det(D) \equiv A_{44} - A_{43}$ must be negative. (Q.E.D.)